

CLASS X

SUBJECT: MATHEMATICS

TOPIC	POLYNOMIALS
PREREQUISITE KNOWLEDGE	Polynomials: Class IX
OBJECTIVES	The students will be able to: 1) find the zeroes of the polynomials 2) find the roots by middle term splitting. 3) apply long division method to find out Quotient, Remainder, Divisor. 4) to show the relationship between zeroes and the coefficients of the given polynomial. 5) to apply Division Algorithm $P(x) = G(x) \times Q(x) + R(x)$
THE HISTORY OF POLYNOMIALS	In mathematics, a polynomial is an expression consisting of variables (also called indeterminates) and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents of variables. An example of a polynomial of a single indeterminate, x, is $x^2 - 4x + 7$. An example in three variables is $x^3 + 2xyz^2 - yz + 1$. Polynomials appear in many areas of mathematics and science. For example, they are used to form polynomial equations, which encode a wide range of problems, from elementary word problems to complicated problems in the sciences; they are used to define polynomial functions, which appear in settings ranging from basic chemistry and physics to economics and social science; they are used in Calculus and Numerical Analysis to approximate other functions. In Advanced Mathematics, polynomials are used to construct polynomial rings and algebraic varieties, central concepts in Algebra and Algebraic Geometry.
SUMMARY OF THE CHAPTER	In this chapter, students will study: • Zeroes of a polynomial. k is said to be zero of a polynomial $p(x)$ if $p(k) = 0$ • Graph of polynomial. (i) Graph of a linear polynomial $ax + b$ is a straight line. (ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open upwards like $\cup$, if $a > 0$. (iii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open downwards like $\cap$, if $a < 0$. (iv) In general, a polynomial $p(x)$ of degree n crosses the x-axis at atmost n points.

	- Relationship between the zeroes and the coefficients of a Polynomial. (i) If α, β are zeroes / roots of $p(x) = ax^2 + bx + c$, then $\text{Sum of roots } \alpha + \beta = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$ $\text{Product of roots } \alpha\beta = \frac{c}{a} \Rightarrow \alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2}$ (ii) If α, β and γ are zeroes / roots of $p(x) = ax^3 + bx^2 + cx + d$ $\text{Then, sum of roots } = \alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3}$ $\text{Sum of product of roots taken, two at a time} = \frac{c}{a} = \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$ $\text{Product of roots } = \frac{-d}{a} \Rightarrow \alpha\beta\gamma = \frac{-(\text{constant term})}{\text{coefficient of } x^3}$ (iii) If α, β are roots of a quadratic polynomial $p(x)$, then $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$ $\Rightarrow p(x) = x^2 - (\text{sum of roots})x + \text{product of roots}$ (iv) If α, β and γ are zeroes of a cubic polynomial $p(x)$, $\text{Then, } p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma)x - (\alpha\beta\gamma)$ $\Rightarrow p(x) = x^3 - (\text{sum of zeroes})x^2 + (\text{sum of product of zeroes / roots taken two at a time})x - (\text{product of zeroes})$
TRANSACTION METHODOLOGY	Strategies used: Warm up session: Begin the session with a simple activity to help students recall their prior knowledge about polynomials. Flash Card Activity: Divide the classroom into a few groups and ask a student from each group to pick up a card and answer the question on the card. The questions should be on the various terms related to polynomials and the categories of polynomials. Write algebraic expressions on a few cards and ask the students to identify its degree and name it. Also, ask the students to find the zeroes of the polynomial. Graph Activity-1: In this activity, students will be shown a few graphs. They need to study these graphs and find the zeroes of the polynomials as well as the degree of each polynomial.

	Divide the class into a few groups. Show the graph of a polynomial to one of the groups and ask the students to discuss the graph amongst themselves. Now, ask a student from the group to name the zeroes of the polynomial and cite the degree of the polynomial. Now show the graphs of linear, quadratic and cubic polynomials to the students. The team that gets maximum correct answers will be the winner. 3) Graph Activity-2: In this activity, students will find the zeroes of the given polynomials by plotting the graphs of the polynomials. Write down a linear, quadratic and cubic polynomial on the board and ask each student to plot the three polynomials on different graph papers. Once the students have plotted the graphs, ask them to find the zeroes of the polynomials. 4) Finding the Zeroes of the Quadratic Polynomials: In this activity, students need to find the zeroes of a given quadratic polynomials and verify the relationship between the zeroes and the coefficients of the polynomials. • Write down a quadratic polynomial on the board and ask the students to find it's zeroes. Once done, ask them to verify the relationship between the zeroes and the coefficients of the polynomial by finding the sum of the zeroes and the product of the zeroes. • Write down a cubic polynomial on the board along with it's zeroes. Ask the students to find the sum of the zeroes, product of zeroes and sum of the products of zeroes taken two at a time for the given cubic polynomial. Once done, ask them to verify the relationship between the zeroes and the coefficients of the polynomial. 5) Division Algorithm Activity: In this activity, students need to verify the Division Algorithm after performing the Polynomial Division. Discuss the Division Algorithm in the class. Then, write down two polynomials on the board and ask the students divide one by the other. Once done with the division, ask the students to verify the Division Algorithm. Each student should do the division and the verification in his/her notebook.
EXPECTED LEARNING OUTCOMES	Students will be able to: • find the zero or zeroes of a polynomial by studying its graph. • verify the relationship between the zeroes and the coefficients of linear, quadratic and cubic polynomials. • verify the Division Algorithm.
PRACTICE QUESTIONS/ ASSESSMENTS	1. If one zero of the polynomial $5z^2 + 13z - p$ is reciprocal of the other, then find p. 2. If the product of two zeroes of polynomial $2x^3 + 3x^2 - 5x - 6$ is 3, then find its third zero.

	3. Find the polynomial of least degree which should be subtracted from the polynomial $x^4 + 2x^3 - 4x^2 + 6x - 3$ so that it is exactly divisible by $x^2 - x + 1$. 4. Is polynomial $y^4 + 4y^2 + 5$ have zeroes or not? 5. Write a quadratic polynomial, sum of whose zeroes is $2\sqrt{3}$ and product is 5. 6. Write the zeroes of the polynomial $x^2 + 2x + 1$. 7. If the zeroes of the polynomial $f(x) = x^3 - 12x^2 + 39x + a$ are in AP, find the value of a. 8. A polynomial $g(x)$ of degree zero is added to the polynomial $2x^3 + 5x^2 - 14x + 10$ so that it becomes exactly divisible by $2x - 3$. Find the $g(x)$. 9. Find the zeroes of the quadratic polynomial $x^2 + 5x + 6$ and verify the relationship between the zeroes and the coefficients. 10. If the zeroes of polynomial $x^3 - ax^2 + bx - c$ are in AP then show that $2a^3 - 9ab + 27c = 0$ 11. If 1 and -1 are zeroes of polynomial $Lx^4 + Mx^3 + Nx^2 + Rx + P$, show that $L + N + P = M + R = 0$ 12. Draw graph of the function $f(x) = -2x^2 + 4x$. 13. If $x + a$ is a factor of the polynomial $x^2 + px + q$ and $x^2 + mx + n$ prove that $a = \frac{n-q}{m-p}$. 14. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time and product of its zeroes are $3, \frac{-1}{2}, \frac{5}{4}$ respectively. 15. Write cubic polynomial whose zeroes are $\frac{2+\sqrt{5}}{2}, \frac{2-\sqrt{5}}{2}, 4$. 16. α, β, γ are zeroes of cubic polynomial $kx^3 - 5x + 9$. If $\alpha^3 + \beta^3 + \gamma^3 = 27$, find the value of k. 17. α, β, γ are zeroes of cubic polynomial $x^3 - 12x^2 + 44x + c$. If α, β, γ are in AP, find the value of c. 18. Two zeroes of cubic polynomial $ax^3 + 3x^2 - bx - 6$ are -1 and -2. Find the third zero and value of a and b. 19. α, β, γ are zeroes of cubic polynomial $x^3 - 2x^2 + qx - r$. If $\alpha + \beta = 0$ then show that $2q = r$. 20. α, β, γ are zeroes of polynomial $x^3 + px^2 + qx + 2$ such that $\alpha\beta + 1 = 0$. Find the value of $2p + q + 5$.
EVALUATION	1) Textual learning and understanding assessed through Pen Paper Test. 2) Assignment project to demonstrate the learning and clarity of topics. 3) Classroom connection to be assessed through oral questionnaire. 4) Peer coordination and collaboration to be assessed through Lab Manual Activities and Quiz.