

Definition

What is a set? Well, simply put, it's a **collection**.

First we specify a common property among "things" (we define this word later) and then we gather up all the "things" that have this common property.



For example, the items you wear: shoes, socks, hat, shirt, pants, and so on.

I'm sure you could come up with at least a hundred.

This is known as a **set**.

Or another example is **types of fingers**.

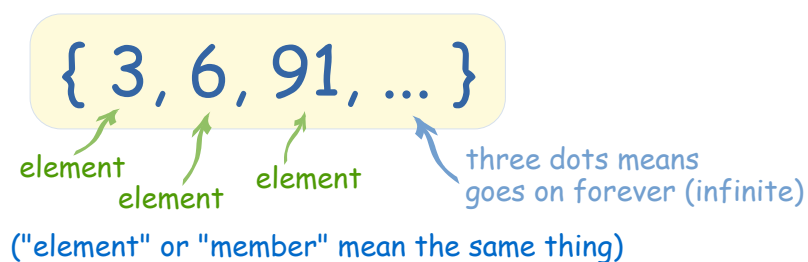
This set includes index, middle, ring, and pinky.



So it is just things grouped together with a certain property in common.

Notation

There is a fairly simple notation for sets. We simply list each element (or "member") separated by a comma, and then put some curly brackets around the whole thing:



The curly brackets { } are sometimes called "set brackets" or "braces".

This is the notation for the two previous examples:

{socks, shoes, watches, shirts, ...}

{index, middle, ring, pinky}

Notice how the first example has the "..." (three dots together).

The three dots ... are called an ellipsis, and mean "continue on".

So that means the first example continues on ... for infinity.

(OK, there isn't **really** an infinite amount of things you could wear, but I'm not entirely sure about that! After an hour of thinking of different things, I'm still not sure. So let's just say it is infinite for this example.)

So:

- The first set {socks, shoes, watches, shirts, ...} we call an **infinite set**,
- the second set {index, middle, ring, pinky} we call a **finite set**.

But sometimes the "..." can be used in the middle to save writing long lists:

Example: the set of letters:

{a, b, c, ..., x, y, z}

In this case it is a **finite set** (there are only 26 letters, right?)

Numerical Sets

So what does this have to do with mathematics? When we define a set, all we have to specify is a common characteristic. Who says we can't do so with numbers?

Set of even numbers: {..., -4, -2, 0, 2, 4, ...}

Set of odd numbers: {..., -3, -1, 1, 3, ...}

Set of prime numbers: {2, 3, 5, 7, 11, 13, 17, ...}

Positive multiples of 3 that are less than 10: {3, 6, 9}

And the list goes on. We can come up with all different types of sets.

There can also be sets of numbers that have no common property, they are just **defined** that way.

For example:

{2, 3, 6, 828, 3839, 8827}

{4, 5, 6, 10, 21}

{2, 949, 48282, 42882959, 119484203}

Are all sets that I just randomly banged on my keyboard to produce.

Why are Sets Important?

Sets are the fundamental property of mathematics. Now as a word of warning, sets, by themselves, seem pretty pointless. But it's only when we apply sets in different situations do they become the powerful building block of mathematics that they are.

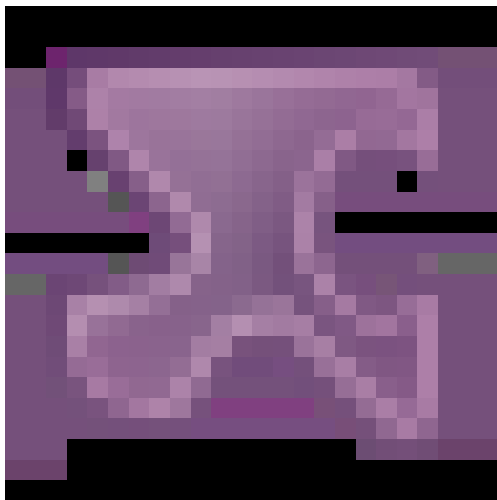
*Math can get amazingly complicated quite fast. Graph Theory, Abstract Algebra, Real Analysis, Complex Analysis, Linear Algebra, Number Theory, and the list goes on. But there is one thing that all of these share in common: **Sets**.*

set



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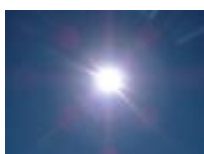


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SET is also an acronym for [Secure Electronic Transaction](#).

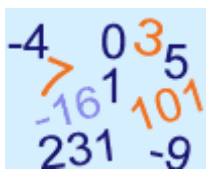
A set is a group or collection of objects or numbers, considered as an entity unto itself. Sets are usually symbolized by uppercase, italicized, boldface letters such as *A*, *B*, *S*, or *Z*. Each object or number in a set is called a member or element of the set. Examples include the set of all computers in the world, the set of all apples on a tree, and the set of all irrational numbers between 0 and 1.

Universal Set



At the start we used the word "things" in quotes.

We call this the **universal set**. It's a set that contains everything. Well, not *exactly* everything. **Everything that is relevant to our question.**



Then our sets included [integers](#). The universal set for that is **all the integers**.

In fact, when doing Number Theory, this is almost always what the universal set is, as Number Theory is simply the study of integers.



But in [Calculus](#) (also known as real analysis), the universal set is almost always the [real numbers](#).

And in complex analysis, you guessed it, the universal set is the [complex numbers](#).

Some More Notation

A =

When talking about sets, it is fairly standard to use Capital Letters to represent the set, and lowercase letters to represent an element in that set.

{a, ...}

So for example, A is a set, and a is an element in A. Same with B and b, and C and c.

Now you don't have to listen to the standard, you can use something like **m** to represent a set without breaking any mathematical laws (watch out, you can get **π** years in math jail for dividing by 0), but this notation is pretty nice and easy to follow, so why not?

Also, when we say an element **a** is in a set **A**, we use the symbol $\in$ to show it.

And if something is not in a set use $\notin$.

Example: Set **A** is {1,2,3}. We can see that **1** $\in$ **A**, but **5** $\notin$ **A**

Equality

Two sets are equal if they have precisely the same members. Now, at first glance they may not seem equal, so we may have to examine them closely!

Example: Are A and B equal where:

- A is the set whose members are the first four positive whole numbers
- B = {4, 2, 1, 3}

Let's check. They both contain 1. They both contain 2. And 3, And 4. And we have checked every element of both sets, so: **Yes, they are equal!**

And the equals sign (=) is used to show equality, so we write:

$$A = B$$

Subsets

When we define a set, if we take pieces of that set, we can form what is called a **subset**.

Example: the set {1, 2, 3, 4, 5}

A **subset** of this is {1, 2, 3}. Another subset is {3, 4} or even another is {1}, etc.

But $\{1, 6\}$ is **not** a subset, since it has an element (6) which is not in the parent set.

In general:

A is a subset of B if and only if every element of A is in B .

So let's use this definition in some examples.

Example: Is A a subset of B , where $A = \{1, 3, 4\}$ and $B = \{1, 4, 3, 2\}$?

1 is in A , and 1 is in B as well. So far so good.

3 is in A and 3 is also in B .

4 is in A , and 4 is in B .

That's all the elements of A , and every single one is in B , so we're done.

Yes, A is a subset of B

Note that 2 is in B , but 2 is not in A . But remember, that doesn't matter, we only look at the elements in A .

Let's try a harder example.

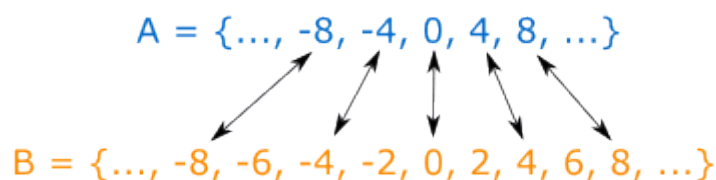
**Example: Let A be all multiples of 4 and B be all multiples of 2.
Is A a subset of B ? And is B a subset of A ?**

Well, we can't check every element in these sets, because they have an infinite number of elements. So we need to get an idea of what the elements look like in each, and then compare them.

The sets are:

- $A = \{\dots, -8, -4, 0, 4, 8, \dots\}$
- $B = \{\dots, -8, -6, -4, -2, 0, 2, 4, 6, 8, \dots\}$

By pairing off members of the two sets, we can see that every member of A is also a member of B , but not every member of B is a member of A :



So:

A is a subset of B , but B is not a subset of A

Proper Subsets

If we look at the definition of subsets and let our mind wander a bit, we come to a weird conclusion.

Let A be a set. Is every element in A an element in A ? (Yes, I wrote that correctly.)

Well, umm, *yes of course*, right?

So doesn't that mean that ***A is a subset of A***?

This doesn't seem very *proper*, does it? We want our subsets to be *proper*. So we introduce (what else but) **proper subsets**.

A is a **proper** subset of B if and only if every element in A is also in B, and there exists **at least one element** in B that is **not** in A.

This little piece at the end is only there to make sure that A is not a proper subset of itself. Otherwise, a proper subset is exactly the same as a normal subset.

Example:

{1, 2, 3} is a **subset** of {1, 2, 3}, but is **not** a **proper subset** of {1, 2, 3}.

Example:

{1, 2, 3} is a **proper subset** of {1, 2, 3, 4} because the element 4 is not in the first set.

Notice that if A is a proper subset of B, then it is also a subset of B.

Even More Notation

When we say that A is a subset of B, we write $A \subseteq B$.