

CONGRUENCE OF TRIANGLES



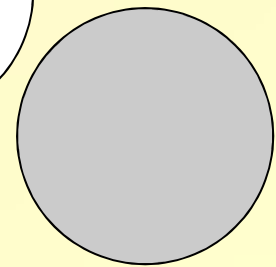
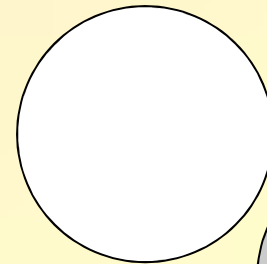
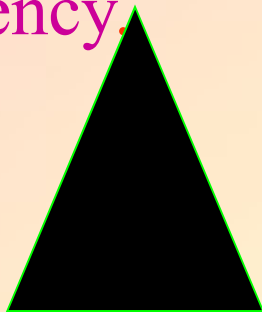
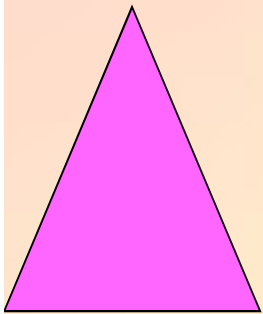
Congruent Figures



In geometry, we study figures, their properties and relations between them. Every figure has a shape, size and position.

Two figures are congruent, if they have the same shape and size.

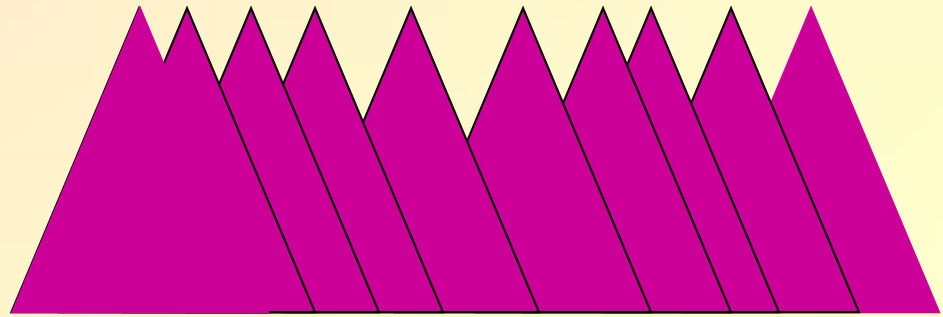
The relation of two figures being congruent, is called congruency.



Method of Superposition.

Suppose we want to compare for congruence two given figures F_1 and F_2 . Let us cut out F_2 and move it in order to cover F_1 . We permit only three types of motion:

- Sliding or translating, that is, moving in the plane without rotation.
- Rotation about a point in the plane, or
-
- Going out of the plane and inverting upside down. But, stretching and tearing is not allowed.



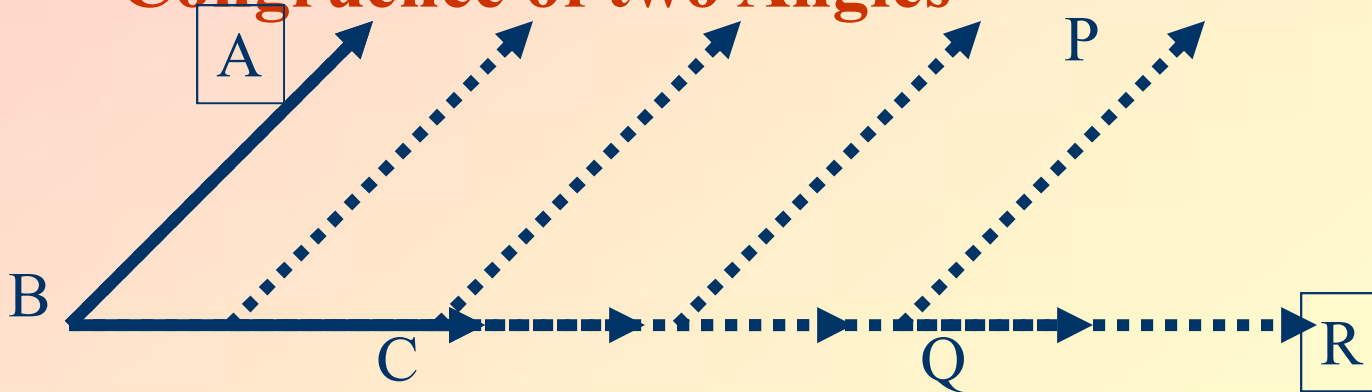
Comparing For Congruence

Congruence of Line Segments



Two line segments are congruent, if they have the same length.

Congruence of two Angles



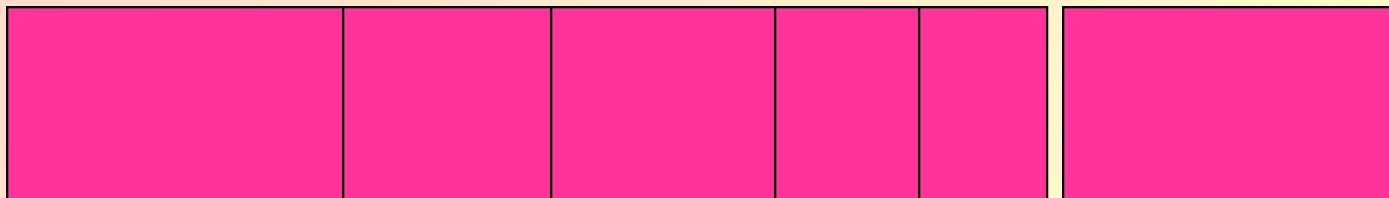
Two angles are congruent, if they have the same measure.

Congruence of two Squares



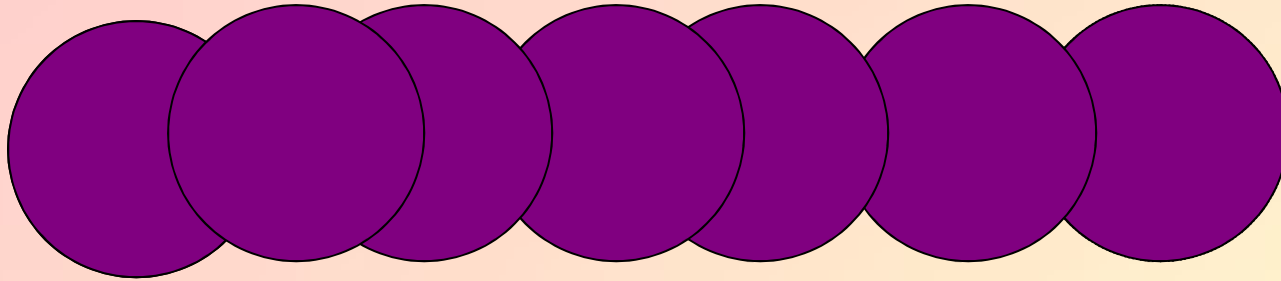
Two squares are, congruent if they have the same side length

Congruence of two Rectangles



Two rectangles are congruent, if they have the same length and breadth.

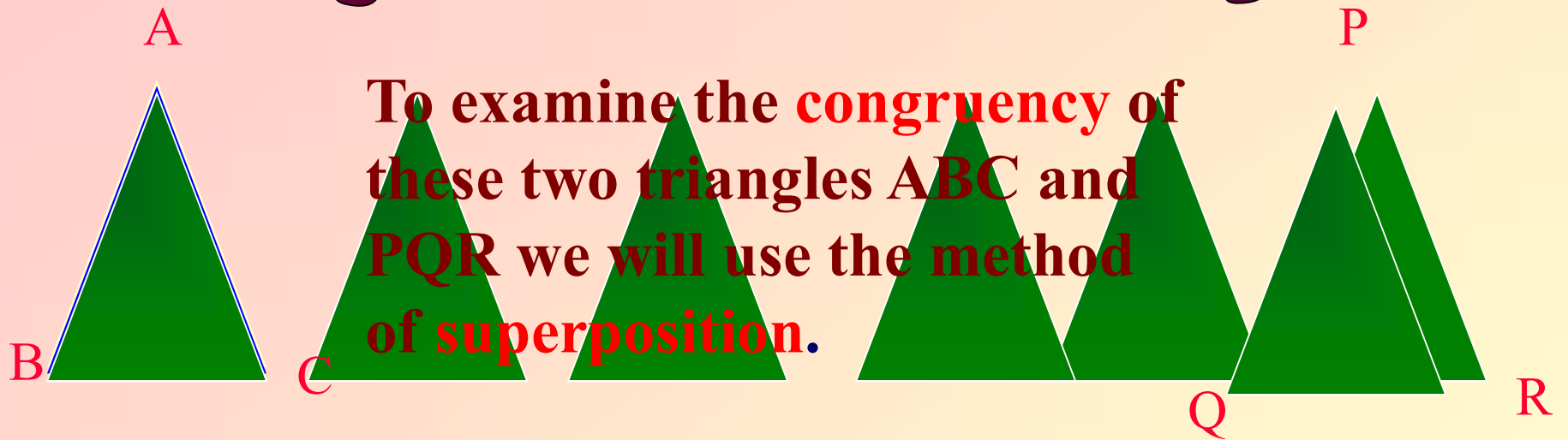
Congruence of two Circles



Two circles are congruent if they have the same radius.

Thus if two figures **cover each other exactly**, then surely they are of the **same shape** and, therefore, congruent. This method of comparing two figures is called the method of **superposition**.

Congruence of Triangles



When the two triangles cover each other exactly, the **vertices** of one triangle will **coincide** with the vertices of the other triangle. Thus there will be **matching** between the **vertices** of the two triangles.

Matching Between the Vertices, Matching Parts

There are six possible ways in which the vertices of the two triangles ABC and PQR may be matched. The diagram below shows one such matching:



This may also be written as

$$A \leftrightarrow P, B \leftrightarrow Q, C \leftrightarrow R$$

And, more briefly, as

$$ABC \leftrightarrow PQR$$

Each matching between the vertices of the two triangles determines a matching of their parts as well. Thus, the matching

$ABC \longleftrightarrow PQR$ leads to the matching pairs

$(\text{side } AB) \longleftrightarrow (\text{side } PQ), (\text{side } BC) \longleftrightarrow (\text{side } QR)$

$(\text{side } AC) \longleftrightarrow (\text{side } PR)$

$\angle A \longleftrightarrow \angle P, \angle B \longleftrightarrow \angle Q, \angle C \longleftrightarrow \angle R.$

Matching parts are also called the *corresponding parts* of the two triangles.

When we write

$\triangle ABC \overset{\sim}{=} \triangle PQR$

It means that the two triangles ABC and PQR are **congruent** under the matching between the vertices. $ABC \longleftrightarrow PQR$

Congruent Triangles and their Parts

Suppose we have two triangles ABC and PQR such that

$$\triangle ABC \cong \triangle PQR$$

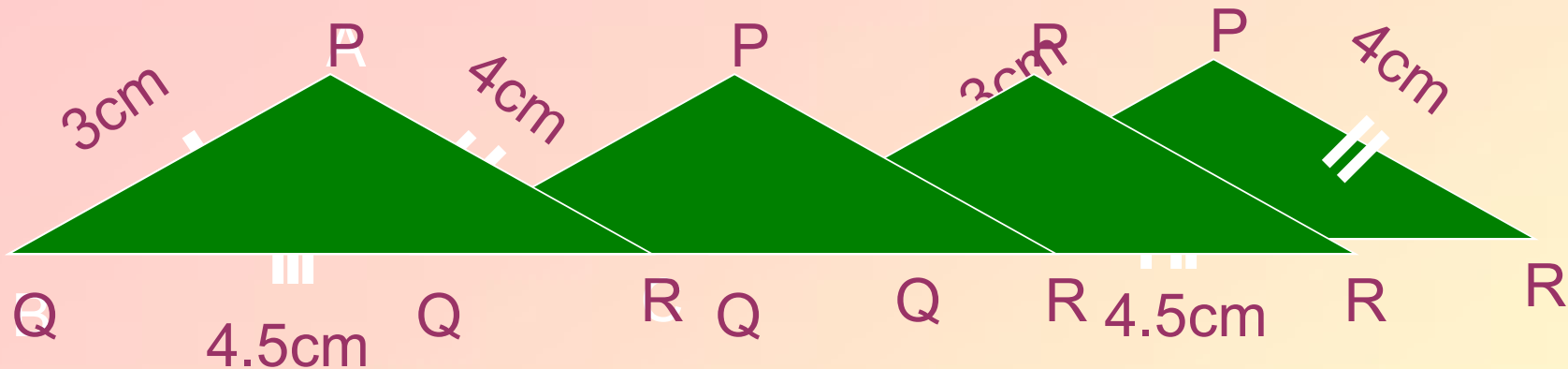
We know that if we **superpose** $\triangle PQR$ on $\triangle ABC$ With P on A, Q on B, and R on C, then **two triangles cover** each other **exactly**. It follows immediately that

$$AB = PQ, \quad BC = QR \quad \text{and} \quad AC = PR$$

$$\angle A = \angle P, \quad \angle B = \angle Q \quad \angle C = \angle R$$

Thus, we say that **matching parts** (corresponding parts) of **congruent triangles** are equal.

The SSS Congruence Condition

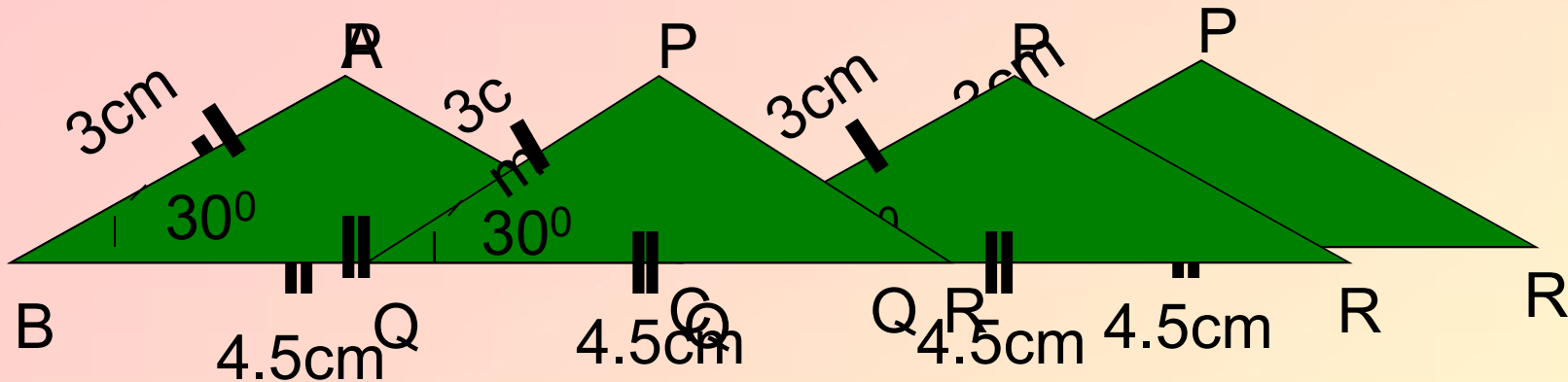


In $\triangle ABC$ and $\triangle PQR$

$AB = PQ$, $BC = QR$ and $AC = PR$

$\triangle ABC \cong \triangle PQR$ (SSS congruency)

The SAS Congruence Condition

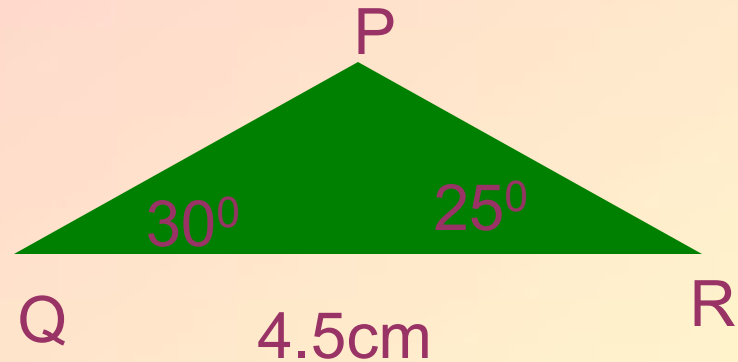
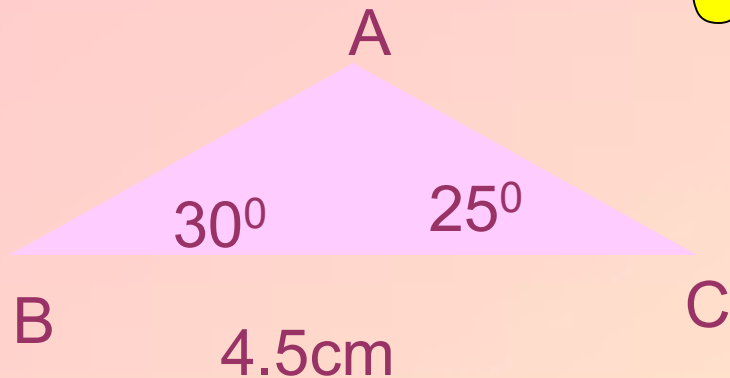


In $\triangle ABC$ and $\triangle PQR$

$AB = PQ$, $BC = QR$ and $\angle B = \angle Q$

$\triangle ABC \cong \triangle PQR$ (SAS congruency)

The ASA Congruence Condition

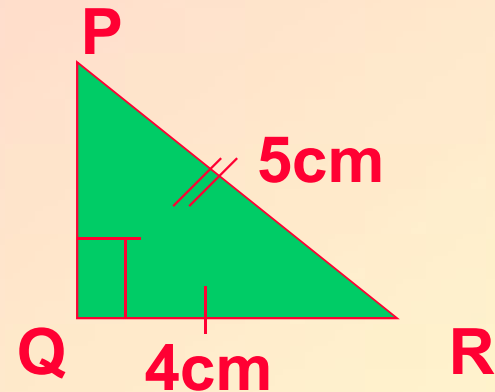
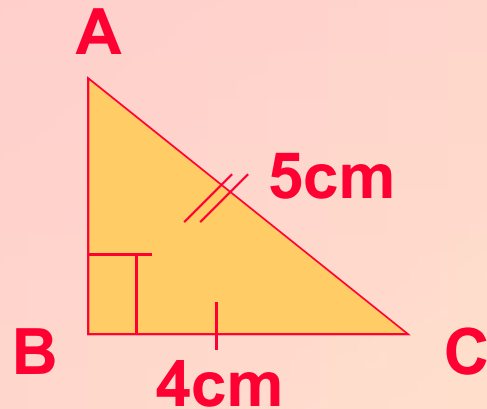


In $\triangle ABC$ and $\triangle PQR$

$$\angle C = \angle R \quad BC = QR \quad \text{and} \quad \angle B = \angle Q$$

$$\triangle ABC \cong \triangle PQR \quad (\text{ASA congruency})$$

The RHS Congruence Condition



In $\triangle ABC$ and $\triangle PQR$

$AC = PR$ $BC = QR$ and $\angle B = \angle Q$

$\triangle ABC \cong \triangle PQR$ (RHS congruency)

ASSIGNMENT

- LIST 10 FIGURES FROM YOUR SURROUNDINGS WHICH ARE CONGRUENT FIGURES
- DRAW 10 PAIRS OF FIGURES AND TRACE THEM AND CHECK IF THEY ARE CONGRUENT FIGURES