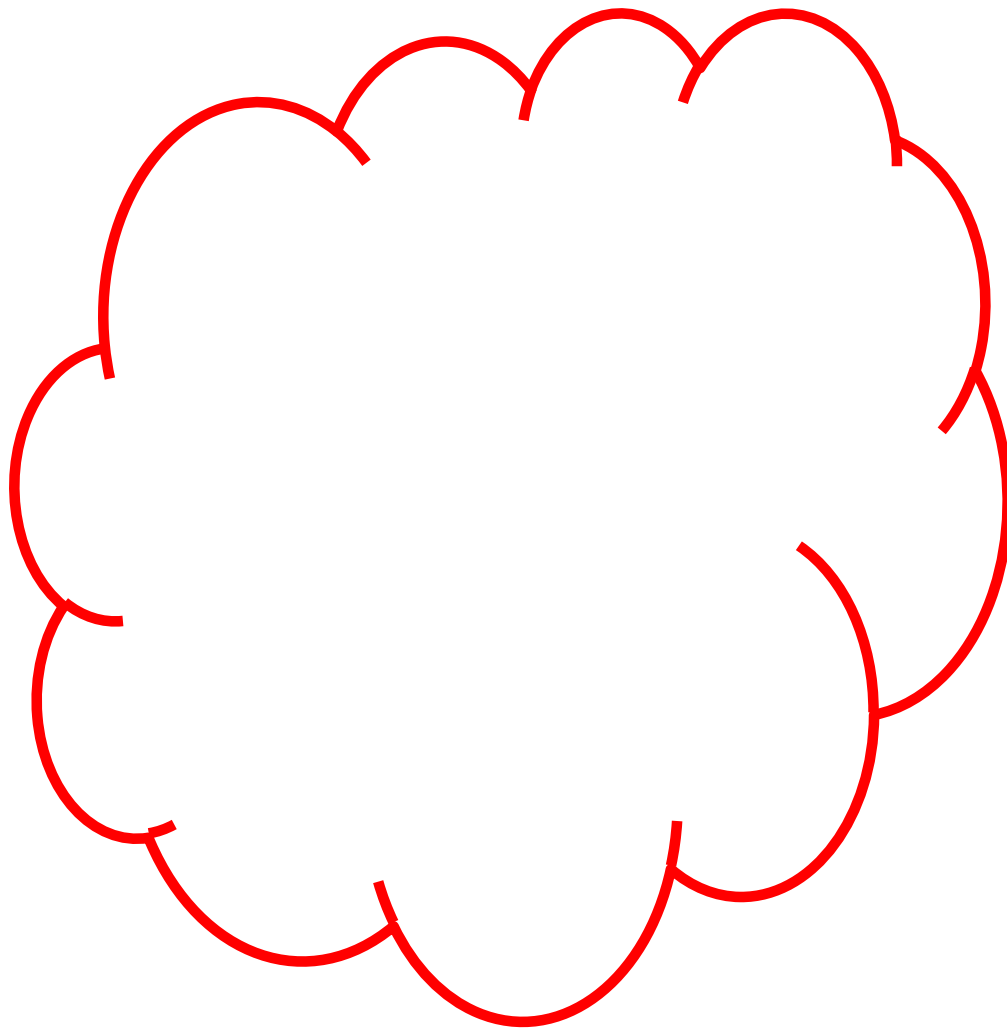




Coordinate Geometry

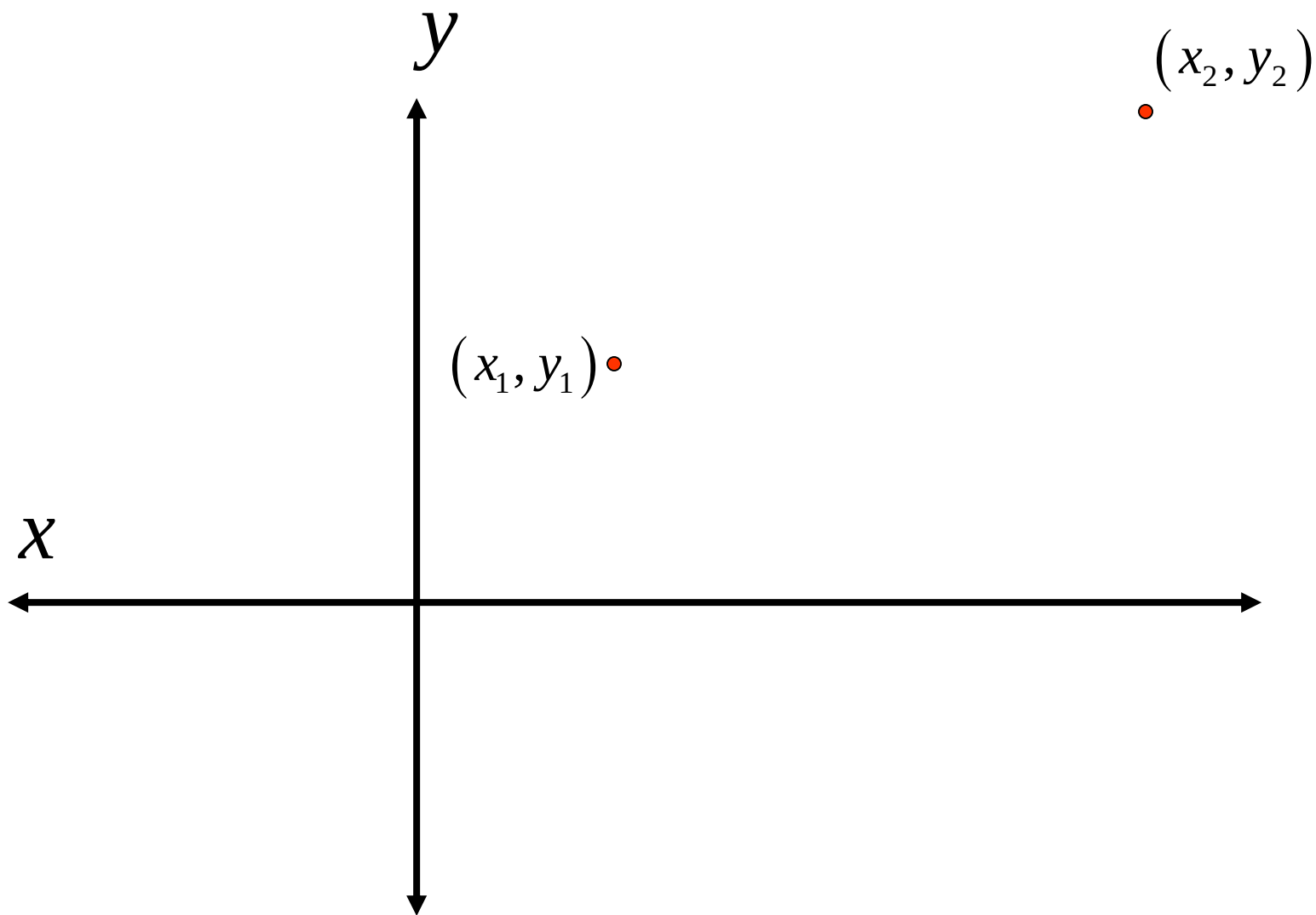
René Descartes



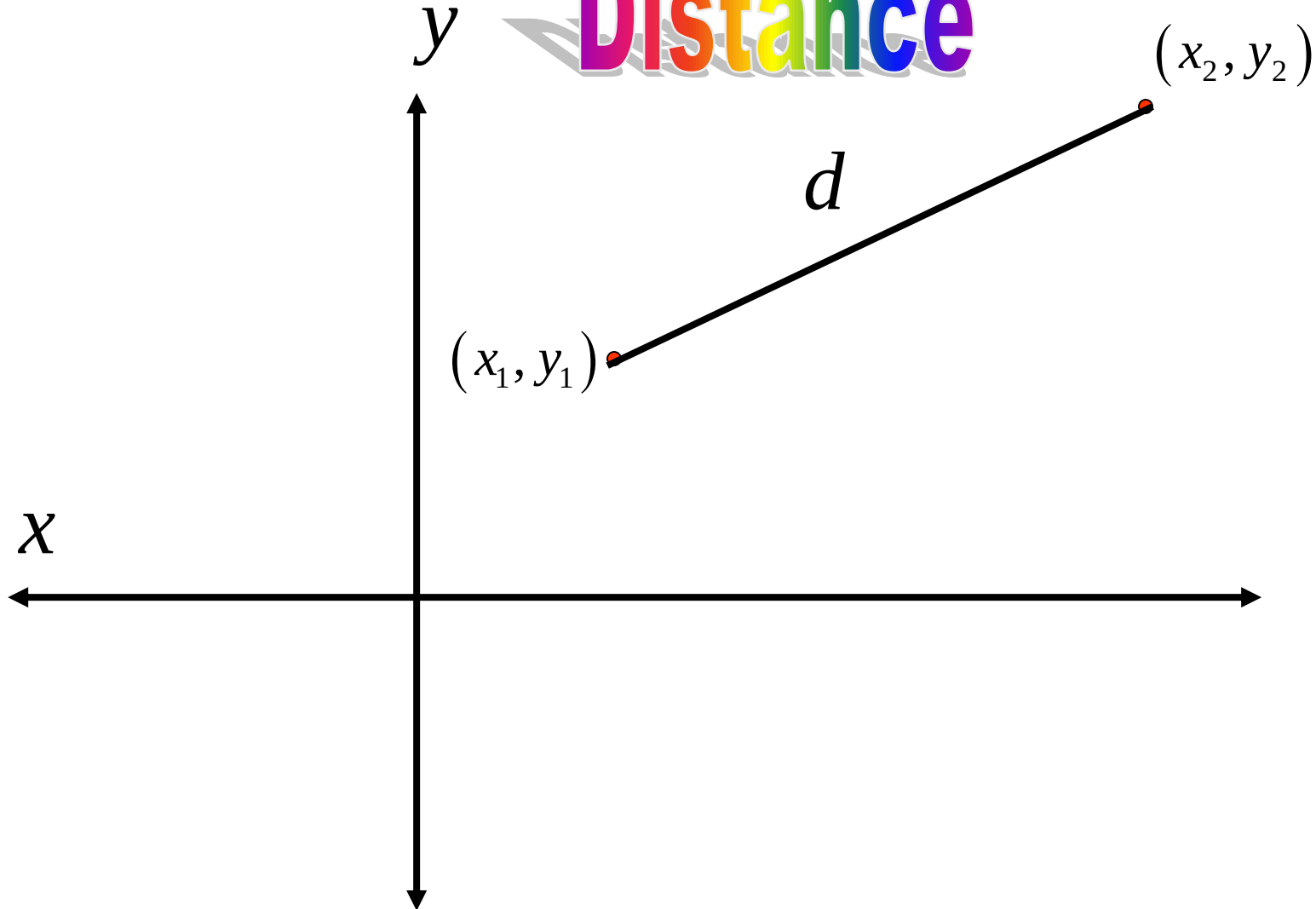
Session Objectives

1. Distance formula
2. Area of a triangle
3. Collinearity of three points
4. Section formula

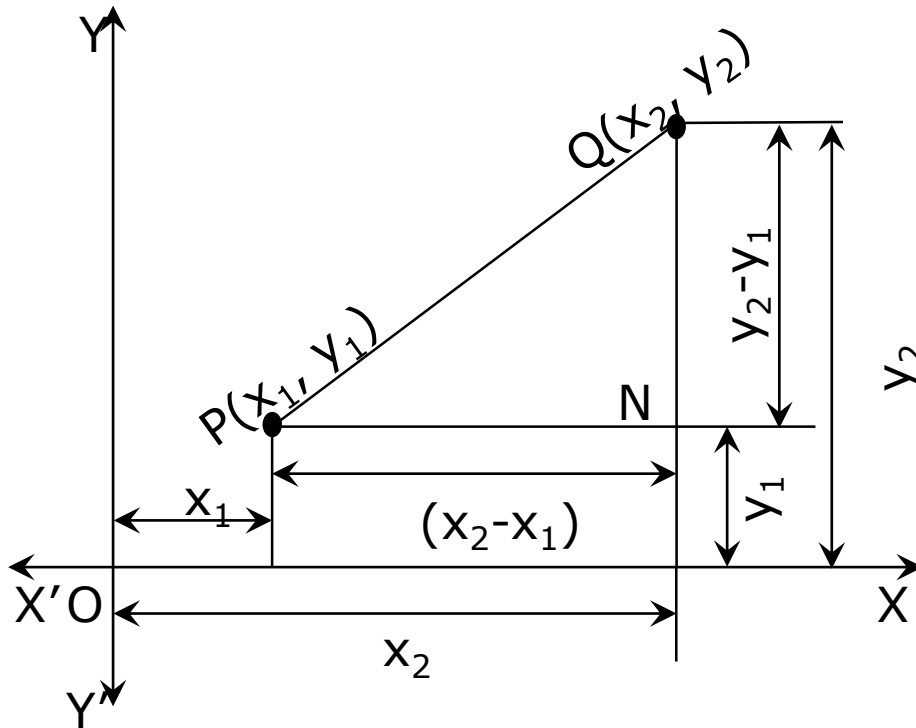
The Distance Formula



Distance



Distance Formula



PQN is a right angled Δ .

$$\therefore PQ^2 = PN^2 + QN^2$$

$$\therefore PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance From Origin

Distance of $P(x, y)$ from the origin is

$$= \sqrt{(x - 0)^2 + (y - 0)^2}$$

$$= \sqrt{x^2 + y^2}$$

Example 1

$$\left(\overset{x_1}{3}, \overset{y_1}{1} \right) \left(\overset{x_2}{2}, \overset{y_2}{4} \right)$$

$$d = \sqrt{\left(x_2 - x_1 \right)^2 + \left(y_2 - y_1 \right)^2}$$

$$d = \sqrt{}$$

Example 1

$$\begin{pmatrix} x_1 & y_1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x_2 & y_2 \\ 2 & 4 \end{pmatrix}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

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$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(2 - 3)^2 + (4 - 1)^2}$$

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$$\begin{pmatrix} x_1 & y_1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x_2 & y_2 \\ 2 & 4 \end{pmatrix}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(2 - \quad \quad \quad)}$$

Example 1

$$\begin{pmatrix} x_1 & y_1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x_2 & y_2 \\ 2 & 4 \end{pmatrix}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(2 - 3)}$$

Example 1

$$\left(\overset{x_1}{3}, \overset{y_1}{1} \right) \left(\overset{x_2}{2}, \overset{y_2}{4} \right)$$

$$d = \sqrt{\left(x_2 - x_1 \right)^2 + \left(y_2 - y_1 \right)^2}$$

$$d = \sqrt{\left(2 - 3 \right)^2 + \quad}$$

Example 1

$$\begin{pmatrix} x_1 \\ 3 \end{pmatrix} \begin{pmatrix} y_1 \\ 1 \end{pmatrix} \begin{pmatrix} x_2 \\ 2 \end{pmatrix} \begin{pmatrix} y_2 \\ 4 \end{pmatrix}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(2 - 3)^2 + (\quad)^2}$$

Example 1

$$\begin{pmatrix} x_1 \\ 3 \end{pmatrix}, \begin{pmatrix} y_1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} x_2 \\ 2 \end{pmatrix}, \begin{pmatrix} y_2 \\ 4 \end{pmatrix}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(2 - 3)^2 + (4 - 1)^2}$$

Example 1

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(2 - 3)^2 + (4 -$$

Example 1

$$\begin{pmatrix} x_1 \\ 3 \end{pmatrix}, \begin{pmatrix} y_1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} x_2 \\ 2 \end{pmatrix}, \begin{pmatrix} y_2 \\ 4 \end{pmatrix}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(2 - 3)^2 + (4 - 1)^2}$$

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$$d = \sqrt{\left(2 - 3 \right)^2 + \left(4 - 1 \right)^2}$$

$$d = \sqrt{\left(-1 \right)^2}$$

Example 1

$$\left(\overset{x_1}{3}, \overset{y_1}{1} \right) \left(\overset{x_2}{2}, \overset{y_2}{4} \right)$$

$$d = \sqrt{\left(x_2 - x_1 \right)^2 + \left(y_2 - y_1 \right)^2}$$

$$d = \sqrt{\left(2 - 3 \right)^2 + \left(4 - 1 \right)^2}$$

$$d = \sqrt{\left(-1 \right)^2 + \left(3 \right)^2}$$

$$d = \sqrt{(-1)^2 + (3)^2}$$

$$d = \sqrt{1 + 9}$$

$$d = \sqrt{10}$$

Q. Check whether $(5, -2)$, $(6, 4)$ and $(7, -2)$ are the vertices of an isosceles triangle.

Sol. Let the points $(5, -2)$, $(6, 4)$, and $(7, -2)$ are representing the vertices A, B, and C of the given triangle respectively.

$$AB = \sqrt{(5 - 6)^2 + (-2 - 4)^2}$$

$$AB = \sqrt{1 + 36} = \sqrt{37}$$

$$BC = \sqrt{(6 - 7)^2 + (4 + 2)^2}$$

$$BC = \sqrt{1 + 36} = \sqrt{37}$$

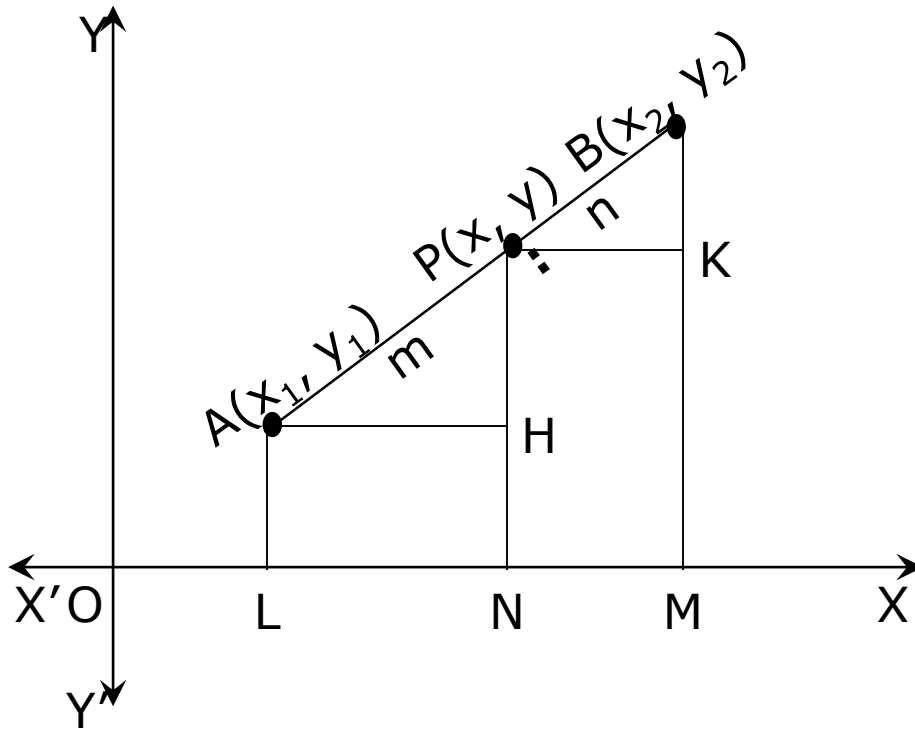
$$CA = \sqrt{(5 - 7)^2 + (-2 - 2)^2}$$

$$CA = \sqrt{4 + 0} = 2$$

$$AB = BC$$

As two sides are equal in length, therefore, ABC is an isosceles triangle.

Section Formula - Internal Division



Clearly $\triangle AHP \sim \triangle PKB$

$$\therefore \frac{AP}{BP} = \frac{AH}{PK} = \frac{PH}{BK}$$

$$\therefore \frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}$$

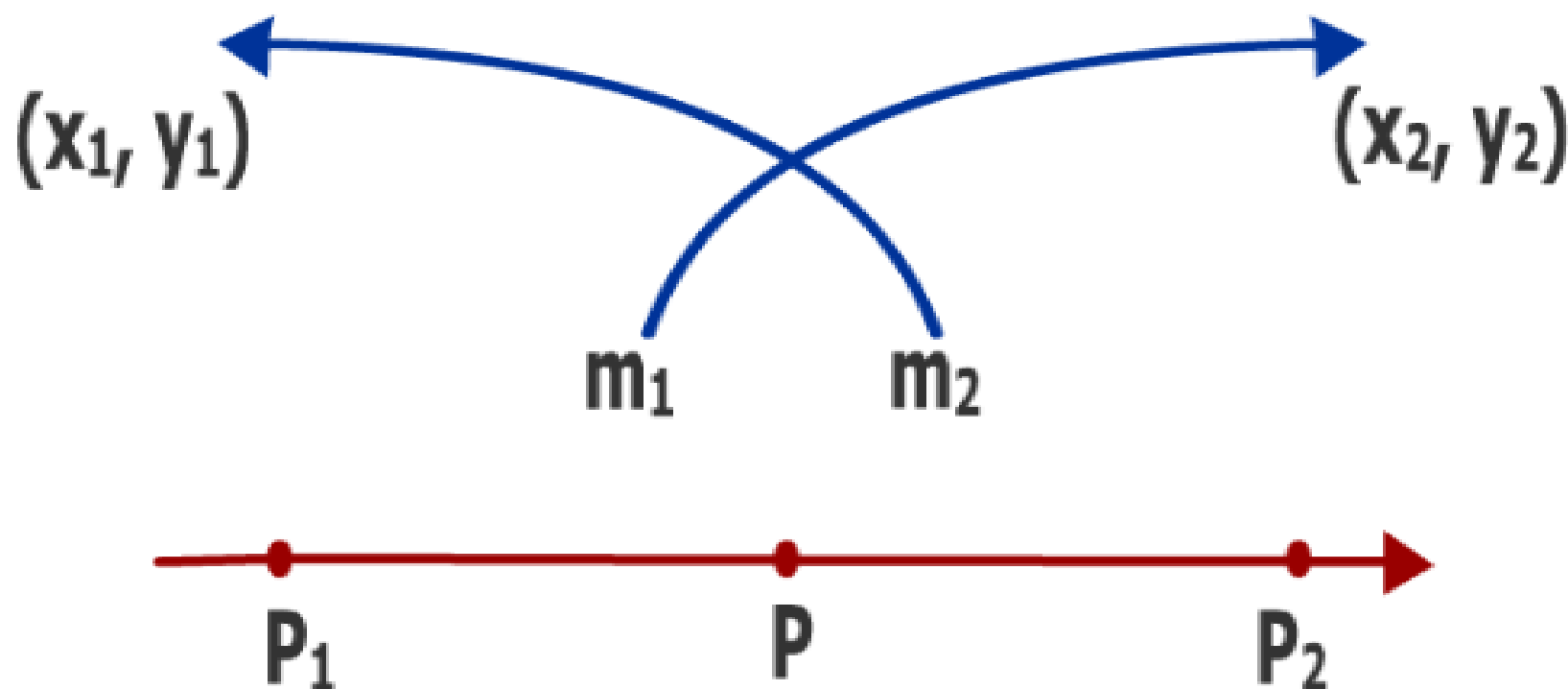
$$\therefore P \equiv \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right)$$

Rules to Write Down the Coordinates of a Point Dividing a Segment in
given ratio
Rule for Internal Division

To divide the line joining the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the ratio $m_1:m_2$.

Step 1

Multiply m_1 with the x-coordinate of P_2 and m_2 with the x-coordinate of P_1 .



Step 2

Add the products obtained in step 1.

Step 3

Divide the sum in Step 2 by $m_1 + m_2$ and put it equal to x .

(This gives x -coordinate of point P)

Step 4

Repeat Step 1 to Step 3 after replacing x by y .

(This gives y -coordinate of point P).

Finding the Ratio of a Point Which Divides The Line Joining Two Given Points.

In case we are interested to find out a ratio, we take it to be $k : 1$ (rather than as $m_1 : m_2$). Note that we now have to find merely one constant k rather than two.

Midpoint Formula

Midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$

$m:n \equiv 1:1$

$$\therefore P \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example

(i) Find the coordinates of the point which divides the line joining the points $(5, -2)$ and $(9, 6)$ internally in the ratio $1:3$.

Solution

(i) The coordinates of the point which divides the line joining the points $(5, -2)$ and $(9, 6)$ internally in the ratio $1:3$ are



$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{1(9) + 3(5)}{1 + 3} = \frac{24}{4} = 6,$$
$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1(6) + 3(-2)}{1 + 3} = \frac{0}{4} = 0.$$

Thus the coordinates of the required point are $(6, 0)$.

Example : Find the ratio in which the y-axis divides the line segment joining the points $(5, -6)$ and $(-1, -4)$. Also find the point of intersection.

Solution : Let the ratio be $k : 1$. Then by the section formula, the coordinates of the

point which divides AB in the ratio $k : 1$ are $\left(\frac{-k + 5}{k + 1}, \frac{-4k - 6}{k + 1} \right)$.

This point lies on the y-axis, and we know that on the y-axis the abscissa is 0.

Therefore,
$$\frac{-k + 5}{k + 1} = 0$$

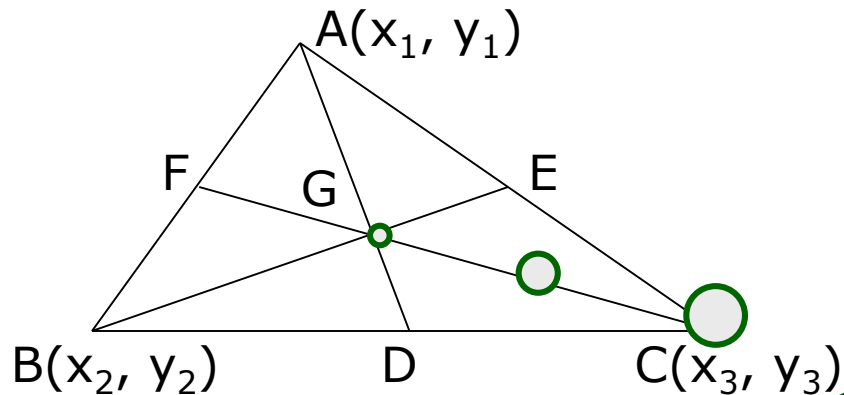
So,
$$k = 5$$

That is, the ratio is $5 : 1$. Putting the value of $k = 5$, we get the point of intersection as

$$\left(0, \frac{-13}{3} \right).$$

Centroid

Intersection of medians of a triangle is called the centroid.



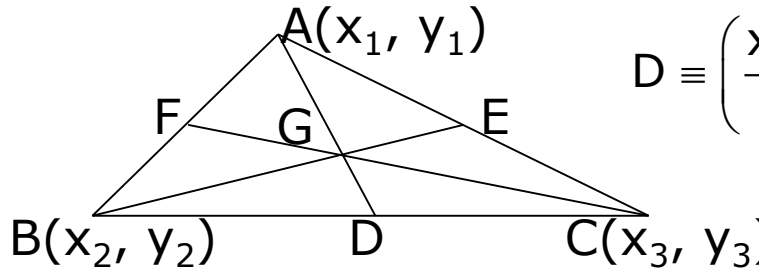
Centroid is always denoted by G .

$$D \equiv \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right)$$

$$E \equiv \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right)$$

$$F \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Centroid

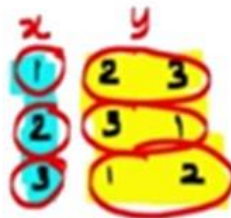
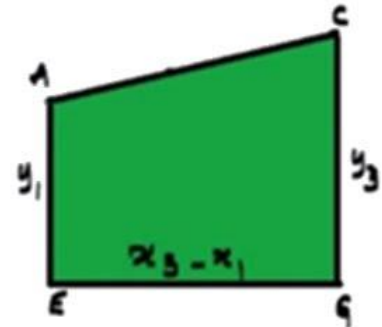
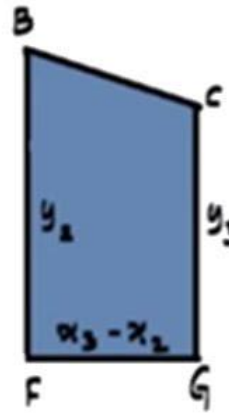
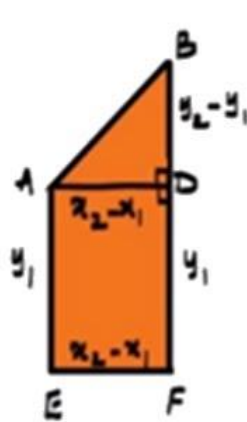
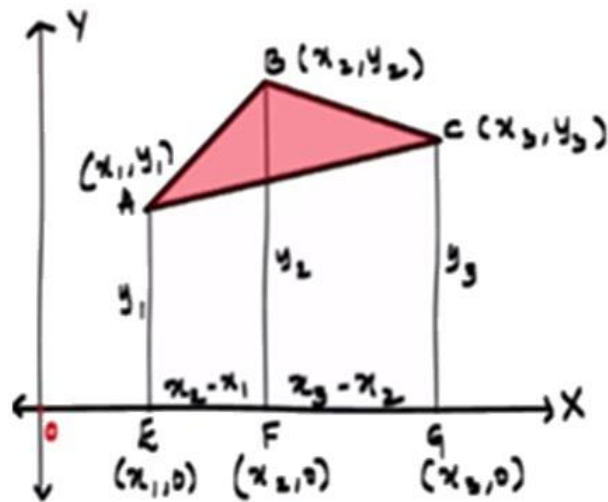

$$D \equiv \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right)$$
$$E \equiv \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right)$$
$$F \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Point G divides AD, BE and CF respectively in the ratio 2:1

$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Area of Triangle

(Coordinate Geometry)



$$A(\Delta ABC) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$A(\Delta ABC) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Example : Find the area of a triangle whose vertices are (1, -1), (-4, 6) and (-3, -5).

Solution : The area of the triangle formed by the vertices A(1, -1), B(-4, 6) and C(-3, -5), by using the formula above, is given by

$$\begin{aligned} & \frac{1}{2} [1(6 + 5) + (-4)(-5 + 1) + (-3)(-1 - 6)] \\ &= \frac{1}{2} (11 + 16 + 21) = 24 \end{aligned}$$

So, the area of the triangle is 24 square units.

Example : Find the area of a triangle formed by the points A(5, 2), B(4, 7) and C(7, -4).

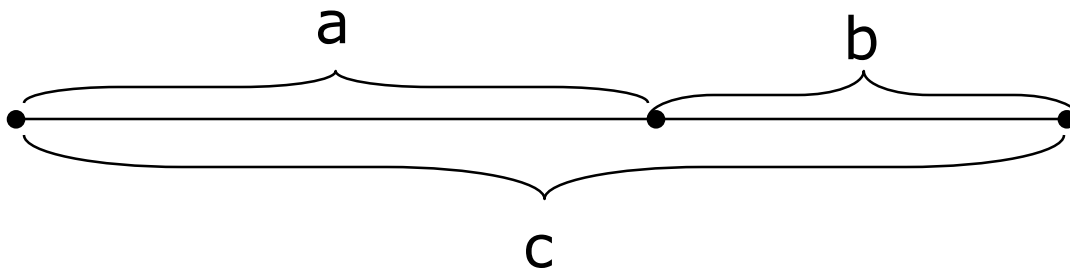
Solution : The area of the triangle formed by the vertices A(5, 2), B(4, 7) and C(7, -4) is given by

$$\begin{aligned} & \frac{1}{2} [5(7 + 4) + 4(-4 - 2) + 7(2 - 7)] \\ &= \frac{1}{2} (55 - 24 - 35) = \frac{-4}{2} = -2 \end{aligned}$$

Collinearity of Three Points

Method I :

Use Distance Formula



Show that $a + b = c$

Use distance formula to verify that the points A(0,7), B(3,-5) and C(-2,15) are collinear.

$$\begin{aligned}\text{Distance CB: } & \sqrt{((-2-3)^2 + (15-(-5))^2)} \\ & \sqrt{(25+400)} \\ & \sqrt{425} \\ & 5\sqrt{17}\end{aligned}$$

$$\begin{aligned}\text{Distance CA: } & \sqrt{((-2-0)^2 + (15-7)^2)} \\ & \sqrt{(4+64)} \\ & \sqrt{68} \\ & 2\sqrt{17}\end{aligned}$$

$$\begin{aligned}\text{Distance AB: } & \sqrt{((3-0)^2 + (-5-7)^2)} \\ & \sqrt{(9+144)} \\ & \sqrt{153} \\ & 3\sqrt{17}\end{aligned}$$

Does distance CB = distance CA + distance AB ?

$$5\sqrt{17} = 2\sqrt{17} + 3\sqrt{17}$$

$5\sqrt{17} = 5\sqrt{17}$? yes, points A, B, and C are collinear.

Collinearity of Three Points

Method II :

Use Area of Triangle

$$A \equiv (x_1, y_1)$$

$$B \equiv (x_2, y_2)$$

$$C \equiv (x_3, y_3)$$

Show that area of triangle is zero

Example

Find the value of p for which the points $(-5,1)$, $(1,p)$ and $(4, -2)$ are collinear.

Solution

The given points are $A(-5,1), B(1,p)$ and $C(4, -2)$

The given points A , B and C are collinear

$$\Leftrightarrow x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Leftrightarrow (-5).(p + 2) + 1.(-2-1) + 4.(1-p) = 0$$

$$\Leftrightarrow (-5p - 10 - 3 + 4 - 4p) = 0$$

$$\Leftrightarrow (-9p - 9) = 0$$

$$\Leftrightarrow 9p = -9 \quad p = -1$$

Hence $p = -1$.

Summary

1. The distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

The distance of a point $P(x, y)$ from the origin is $\sqrt{x^2 + y^2}$.

The coordinates of the point $P(x, y)$ which divides the line segment

joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right).$$

4. The coordinates of the point $P(x, y)$ which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1 x_2 - m_2 x_1}{m_1 - m_2}, \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2} \right).$$

5. The mid-point of the line segment joining the points $P(x_1, y_1)$ and

$$Q(x_2, y_2) \text{ is } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

6. The area of the triangle formed by the points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is given by the numerical value of the expression

$$\frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)].$$

7. Three given points are said to be collinear if the area of the triangle is zero.

CONCEPT MAP

