

AREA RELATED TO CIRCLE

INTRODUCTION

You are already familiar with the concept of a circle and some basic terms such as centre, radius, arc, chord etc related to a circle. You have also learnt to find the perimeter and area of a plane figure like square, rectangle, quadrilateral such as a trapezium, parallelogram, rhombus, triangle etc. You also know how to find area and circumference (perimeter) of a circle. Wheel , cake , bangles , etc are some examples.



PERIMETER AND AREA OF A CIRCLE

Recall that the distance covered by going around a circle one time is called its **perimeter** or **circumference**.

- You also know that
- Circumference

Diameter

is a constant, denoted by a Greek letter π (read as “pi”).

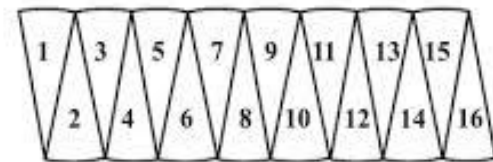
- or Circumference $\text{---} \pi$

Diameter

- or circumference

$$= \pi \times 2r,$$

- where r is the radius of the circle.

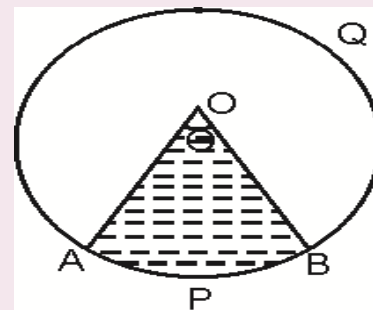


SECTOR OF A CIRCLE



- You know that area of a circle of radius r is πr^2 .
- You can imagine the circular region formed by the circle of radius r as a sector of angle 360° (Because angle at the centre is a complete angle).
- With this assumption, we can calculate the area of the sector OAPB as follows:
- Area of a sector of angle $360^\circ = \pi r^2$
- So, area of a sector of angle $1^\circ = \frac{\pi r^2}{360^\circ}$

$$\begin{aligned} \text{Hence, area of a sector of angle } \theta &= \frac{\pi r^2}{360^\circ} \times \theta \\ &= \frac{\pi r^2 \theta}{360^\circ} \end{aligned}$$



- **Length of the Arc of a sector**

You know that circumference of a circle of radius r is $2\pi r$.

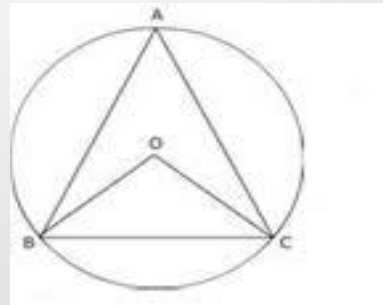
You can calculate the length of the arc of sector OAPB as follows:

Length of the arc of a sector of angle $360^\circ = 2\pi r$

So, length of the arc of a sector of angle $1^\circ = \frac{2\pi r}{360^\circ}$

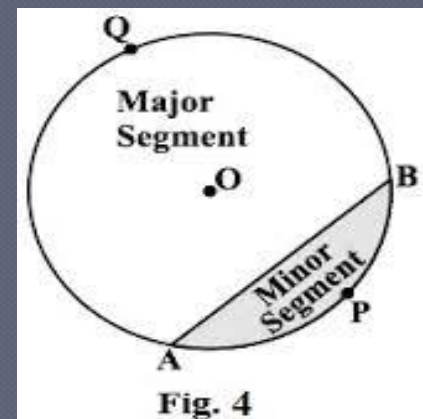
Hence, length of the arc of a sector of angle $\theta =$

$$\frac{\theta}{360} \times 2\pi r$$



Areas of segment of a circle

- Recall that a chord of a circle divides the circular region into two parts. Each part is called a **segment** of the circle.
- There are two parts of area of segment :-
 - ❖ Major Segment
 - ❖ Minor Segment



MINOR SEGMENT

In the figure, APB is the **minor segment** and AQB is the major segment

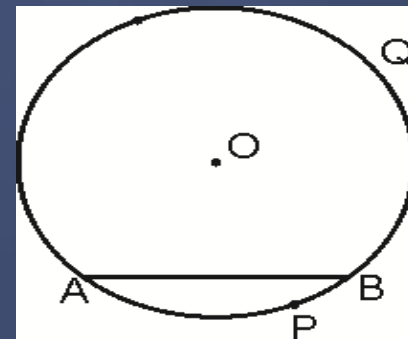
To find area of the **minor segment** APB, join the centre O to A and B.

Let $\angle AOB = \theta$

Area of minor segment APB

$$\square = \text{Area of sector OAPB} - \text{Area of } \triangle OAB$$

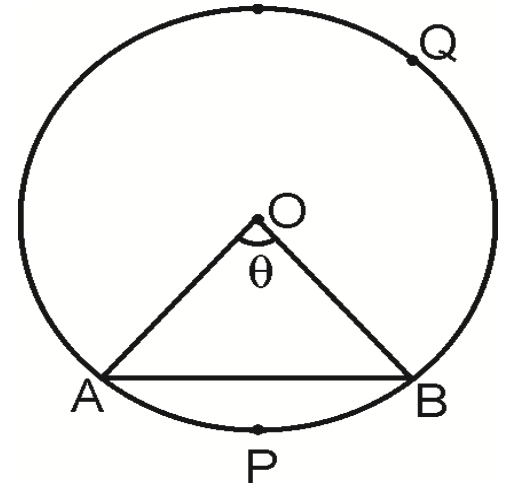
$$\square = \frac{\theta \pi r^2}{360^\circ} - \text{Area of } \triangle OAB$$



Major Segment

- = **Area of sector OAQB + area of ΔOAB**
- $$\frac{\pi r^2(360^\circ - \Theta)}{360^\circ} + \text{Area of } \Delta AOB$$

- **Alternatively**
- Area of major segment AQB
= Area of circle with centre O - Area of minor segment APB.



Areas of Combination of Plane Figure and circles

In daily life, we see many designs which involve circles along with other plane figures such as square, triangle, rectangle etc. We now illustrate the process of calculating areas of such figures/ designs through some examples.



NOW QUESTION ANSWER TIME

Q1:-The radii of two circles are 6cm and 8cm. Find the radius of the circle having its area equal to the sum of the areas of the two circles ?

Q2:-The radii of two circles are 12cm and 21cm. Find the radius of the circle which has circumference equal to the sum of the circumference of the two circles.

ANS1: -Let $r_1 = 6\text{cm}$, $r_2 = 8\text{cm}$.

Area of the circle with radius $r_1 = \pi r_1^2 = \pi(6)^2\text{cm}^2 = 36\pi\text{cm}^2$

Area of the circle with radius $r_2 = \pi r_2^2 = \pi(8)^2\text{cm}^2 = 64\pi\text{cm}^2$

Area of new circle $= \pi R^2 = 36\pi + 64\pi = 100\pi\text{cm}^2$,
where R is the radius of the new circle.

$$\text{Thus } \pi R^2 = 100\pi$$

$$\text{or, } R^2 = 100$$

$$\text{or, } R = 10$$

Hence, the required radius = 10cm .

ANS2:-Let $r_1 = 12\text{cm}$, $r_2 = 21\text{cm}$.

Circumference of the circle with radius $r_1 = 2$

$$\pi r_1 = 2\pi (12) = 24\pi\text{cm}$$

Circumference of the circle with radius $r_2 =$

$$2\pi r_2 = 2\pi(21) = 42\pi\text{ cm}$$

Circumference of the new circle $= 24\pi + 42\pi$
 $= 66\pi$

(where R is the radius of the new circle)

Thus, $2\pi R = 66\pi$

or, $R = 33$

i.e., required radius $= 33\text{cm}$

Q3:-Find the area of a sector of a circle with radius 14cm and of angle 45° . Also, find the length of the corresponding arc of the sector.

Q4:-In a circle of diameter 42cm, an arc subtends an angle of 60° at the centre. Find:

- ⦿ Length of the arc.
- ⦿ Area of the corresponding sector.
- ⦿ Area of the corresponding major sector.
- ⦿ Length of the major sector.

$$\text{ANS3:-Area of the sector} = \frac{\pi r^2 \Theta}{360^\circ}$$

$$= \frac{22 \times 14 \times 14 \times 45^\circ}{7 \times 360^\circ} = \text{cm}^2$$

$$= 11 \times 7 \text{ cm}^2 = 77 \text{ cm}^2$$

$$\text{Length of the arc} = \frac{2\pi r \Theta}{360^\circ}$$

$$= 2 \times \frac{22 \times 14 \times 14 \times 45^\circ}{7 \times 360^\circ} \text{ cm}$$

$$= 11 \text{ cm}$$

$$\text{ANS4:- (I) Length of the arc} = \frac{2\pi r\Theta}{360^\circ}$$

$$= 2 \times \frac{22 \times 21 \times 60^\circ}{7 \times 360^\circ}$$

$$(\text{Diameter} = 42\text{cm, so, } r = \frac{42}{2} = 21)$$

$$= 22\text{cm}$$

$$\text{(ii) Area of the sector} = \frac{\pi r^2 \Theta}{360^\circ}$$

$$= \frac{22 \times 21 \times 21 \times 60^\circ}{7 \times 360^\circ}$$

$$= 231 \text{ cm}^2$$

$$(iii) \text{ Area of the major sector} = \frac{\pi r^2 (360^\circ - \Theta)}{360^\circ}$$

$$= \frac{22 \times 21 \times 21 \times (360^\circ - 60^\circ)}{7 \times 360^\circ} = 22 \times 3 \times 21 \times \frac{300^\circ}{360^\circ}$$

$$= 11 \times 21 \times 5 \text{ cm}^2 = 1155 \text{ cm}^2$$

$$(iv) \text{ Length of the major sector} = \frac{2\pi r^2 (360^\circ - \Theta)}{360^\circ}$$

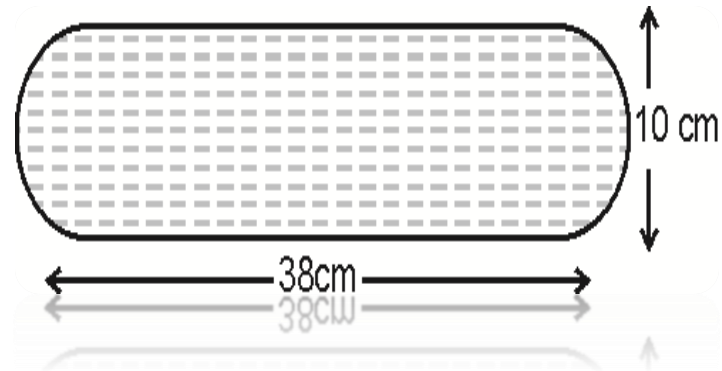
$$= \frac{2 \times 22 \times 21 \times (360^\circ - 60^\circ)}{7 \times 360^\circ} = 2 \times 22 \times 3 \times \frac{300^\circ}{360^\circ}$$

$$= 22 \times 5 = 110 \text{ cm}$$

Q5:-A chord of a circle of radius 10cm subtends a right angle at the centre. Find the area of

- Minor segment
- Major segment (use $\pi = 3.14$)

Q6:-Find the area of a flower bed with semicircular ends ?



ANS5:- Area of minor segment APB

= area of sector OAPB - area of ΔAOB

$$\frac{\pi r^2 \Theta}{360^\circ} - \frac{1}{2} OA \times OB$$

$$= (3.14) \times 10 \times 10 \times \frac{90^\circ}{360^\circ} - \frac{1}{2} \times 10 \times 10$$

$$= (87.50 - 50) \text{ cm}^2$$

$$= 37.50 \text{ cm}^2$$

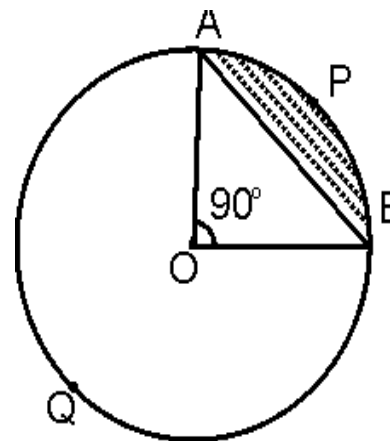
Area of major segment AQB

= area of circle - area of minor segment

$$= (3.14 \times 10 \times 10 - 37.50)$$

$$= (314 - 37.50) \text{ cm}^2$$

$$= 276.50 \text{ cm}^2$$



ANS 6:- The flower bed consists of a rectangle of dimensions 38cm x 10cm and two semicircles
The flower bed consists of a rectangle of dimensions 38cm x 10cm and two semicircles each of radius 10cm.

So, area of the flower bed
= area of the rectangle + area of two semicircles
= $[38 \times 10 + \frac{1}{2} \pi (5)^2 + \frac{1}{2} \pi (5)^2] \text{ cm}^2$
= $[380 + 3.14 \times 25] \text{ cm}^2$
= $(380 + 78.5) \text{ cm}^2$
= 458.5 cm^2

