

CLASS X

SUBJECT: MATHEMATICS

TOPIC	REAL NUMBERS
PREREQUISITE KNOWLEDGE	Rational Numbers: Class VIII Number System: Class IX
OBJECTIVES	The students will be able to: • describe Euclid's Division Lemma • calculate the HCF of two positive integers using Euclid's Division Lemma/ Algorithm • describe the Fundamental Theorem of Arithmetic. • express a number as a product of its prime factors to calculate the HCF and LCM of numbers. • recall the properties of irrational numbers. • prove that if P is prime and P divides a^2, then P divides a, where a is a positive integer. • prove that $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are irrational numbers.
THE HISTORY OF REAL NUMBERS	Simple fractions were used by the Egyptians around 1000 BC; the Vedic "Sulba Sutras" ("The rules of chords") in, c. 600 BC, include what may be the first "use" of irrational numbers. The concept of irrationality was implicitly accepted by early Indian mathematicians since Manava(c. 750–690 BC), who were aware that the square roots of certain numbers such as 2 and 61 could not be exactly determined. Around 500 BC, the Greek mathematicians led by Pythagoras realized the need for irrational numbers, in particular the irrationality of the square root of 2. The Middle Ages brought the acceptance of zero, negative, integral, and fractional numbers, first by Indian and Chinese mathematicians, and then by Arabic mathematicians, who were also the first to treat irrational numbers as algebraic objects, which was made possible by the development of algebra. Arabic mathematicians merged the concepts of "number" and "magnitude" into a more general idea of real numbers. The Egyptian mathematician Abū Kāmil Shujā ibn Aslam (c. 850–930) was the first to accept irrational numbers as solutions to quadratic equations or as coefficients in an equation, often in the form of square roots, cube roots and fourth roots. In the 16th century, Simon Stevin created the basis for modern decimal notation, and insisted that there is no difference between rational and irrational numbers in this regard. In the 17th century, Descartes introduced the term "real" to describe roots of a polynomial, distinguishing them from "imaginary" ones. In the 18th and 19th centuries, there was much work on irrational and transcendental numbers. Johann Heinrich Lambert (1761) gave the first flawed proof that π cannot be rational; Adrien-Marie Legendre (1794) completed the proof, and showed that π is not the square root of a rational number. Paolo Ruffini (1799) and Niels Henrik Abel (1842) both

	constructed proofs of the Abel–Ruffini theorem: that the general quintic or higher equations cannot be solved by a general formula involving only arithmetical operations and roots. The development of calculus in the 18th century used the entire set of real numbers without having defined them cleanly. The first rigorous definition was published by Georg Cantor in 1871. In 1874, he showed that the set of all real numbers is uncountably infinite but the set of all algebraic numbers is countably infinite. Contrary to widely held beliefs, his first method was not his famous diagonal argument, which he published in 1891. See Cantor's first uncountability proof.
SUMMARY OF THE CHAPTER	In this chapter, students will study: • Natural numbers: Counting numbers are called Natural numbers. These numbers are denoted by $N = \{1, 2, 3, \dots\}$ • Whole numbers: The collection of natural numbers along with 0 is the collection of Whole number and is denoted by W. • Integers: The collection of natural numbers, their negatives along with the number zero are called Integers. This collection is denoted by Z. • Rational number: The numbers, which are obtained by dividing two integers, are called Rational numbers. Division by zero is not defined. • Coprime: If HCF of two numbers is 1, then the two numbers are called relatively prime or coprime. 1. Euclid’s division lemma: For given positive integers ‘a’ and ‘b’ there exist unique whole numbers ‘q’ and ‘r’ satisfying the relation. Theorem: If a and b are non-zero integers, the least positive integer which is expressible as a linear combination of a and b is the HCF of a and b, i.e., if d is the HCF of a and b, then there exist integers x and y, such that d = ax + by and d is the smallest positive integer which is expressible in this form. d is the HCF and is denoted by HCF. 2. Euclid’s division algorithms: HCF of any two positive integers a and b. With $a > b$ is obtained as follows: Step 1: Apply Euclid’s division lemma to a and b to find q and r such that $b = aq + r$ where $0 \leq r < b$ b = Divisor q = Quotient r = Remainder Step II: If $r = 0$, HCF (a,b)=b if $a \geq b$, apply Euclid’s lemma to b and r. Step III: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

	3. The Fundamental Theorem of Arithmetic: Every composite number can be expressed (factorized) as a product of primes and this factorization is unique, apart from the order in which the prime factors occur. 4. If a and b are two positive integers, then $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$ i.e., (HCF $\times$ LCM) of two integers = Product of integers. 5. A rational number which when expressed in the lowest term, has factors 2 or 5 in the denominator can be written as a terminating decimal otherwise a non-terminating recurring decimal. 6. Let $x = p/q$ be a rational number, such that the prime factorization of 'q' is of the form $2^m 5^n$, where m, n are non-negative integers. Then x has a decimal expansion which is terminating. 7. Let $x = p/q$ be a rational number, such that the prime factorization of q is not of the form $2^m 5^n$, where m, n are non-negative integers. Then x has a decimal expansion which is non-terminating repeating. 8. We conclude that every rational number can be represented in the form of terminating or non-terminating recurring decimal.
TRANSACTION METHODOLOGY	Strategies used: 1) Warm up session: Begin the lesson by recalling the concepts pertaining to the real number system. Show the diagram of the real number system and ask the students the definitions of different types of numbers. Conduct a quiz covering the concepts, such as the properties of rational and irrational numbers and representing them on a number line. 2) Euclid's Division Lemma or Division Algorithm: Practice Questions: In this activity, students will learn to calculate the HCF of given numbers using Euclid's Division Lemma. Provide students the historical and biographical details about Euclid. He was a Greek Mathematician, who lived around 300 BC and was popularly known as Euclid of Alexandria. Euclid is also known as the Father of Geometry owing to his significant contribution to the subject. He wrote a treatise consisting of 13 books called 'Elements', which served as the main textbook for teaching mathematics even till the early 20th century. "Elements" is mainly known for its geometric results and numbers. 3) Presentations: Rational and Irrational numbers In this activity students will make presentations on the following theorems:

	• Group A: Prove that if P is prime and P divides a^2, then P divides a, where a is a positive integer. • Group B: Prove that $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are irrational numbers. • Group C: Theorem on Terminating Decimal Expansion: Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form of p/q, where p and q are coprime, and the prime factorization of q is in the form $2^n 5^m$, where n and m are non-negative integers. • Ask the students to find out about RSA algorithm, an internet encryption and authentication system, and the use of prime numbers in it.
EXPECTED LEARNING OUTCOMES	Students will be able to: • explain Euclid's Division Lemma and the Fundamental Theorem of Arithmetic. • calculate the HCF of numbers using Euclid's Division Lemma and HCF & LCM of numbers using the prime factorization method. • recall the properties of rational and irrational numbers and prove a few theorems pertaining to these numbers.
PRACTICE QUESTIONS/ ASSESSMENTS	1. Find the HCF of 52 and 117 and express it in form $52x + 117y$. 2. Prove that $x^2 - x$ is divisible by 2 for all positive integer x. 3. If m and n are odd positive integers, then $m^2 + n^2$ is even, but not divisible by 4. Justify. 4. If $\text{HCF}(6, a) = 2$ and $\text{LCM}(6, a) = 60$, then find a. 5. If remainder of $\frac{(5m+1)(5m+3)(5m+4)}{5}$ is a natural number, then find it. 6. If n is any prime number and a^2 is divisible by n, then n will also divide a. Justify. 7. Find the missing numbers in prime factors tree. <pre> graph TD A[] --- B[2] A --- C[] C --- D[2] C --- E[] E --- F[2] E --- G[13] </pre> 8. Find the greatest number of 5 digits exactly divisible by 12, 15 and 36. 9. Find the smallest number which when increased by 20 is exactly divisible by 90 and 144. 10. Find the smallest number which leaves remainder 8 and 12 when divided by 28 and 32 respectively. 11. Floor of a room is to be fitted with square marble tiles of the largest possible size. The size of the room is $10\text{ m} \times 7\text{ m}$. What should be the size of tiles required that has to be cut and how many such tiles are required?

	12. If the HCF of 408 and 1032 is expressible in the form $1032p - 408 \times 5$ find p.
EVALUATION	1) Textual learning and understanding assessed through Pen Paper Test. 2) Assignment project to demonstrate the learning and clarity of topics. 3) Classroom connection to be assessed through an oral questionnaire. 4) Peer coordination and collaboration to be assessed through Lab Manual Activities and Quiz.