

CLASS X

SUBJECT: MATHEMATICS

TOPIC	QUADRATIC EQUATIONS
PREREQUISITE KNOWLEDGE	Knowledge of Polynomials
OBJECTIVES	The students will be able to: • check if the given equation is a quadratic equation. • find the roots of a quadratic equation by factorization. • find the roots of the quadratic equation by the method of completing the square. • find the roots of the quadratic equation by using the Quadratic Formula. • find the nature of the roots of a quadratic equation.
THE HISTORY OF QUADRATIC EQUATIONS	Many people believe that Babylonians were the first to solve quadratic equations. For instance, they knew how to find two positive numbers with a given positive sum and a given positive product, and this problem is equivalent to solving a quadratic equation of the form $x^2 - px + q = 0$. Greek mathematician Euclid developed a geometrical approach for finding out lengths which, in our present-day terminology, are solutions of quadratic equations. Solving of quadratic equations, in general form, is often credited to ancient Indian mathematicians. In fact, Brahmagupta (C.E.598–665) gave an explicit formula to solve a quadratic equation of the form $ax^2 + bx = c$. Later, Sridharacharya (C.E. 1025) derived a formula, now known as the quadratic formula, (as quoted by Bhaskara II) for solving a quadratic equation by the method of completing the square. An Arab mathematician Al-Khwarizmi (about C.E. 800) also studied quadratic equations of different types. Abraham bar Hiyya Ha-Nasi, in his book ‘Liber Embadorum’ published in Europe in C.E. 1145 gave complete solutions of different quadratic equations.
SUMMARY OF THE CHAPTER	In this chapter, students will study quadratic equations, and various ways of finding their roots. They will also see some applications of quadratic equations in daily life situations. $ax^2 + bx + c = 0$, $a \neq 0$ is called the standard form of a quadratic equation. Solution of a Quadratic Equation by Factorisation. Consider the quadratic equation $2x^2 - 3x + 1 = 0$. If we replace x by 1 on the LHS of this equation, we get $(2 \times 1^2) - (3 \times 1) + 1 = 0 = \text{RHS}$ of the equation. We say that 1 is a root of the quadratic equation $2x^2 - 3x + 1 = 0$. This also means that 1 is a zero of the quadratic polynomial $2x^2 - 3x + 1$.

	In general, a real number α is called a root of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ if $a\alpha^2 + b\alpha + c = 0$. We also say that $x = \alpha$ is a solution of the quadratic equation, or that α satisfies the quadratic equation. Note that the zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same. Solution of a Quadratic Equation by Completing the Square. In the previous section, you have learnt one method of obtaining the roots of a quadratic equation. In this section, we shall study another method. Consider the following situation: The product of Sunita's age (in years) two years ago and her age four years from now is one more than twice her present age. What is her present age? To answer this, let her present age (in years) be x. Then the product of her ages two years ago and four years from now is $(x - 2)(x + 4)$. So, $x^2 + 2x - 8 = 0$ can be written as $(x + 1)^2 - 9 = 0$ by this process of completing the square. This is known as the method of completing the square. Quadratic Formula: The roots of a quadratic equation $ax^2 + bx + c = 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, provided $b^2 - 4ac > 0$ or $b^2 - 4ac = 0$ Nature of Roots Since $b^2 - 4ac$ determines whether the quadratic equation $ax^2 + bx + c = 0$ has real roots or not, $b^2 - 4ac$ is called the discriminant of the quadratic equation. So, a quadratic equation $ax^2 + bx + c = 0$ has 1) Two distinct real roots, if $b^2 - 4ac > 0$, 2) Two equal real roots, If $b^2 - 4ac = 0$, 3) No real roots, If $b^2 - 4ac < 0$
TRANSACTION METHODOLOGY	Strategies used: 1) Warm up session: a) By talking briefly about the history of quadratic equations. b) Identification of the different types of polynomials: Linear, Quadratic and Cubic on board. 2) Chit Activity: a) Divide the class in groups. Create chits with equations written on them. Tell the students of each group to pick one chit and ask them to identify the equations. Thereafter tell them to write it on board and simplify it to get the roots of the equation.

	b) Once the activity is done, present a few situations to the students and ask them to represent these situations in the form of a quadratic equation. Write down the situation on the board with four options as the possible answers and ask them to identify the correct equation for the given situation. 3) Finding Roots of Quadratic Equations: In this activity the students will solve the problems in their notebooks. Once they will find the roots of an equation, ask them to verify the roots. 4) Quadratic Formula Activity: Write a few quadratic equations on flash cards (one equation per card). Divide the class into a few groups and then show a flash card to one of the groups. Thereafter, ask a student from the group to write the equation on the board and find it's roots using the quadratic formula. Continue the activity with the other groups. 5) Nature of the roots: Write a quadratic equation on the board and ask a student to find the discriminant of the equation. Once the discriminant is found, discuss the nature of the roots. If the equation has real roots, ask the students to find the roots and then find the sum and the product of the roots. Write several other quadratic equations to the students and ask them to find the nature of the roots, if the roots are real students can solve the equations in their notebooks. 6) Ask the students to research on the uses of quadratic equation.
EXPECTED LEARNING OUTCOMES	Students will be able to: • identify a quadratic equation. • find the roots of the quadratic equation using the methods of factorization and completing the square. • use the quadratic formula to find the roots of a quadratic equation. • Find the nature of the roots of the quadratic equation.
PRACTICE QUESTIONS/ ASSESSMENTS	1) How many real roots does the equation $x^2+3x+4=0$ have? 2) The difference of two positive integers is 3 and the sum of their squares is 117; find the numbers? 3) For every litre of petrol, one car for every litre of petrol, one car travels x km and another car travels 5 km more than the first. If the first car uses 4 litres more than the second car in converting 400 km, frame an equation for the statement to find x. What is the value of x? 4) The product of two positive consecutive even integer is 168. Assuming the smaller integer to be x, frame an equation for the statement and find the numbers.

	5) A two-digit number is made of two consecutive digits such that the sum of their squares is 4 less than the number. Find the two-digit number. 6) Mrs Tendon has two sons, one being exactly one year older than the other. At percentage, her age is equal to the sum of the squares of the ages of her sons. If 4 years hence her age becomes five times the age of the elder son then find the percent ages of her sons. 7) The base of a triangle exceeds twice its altitude by 1 8m. If the area of the triangle be 360 sq. m., what is its altitude? 8) Solve the quadratic equation $6m^2 - 7m + 2 = 0$ by factorization method. 9) Find the discriminant of the following equation: $x(x - 2) + 1 = 0$ 10) The equation $3x^2 - 12x + z - 5 = 0$ has equal roots. Find the value of z.
EVALUATION	1) Textual learning and understanding assessed through Pen Paper Test. 2) Assignment project to demonstrate the learning and clarity of topics. 3) Classroom connection to be assessed through an oral questionnaire. 4) Peer coordination and collaboration to be assessed through Lab Manual Activities and Quiz.