

ARITHMETIC PROGRESSION

INTRODUCTION

- You must have observed that in nature, many things follow a certain pattern, such as the petals of sun flower, the holes of a honeybee comb, the grains on a maize cob, the spirals on a pineapple and on a pine cone etc. We now look for some patterns which occur in our daily life.
- For example: Mohit applied for a job and got selected. He has been offered a job with a starting monthly salary Rs 8000, with an annual increment of Rs 500 in his salary. His salary for the 1st, 2nd, 3rd, ... years will be, respectively 8000, 8500, 9000,

- Consider the following lists of numbers:

1) $1, 2, 3, 4, \dots$

2) $100, 70, 40, 10, \dots$

3) $-3, -2, -1, 0, \dots$

4) $3, 3, 3, 3, \dots$

5) $-1.0, -1.5, -2.0, -2.5, \dots$



Each of the numbers in the list is called a term.

- In all the lists above, we see that the successive terms are obtained by adding a fixed number to the preceding terms. Such lists are called **ARITHMETIC PROGRESSIONS** (or) **AP**.
- So, An Arithmetic Progression is a list of numbers in which each term is obtained by adding a fixed number preceding term except the first term.
- This fixed number is called the common difference of the AP.
- Remember that it can be positive(+), negative(-) or zero(o)

- Let us denote the first term of an **Arithmetic Progression** by (a_1) second term by (a_2) , nth term by (a_x) and the common **difference** by d .
- The general form of an Arithmetic Progression is :
- $a, a + d, a + 2d, a + 3d, \dots, a + (n-1)d$
- Now, let us consider the situation again in which Mohit applied for a job and been selected. He has been offered a starting monthly salary of Rs8000, with an annual increment of Rs500. what would be his salary for the fifth year?

Find the nth Term of an A.P.

Let us consider an A.P. with first term 'a' and common difference 'd', then

- The first term = $a_1 = a + 0d = a + (1-1)d$
- The second term = $a_2 = a + d = a + (2-1)d$
- The third term = $a_3 = a + 2d = a + (3-1)d$
- The fourth term = $a_4 = a + 3d = a + (4-1)d$

$$\text{The } n\text{th term} = a_n = a + (n-1)d$$

To check that a given term is in
A.P. or not.

2, 6, 10, 14....

(i) Here , first term $a = 2$,
find differences in the next terms

$$a_2 - a_1 = 6 - 2 = 4$$

$$a_3 - a_2 = 10 - 6 = 4$$

$$a_4 - a_3 = 14 - 10 = 4$$

Since the differences are common.

Hence the given terms are in A.P.

**Problem . Find number of terms of
A.P. 100, 105, 110, 115,,.....**

500
Solution.

First term is $a = 100$, $a_n = 500$

Common difference is $d = 105 - 100 = 5$

n th term is $a_n = a + (n-1)d$

$$500 = 100 + (n-1)5$$

$$500 - 100 = 5(n - 1)$$

$$400 = 5(n - 1)$$

$$5(n - 1) = 400$$



$$5(n - 1) = 400$$

$$n - 1 = 400/5$$

$$n - 1 = 80$$

$$n = 80 + 1$$

$$n = 81$$

Hence the no. of terms are 81.



Problem : Find the value of k for which the given series is in A.P. 4, $k - 1$, 12

Solution : Given A.P. is 4, $k - 1$, 12.....

If series is A.P. then the differences will be common.

$$d_1 = d_1$$

$$a_2 - a_1 = a_3 - a_2$$

$$k - 1 - 4 = 12 - (k - 1)$$

$$k - 5 = 12 - k + 1$$

$$k + k = 12 + 1 + 5$$



The **sum of n terms**, we find as,

$$\text{Sum} = n \times [(\text{first term} + \text{last term}) / 2]$$

Now **last term** will be $= a + (n-1) d$

Therefore,

$$\begin{aligned} \text{Sum}(S_n) &= n \times [\{ a + a + (n-1) d \} / 2] \\ &= n/2 [2a + (n+1)d] \end{aligned}$$



DERIVATION

The sum to n terms is given by:

$$S_n = a + (a + d) + (a + 2d) + \dots + (a + (n - 1)d) \rightarrow (1)$$

If we write this out backwards, we get:

$$S_n = (a + (n - 1)d) + (a + (n - 2)d) + \dots + a \rightarrow (2)$$

Now let's add (1) and (2):

$$2S_n = [2a + (n - 1)d] + [2a + (n - 1)d] + \dots + [2a + (n - 1)d]$$



$$\text{So, } S_n = n/2 [2a + (n - 1)d]$$

Problem . Find the sum of 30 terms of given A.P. ,12 , 20 , 28 , 36.....

Solution : Given A.P. is 12 , 20, 28 , 36

Its first term is $a = 12$

Common difference is $d = 20 - 12 = 8$

The sum to n terms of an arithmetic progression

$$S_n = n/2 [2a + (n - 1)d]$$

$$= \frac{1}{2} \times 30 [2 \times 12 + (30-1) \times 8]$$

$$= 15 [24 + 29 \times 8]$$



$$= 15[24 + 232]$$

$$= 15 \times 246$$

$$= 3690$$

THE SUM OF TERMS IS 3690



Problem . Find the sum of terms in given A.P.

2 , 4 , 6 , 8 , 200

Solution: Its first term is $a = 2$

Common difference is $d = 4 - 2 = 2$

nth term is $a_n = a + (n-1)d$

$$200 = 2 + (n-1)2$$

$$200 - 2 = 2(n - 1)$$

$$2(n - 1) = 198$$

$$n - 1 = 99, \quad n = 100$$



The sum to n terms of an arithmetic progression

$$S_n = n/2 [2a + (n - 1)d]$$

$$S_{100} = 100/2 [2 \times 2 + (100-1) \times 2]$$

$$= 50 [4 + 198]$$

$$= 50[202]$$

$$= 10100$$



Some convenient methods to determine the AP

- It is always convenient to make a choice of
 - i. 3 numbers in an AP as $(a-d)$, a , $(a+d)$;
 - ii. 4 numbers in an AP as $(a-3d)$, $(a-d)$, a , $(a+d)$, $(a+3d)$;
 - iii. 5 numbers in an AP as $(a-2d)$, $(a-d)$, a , $(a+d)$, $(a+2d)$.

CONCEPT MAP

