

Name Of topic

Arithmetic Progression

Class : X

Arithmetic Progression

An Arithmetic Progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except first term.

For example : 5, 10, 15, 20, 25.....

In this each term is obtained by adding 5 to the preceding term except first term

The general form of an Arithmetic Progression is

$a, a + d, a + 2d, a + 3d, \dots, a + (n-1)d$

Where 'a' is first term and

'd' is called common difference.

nth Term of an A.P. is denoted by A_n

$$A_n = a + (n-1)d$$

To find the nth Term of an A.P.

Let us consider an A.P. with first term 'a' and common difference 'd', then

$$\text{The first term} = a_1 = a + 0d = a + (1-1)d$$

$$\text{The second term} = a_2 = a + d = a + (2-1)d$$

$$\text{The third term} = a_3 = a + 2d = a + (3-1)d$$

$$\text{The fourth term} = a_4 = a + 3d = a + (4-1)d$$

$$\text{The nth term} = a_n = a + (n-1)d$$

To check that a given term is in A.P. or not.

(i) 2, 6, 10, 14....

(i) Here first term $a = 2$,
find differences in the next terms

$$a_2 - a_1 = 6 - 2 = 4$$

$$a_3 - a_2 = 10 - 6 = 4$$

$$a_4 - a_3 = 14 - 10 = 4$$

Since the differences are common.

Hence the given terms are in A.P.

Which of the following are in A.P. ?

If they are in A.P. find their first term and common difference

(i) $1, 3, 9, 27, \dots$

(ii) $a, 2a, 3a, 4a, \dots$

(iii) $-6, -2, 2, 6, \dots$

(iv) $1^2, 2^2, 3^2, 4^2, \dots$

Problem : Find the value of k for which the given series is in A.P. $4, k - 1, 12$

Solution : Given A.P. is $4, k - 1, 12$

If series is A.P. then the differences will be common

$$d_1 = d_1$$

$$a_2 - a_1 = a_3 - a_2$$

$$k - 1 - 4 = 12 - (k - 1)$$

$$k - 5 = 12 - k + 1$$

$$k + k = 12 + 1 + 5$$

$$2k = 18 \quad \text{or} \quad k = 9$$

Problem 1. Find 10th term of A.P. 12, 18, 24, 30.. ...

Solution. Given A.P. is 12, 18, 24, 30..

First term is $a = 12$

Common difference is $d = 18 - 12 = 6$

n th term is $a_n = a + (n-1)d$

Put $n = 10$, $a_{10} = 12 + (10-1)6$

$$= 12 + 9 \times 6$$

$$= 12 + 54$$

$$a_{10} = 66$$

Problem 2. Find number of terms of A.P.

100, 105, 110, 115,500

Solution. Given A.P. is 100, 105, 110, 115,500

First term is $a = 100$, $a_n = 500$

Common difference is $d = 105 - 100 = 5$

n th term is $a_n = a + (n-1)d$

$$500 = 100 + (n-1)5$$

$$500 - 100 = 5(n - 1)$$

$$400 = 5(n - 1)$$

$$5(n - 1) = 400$$

Problem 3. Find the number of all three digit numbers divisible by 8

Solution. Smallest three digit number divisible by 8 is 104
& Largest three digit number divisible by 8 is 992

Therefore numbers obtained are
104, 112, 120, 128,992

Which are in A.P.

Now solve using A.P. and find how many are they ?

$$5(n - 1) = 400$$

$$n - 1 = 400/5$$

$$n - 1 = 80$$

$$n = 80 + 1$$

$$n = 81$$

Hence the no. of terms are 81.

Let us solve some problems based on Arithmetic Progression

1. Find 50th term of A.P. 5, 10, 15, 20...
2. Find first three terms of an A.P. whose 3rd term is 10 and 6th term is 19.
3. Find number of terms of an A.P. 3, 6, 9,99.

Sum of n terms of an Arithmetic Progression

Its formula is

$$S_n = \frac{1}{2} n [2a + (n - 1)d]$$

It can also be written as

$$S_n = \frac{1}{2} n [a + a_n]$$

The sum to n terms of an arithmetic progression

$$S_n = \frac{1}{2} n [2a + (n - 1)d]$$

You may need to be able to prove this formula. It is derived as follows:

The sum to n terms is given by:

$$S_n = a + (a + d) + (a + 2d) + \dots + (a + (n - 1)d) \quad (1)$$

If we write this out backwards, we get:

$$S_n = (a + (n - 1)d) + (a + (n - 2)d) + \dots + a \quad (2)$$

Now let's add (1) and (2):

$$2S_n = [2a + (n - 1)d] + [2a + (n - 1)d] + \dots \\ \dots\dots\dots + [2a + (n - 1)d]$$

$$\text{So } S_n = \frac{1}{2} n [2a + (n - 1)d]$$

Problem 1. Find the sum of 30 terms of given A.P.

$$12 + 20 + 28 + 36 \dots\dots\dots$$

Solution : Given A.P. is 12 , 20, 28 , 36

Its first term is $a = 12$

Common difference is $d = 20 - 12 = 8$

The sum to n terms of an arithmetic progression

$$\begin{aligned} S_n &= \frac{1}{2} n [2a + (n - 1)d] \\ &= \frac{1}{2} \times 30 [2 \times 12 + (30-1) \times 8] \\ &= 15 [24 + 29 \times 8] \\ &= 15 [24 + 232] \\ &= 15 \times 246 \\ &= 3690 \end{aligned}$$

Problem 2. Find the sum given A.P.

$$2 + 4 + 6 + 8 + \dots + 200$$

Solution : Given A.P. is 2 , 4 , 6 , 8200

Its first term is $a = 2$

Common difference is $d = 4 - 2 = 2$

n th term is $a_n = a + (n-1)d$

$$200 = 2 + (n-1)2$$

$$200 - 2 = 2(n - 1)$$

$$198 = 2(n - 1)$$

$$2(n - 1) = 198$$

$$2(n - 1) = 198$$

$$n - 1 = 198/2$$

$$n - 1 = 99$$

$$n = 99 + 1$$

$$n = 100$$

The sum to n terms of an arithmetic progression

$$S_n = \frac{1}{2} n [2a + (n - 1)d]$$

$$S_{100} = \frac{1}{2} \times 100 [2 \times 2 + (100-1) \times 2]$$

$$= 50 [4 + 198]$$

$$= 50[202]$$

$$= 10100$$

Let us solve some problems based on sum of terms of Arithmetic Progression

1. Find sum of 40 term of A.P. 5, 10, 15, 20...
2. Find sum of 100 term of A.P. -70, -67, -64...
3. Find the sum of series $10 + 20 + 30 + \dots$
 $\dots\dots\dots + 1000$
4. Find the sum of series $-5 + (-15) + (-25) + \dots$
 $\dots\dots\dots + (-525)$

THANK YOU