

CLASS X

SUBJECT: MATHEMATICS

TOPIC	INTRODUCTION TO TRIGONOMETRY
PREREQUISITE KNOWLEDGE	Knowledge of Right Angle Triangle and Pythagoras Theorem
OBJECTIVES	The students will be able to understand the following : • Angles and sides relation of a right triangle • Trigonometric ratios and their relationships. • They will be able to find the trigonometric ratios for some particular angles. • Trigonometric ratios of complementary angles. • Trigonometric identities.
THE HISTORY OF TRIGONOMETRY	<i>There is perhaps nothing which so occupies the middle position of mathematics as trigonometry.</i> – J.F. Herbart (1890) The first use of the idea of ‘sine’ in the way we use it today was in the work Aryabhatiyam by Aryabhata, in A.D. 500. Aryabhata used the word ardha-jya for the half-chord, which was shortened to jya or jivain due course. When the Aryabhatiyam was translated into Arabic, the word jivawas retained as it is. The word jivawas translated into sinus, which means curve, when the Arabic version was translated into Latin. Soon the word sinus, also used as sine, became common in mathematical texts throughout Europe. An English Professor of astronomy Edmund Gunter (1581–1626), first used the abbreviated notation ‘sin’. The origin of the terms ‘cosine’ and ‘tangent’ was much later. The cosine function arose from the need to compute the sine of the complementary angle. Aryabhata called it kotijya. The name cosinus originated with Edmund Gunter. In 1674, the English Mathematician Sir Jonas Moore first used the abbreviated notation ‘cos’.
SUMMARY OF THE CHAPTER	You have already studied about triangles, and in particular, right triangles, in your earlier classes. Let us take some examples from our surroundings where right triangles can be imagined to be formed. For instance: 1. Suppose the students of a school are visiting Qutub Minar. Now, if a student is looking at the top of the Minar, a right triangle can be imagined to be made. Can the student find out the height of the Minar without actually measuring it? 2. Suppose a girl is sitting on the balcony of her house located on the bank of a river. She is looking down at a flower pot placed on a stair of a temple situated near by on the other bank of the river. A right triangle is imagined to be made in this situation. If you know the height at which the person is sitting, can you find the width of the river? 3. Suppose a hot air balloon is flying in the air. A girl happens to spot the balloon in the sky and runs to her mother to tell her about it.

	Her mother rushes out of the house to look at the balloon. Now when the girl had spotted the balloon initially it was at point A. When both the mother and daughter came out to see it, it had already travelled to another point B. Can you find the altitude of B from the ground? In all the situations given above, the distances or heights can be found by using some mathematical techniques, which come under a branch of mathematics called ‘trigonometry’. The word ‘trigonometry’ is derived from the Greek word ‘tri’ (meaning three), ‘gon’ (meaning sides) and ‘metron’ (meaning measure). In fact, trigonometry is the study of relationships between the sides and angles of a triangle. The earliest known work on trigonometry was recorded in Egypt and Babylon. Early astronomers used it to find out the distances of the stars and planets from the Earth. Even today, most of the technologically advanced methods used in Engineering and Physical Sciences are based on trigonometrical concepts. <i>In this chapter</i>, we will study some ratios of the sides of a right triangle with respect to its acute angles, called trigonometric ratios of the angle. We will restrict our discussion to acute angles only. However, these ratios can be extended to other angles also. We will also define the trigonometric ratios for angles of measure 0° and 90°. We will calculate trigonometric ratios for some specific angles and establish some identities involving these ratios, called trigonometric identities. Trigonometric Ratios
TRANSACTION METHODOLOGY	Strategies used: 1) Warm up session: Begin the lesson by recalling the Pythagoras theorem introduce the concept of angle of elevation and angle of depression. Explain how a right angle triangle can be imagined to be formed and used to calculate heights and distances. Explain the concept of trigonometry and talk briefly about its history. Ancient Egyptians used the ideas of Trigonometric function and similar triangles to make the buildings and pyramids in ancient Greece were also made by this concept. Trigonometric functions were used to predict the position of stars, time of the day and period of the year. Trigonometry helps in various fields such as optics and statistics. 2) Flash card Activity: a) In this activity students will calculate the trigonometric ratios of an acute angle of a given right angle triangle. b) Divide the class into two teams and show them a card with a question. The team member will discuss amongst themselves and then one child will answer. The team with maximum correct answer will be the winner.

	c) Two more activities of the similar kind can be performed taking angles and their complements. 3) Chart on Trigonometric Ratios and identities In this activity students will prepare charts on trigonometric ratios and identities. Divide the class into two groups and ask them to prepare chart on the following topics: Group A : Depict trigonometric ratios of an acute angle of a right angle triangle(for e.g $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$) Group B: Make a table depicting the trigonometric values of complementary angles. Make another table of trigonometric identities.
EXPECTED LEARNING OUTCOMES	Students will be able to: • Cite the trigonometric ratios of acute angles. • They will also be able to work out the trigonometric ratios of specific angles. • They will also be able to work out the trigonometric ratios of complementary angles. • They will also be able to work out the trigonometric identities to solve the questions.
PRACTICE QUESTIONS/ ASSESSMENTS	1) Evaluate $\frac{\sin 47^\circ}{\cos 43^\circ}$. 2) Prove that: $\sqrt{(1 - \cos^2 \theta)} \sec^2 \theta = \tan \theta$. 3) Show that: $\tan^4 \theta + \tan^2 \theta = \sec^4 \theta - \sec^2 \theta$. 4) Prove that: $\frac{1 + \sec \theta - \tan \theta}{1 + \sec \theta + \tan \theta} = \frac{1 - \sin \theta}{\cos \theta}$. 5) If $\operatorname{cosec} \theta + \cot \theta = p$, then prove that $\cos \theta = \frac{p^2 - 1}{p^2 + 1}$. 6) Evaluate the following: $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$. 7) If $\sec 4A = \operatorname{cosec} (A - 20^\circ)$, where $4A$ is an acute angle, find the value of A. 8) Prove that : $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$ 9) Evaluate: $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$. 10) Express $\cot 85^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45°.
EVALUATION	Class Test, Extra Questions, Lab Manual Activity, Oral Questions