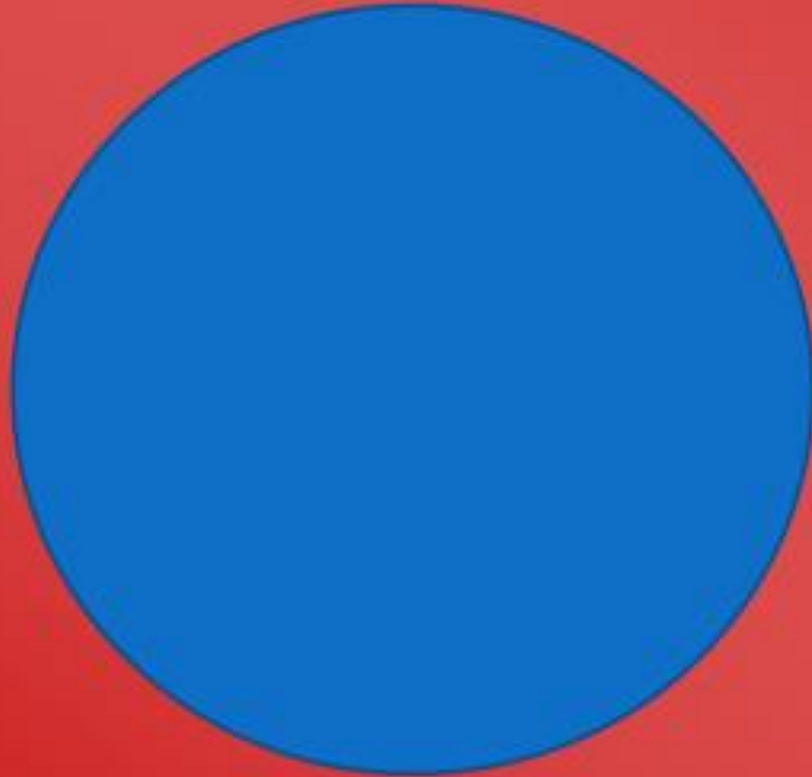


CIRCLE



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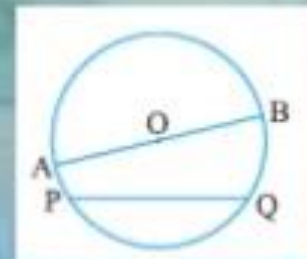
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CIRCLES AND ITS RELATED TERMS.

1.1 CIRCLE:



The collection of all the points in plane, which are at a fixed distance from a fixed point in the plane, is called a circle.



1.2 Radius, Diameter, Center, Chord, Arc And Sector.

The line segment joining the centre and any point on the circle is also called a radius of the circle.

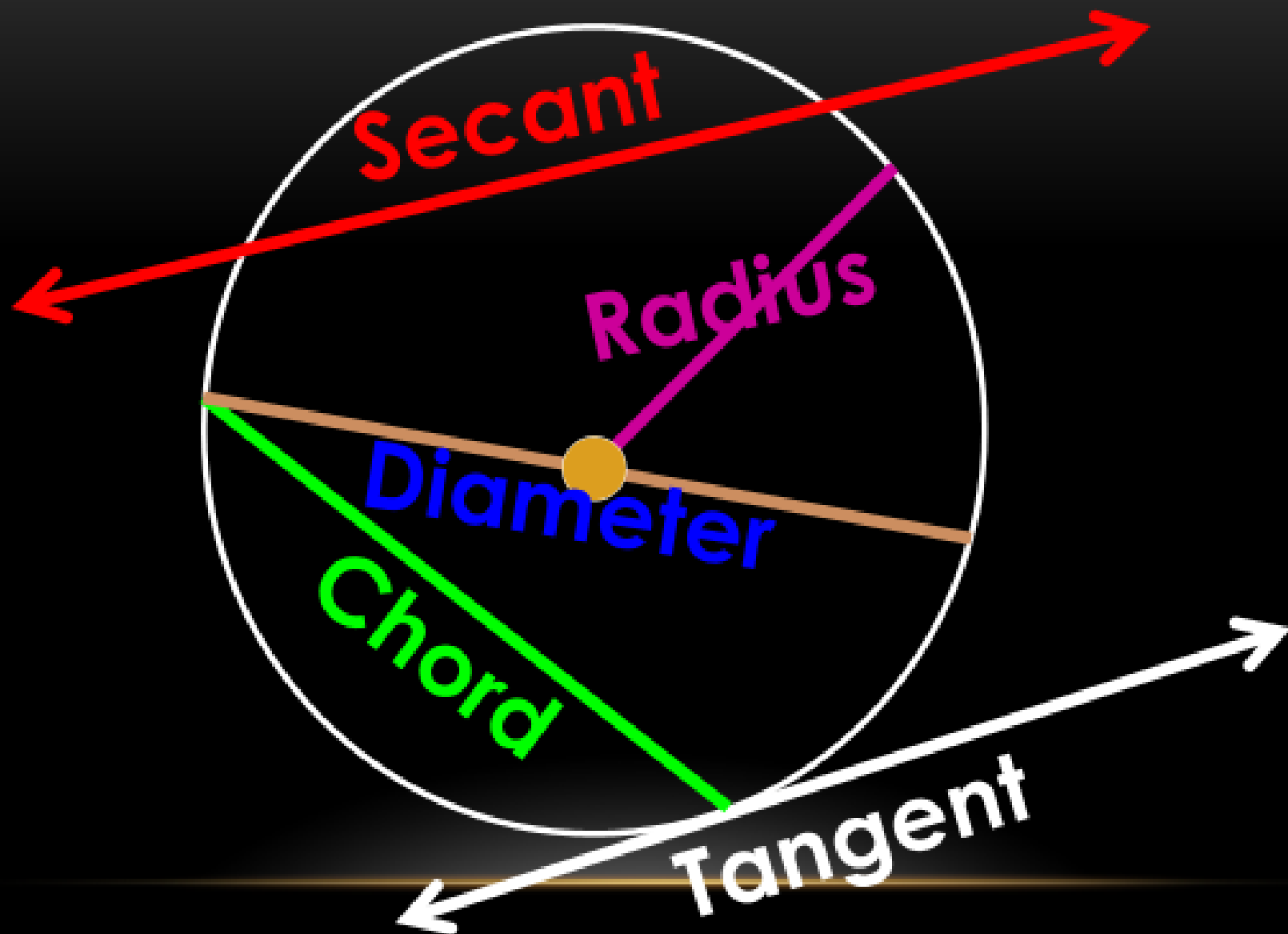
Diameter is two times the radius.

The fixed point is called the centre of the circle.

If we take two points **P** and **Q** on a circle, then the line segment **PQ** is called a chord of the circle.

A piece of a circle between two points is called an arc.

The region between an arc and the two radii, joining the centre to the end points of the arc is called a sector.



Chords of a Circle

We know that the perpendicular from a point to a line segment is the shortest distance between them.

*A line that joins two points on the circumference of a circle is called a **chord**.*

*A chord passing through the center of the circle is called **diameter**.*

*The **longest chord** of the circle is the **diameter**.*

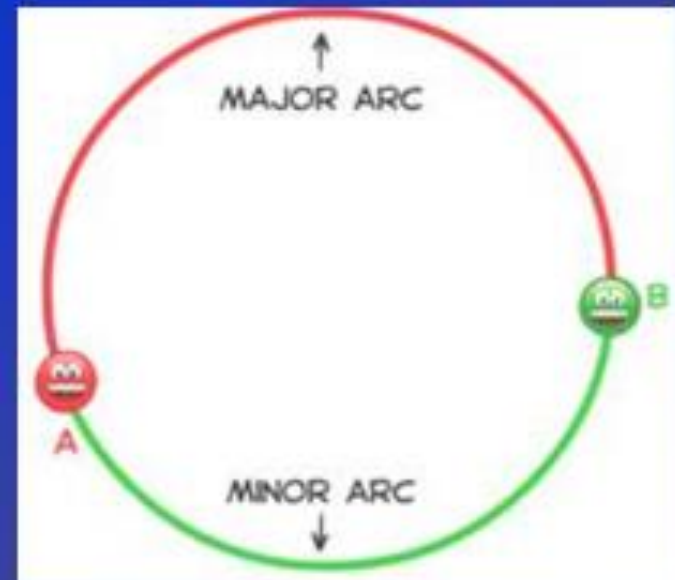
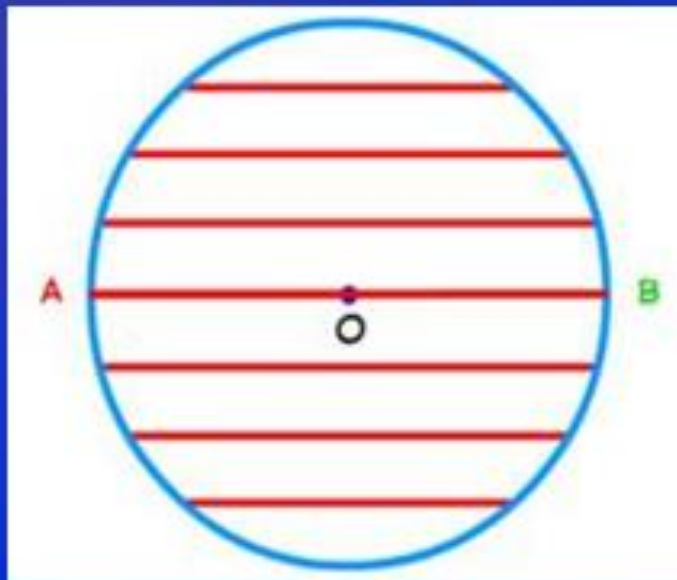
Segments:

A **chord** divides a **circular area** into two parts called **segments** they are **major segment** and **minor segments**.

Sector:

The region between two **radii** of a **circle** and any of the **arcs** between them is called a **sector**.

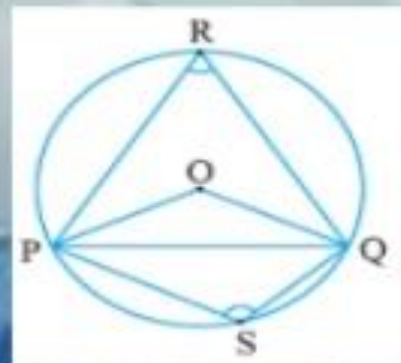
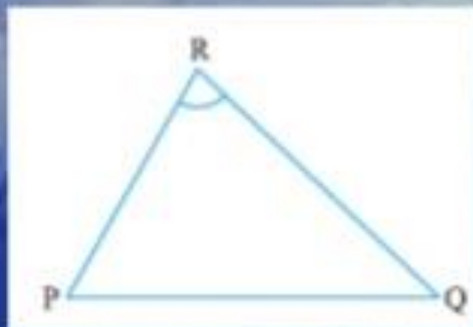
The **diameter** of a **circle** divides it into two **equal segments**



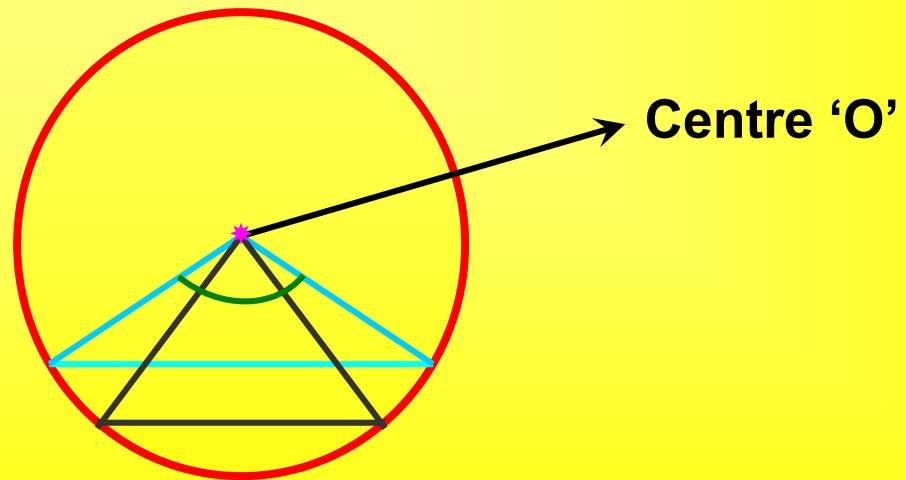
ANGLE SUBTENDED BY A CHORD AT A POINT.

We take a line segment **PQ** and a point **R** not on the line containing **PQ**. Join **PR** and **QR**. Then $\angle PRQ$ is called the angle subtended by the line segment **PQ** at the point **R**.

$\angle POQ$ is the angle subtended by the chord **PQ** at the centre **O**, $\angle PRQ$ and $\angle PSQ$ are respectively the angles subtended by **PQ** at points **R** and **S** on the major and minor arcs **PQ**.



1. Let AB be any chord of the circle with centre O.
2. Then $\angle AOB$ is called the angle subtended by the chord at the centre of the circle.
3. As the chord moves away from the centre, its length and angle subtended by it at the centre decreases and,
4. If it moves closer to the centre its length and angle subtended by it at the centre increases.



THEOREM 1

To Prove: Equal chords of a circle subtend equal angles at the centre.

Proof : We are given two equal chords **AB** and **CD** of a circle with centre **O**. We want to prove that:

$$\angle AOB = \angle COD.$$

In triangles **AOB** and **COD**,

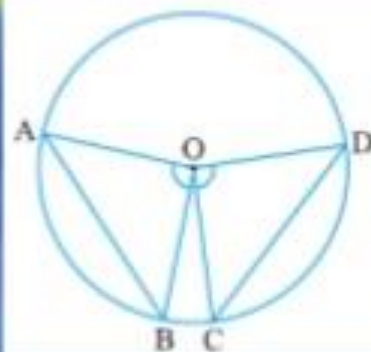
$$OA = OC \text{ (Radii of a circle)}$$

$$OB = OD \text{ (Radii of a circle)}$$

$$AB = CD \text{ (Given)}$$

Therefore, $\Delta AOB \cong \Delta COD$ (SSS rule)

This gives $\angle AOB = \angle COD$ (CPCT)



THEOREM 2

If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

Theorem 2 is converse of the Theorem 1.

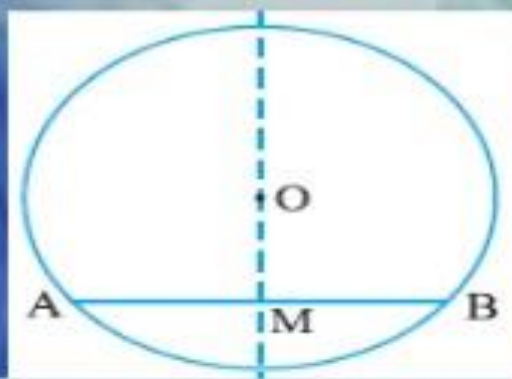
PERPENDICULAR FROM THE CENTRE TO A CHORD

Let there be a circle with **O** as center and let **AB** one of its chord.

Draw a perpendicular through **O** cutting **AB** at **M**.

Then, $\angle OMA = \angle OMB = 90^\circ$ or **OM** is perpendicular to **AB**.

We conclude that the perpendicular from the center of a circle to a chord bisects the chord.

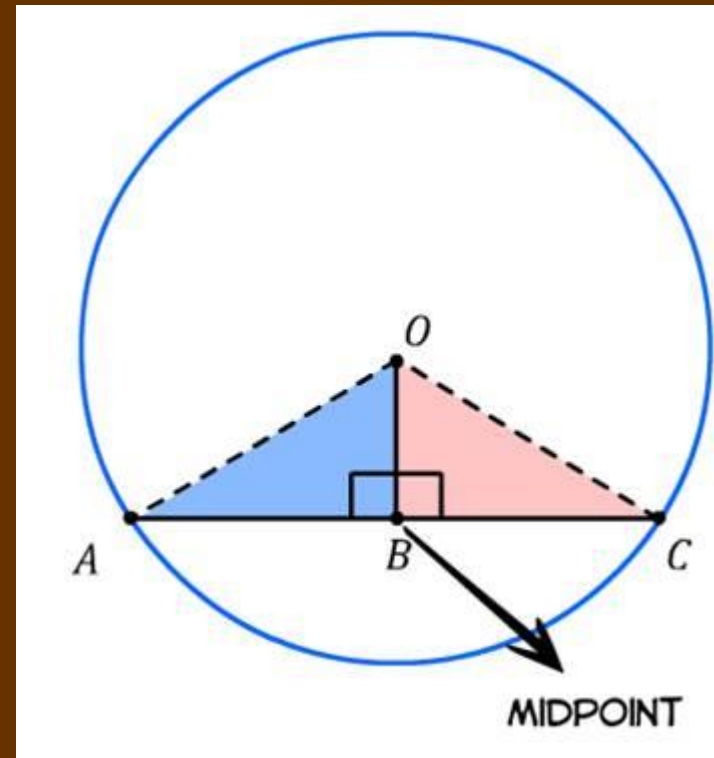


Theorem:

The **perpendicular** from the **centre of a circle** to a chord **bisects** the chord.

Theorem:

The **line drawn** from the **centre of a circle** to bisect a chord is **perpendicular** to the chord



THEOREM 3

TO PROVE: The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

PROOF: Let **AB** be a chord of a circle with centre **O** and **O** is joined to the mid-point **M** of **AB**. We have to prove that **OM** $\perp$ **AB**. Join **OA** and **OB**.

In triangles **OAM** and **OBM**,

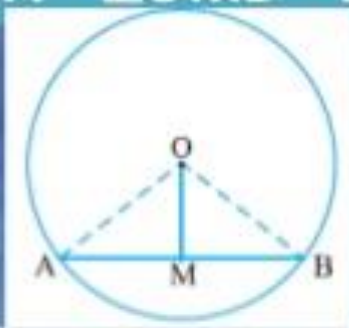
OA = OB (Given)

AM = BM (Given)

OM = OM (Common)

Therefore, **$\Delta OAM \cong \Delta OBM$** (SSS)

This gives **$\angle OMA = \angle OMB = 90^\circ$** (CPCT)



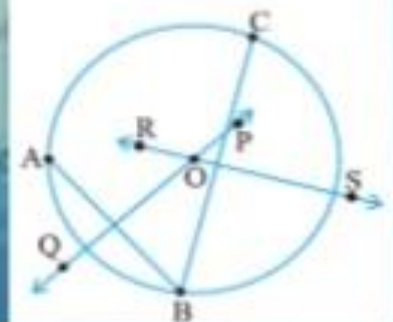
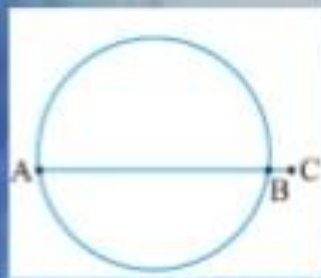
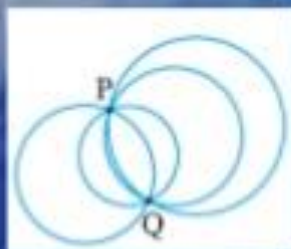
CIRCLE THROUGH THREE POINTS

Infinite number of circle can be drawn through one or two points (Fig 1 And 2) .

In case there are three collinear points, then circle can be drawn but the third point will lie outside the circle (Fig 3).

Let us take three points **A**, **B** and **C**, which are not on the same line (Fig 4).

Draw perpendicular bisectors of **AB** and **BC**, **PQ** and **RS** respectively. Let these perpendicular bisectors intersect at one point **O** (Fig 5).



CONCLUSION

O lies on the perpendicular bisector **PQ** of **AB**, we have **OA = OB**.

Similarly, as **O** lies on the perpendicular bisector **RS** of **BC**, we get **OB = OC**.

So **OA = OB = OC**, which means that the points **A**, **B** and **C** are at equal distances from the point **O**.

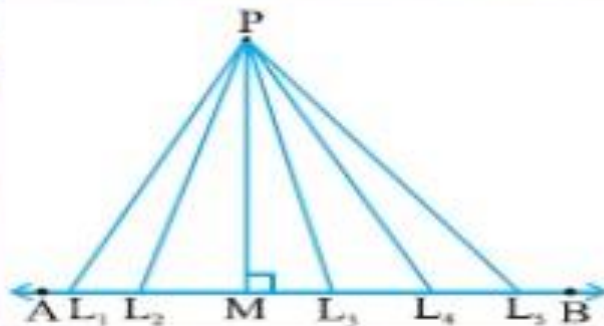
We conclude that there is one and only one circle passing through three given non-collinear points.

EQUAL CHORDS AND THEIR DISTANCES FROM THE CENTRE

Let **AB** be a line and **P** be a point. Since there are infinite numbers of points on a line, if we join these points to **P**, we will get infinitely many line segments **PL₁**, **PL₂**, **PM**, **PL₃**, **PL₄**, etc.

Out of these line segments, the perpendicular from **P** to **AB**, namely **PM** will be the least.

We conclude that the length of the perpendicular from a point to a line is the distance of the line from the point.

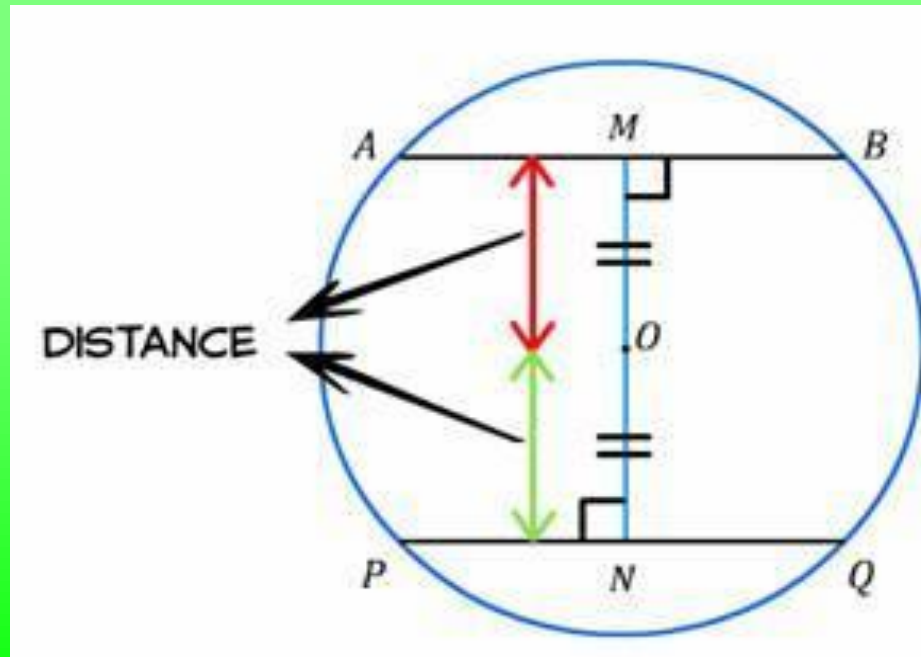


Theorem:

Equal chords of a circle are **equidistant** from the **centre** of the circle.

Theorem:

Chords equidistant from the centre of a circle are **equal in length**.



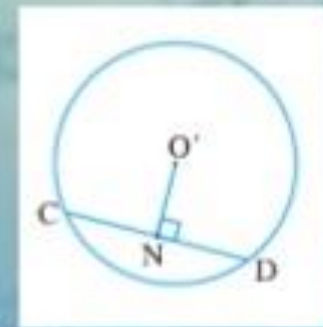
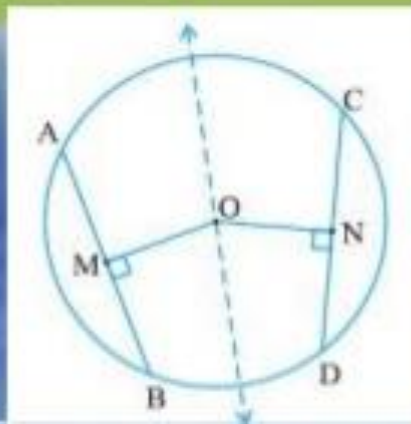
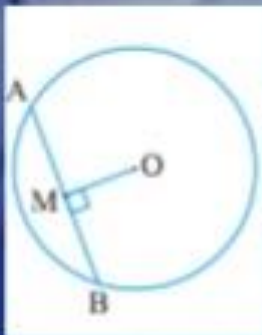
THEOREM 4

TO PROVE: Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centers).

PROOF: Take a circle of any radius.

Draw two equal chords **AB** and **CD** of it and also the perpendiculars **OM** and **ON** on them from the centre **O**. Divide the figure into two so that **D** falls on **B** and **C** falls on **A**.

We observe that **O** lies on the crease and **N** falls on **M**. Therefore, **OM = ON**.



THEOREM 5

Chords equidistant from the centre of a circle are equal in length.

Theorem 5 is converse of the Theorem 4

ANGLE SUBTENDED BY AN ARC OF A CIRCLE

The end points of a chord other than diameter of a circle cuts it into two arcs – one major and other minor.

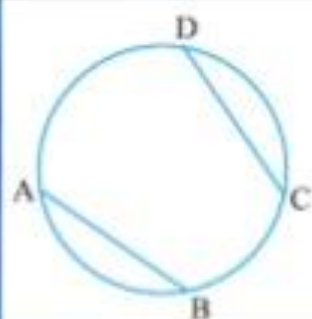
If we take two equal chords,

They are more than just equal in length. They are congruent in the sense that if one arc is put on the other, without bending or twisting, one superimposes the other completely.

We can verify this fact by cutting the arc, corresponding to the chord **CD** from the circle along **CD** and put it on the corresponding arc made by equal chord **AB**. We will find that the arc **CD** superimpose the arc **AB** completely

This shows that equal chords make congruent arcs and conversely congruent arcs make equal chords of a circle.

We conclude that if two chords of a circle are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent, then their corresponding chords are equal.



The angle subtended by an arc at the centre is defined to be angle subtended by the corresponding chord at the centre in the sense that the minor arc subtends the angle and the major arc subtends the reflex angle.

The angle subtended by the minor arc **PQ** at **O** is $\angle POQ$ and the angle subtended by the major arc **PQ** at **O** is reflex angle **POQ**.

We conclude that congruent arcs (or equal arcs) of a circle subtend equal angles at the centre.



THEOREM 6

TO PROVE: The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

PROOF: Given an arc PQ of a circle subtending angles POQ at the centre O and PAQ at a point A on the remaining part of the circle.

Consider the three different cases arc PQ is minor (Fig 1), arc PQ is a semicircle (Fig 2) and in arc PQ is major (Fig 3).

Let us begin by joining AO and extending it to a point B .

In all the cases, $\angle BOQ = \angle OAQ + \angle AQO$ because an exterior angle of a triangle is equal to the sum of the two interior opposite angles.

In $\triangle OAQ$,

$OA = OQ$ (Radii of a circle)

Therefore, $\angle OAQ = \angle OQA$

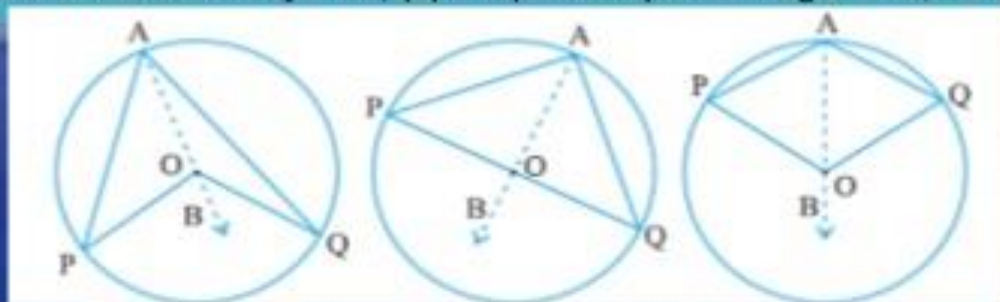
This gives $\angle BOQ = 2 \angle OAQ$ (1)

Similarly, $\angle BOP = 2 \angle OAP$ (2)

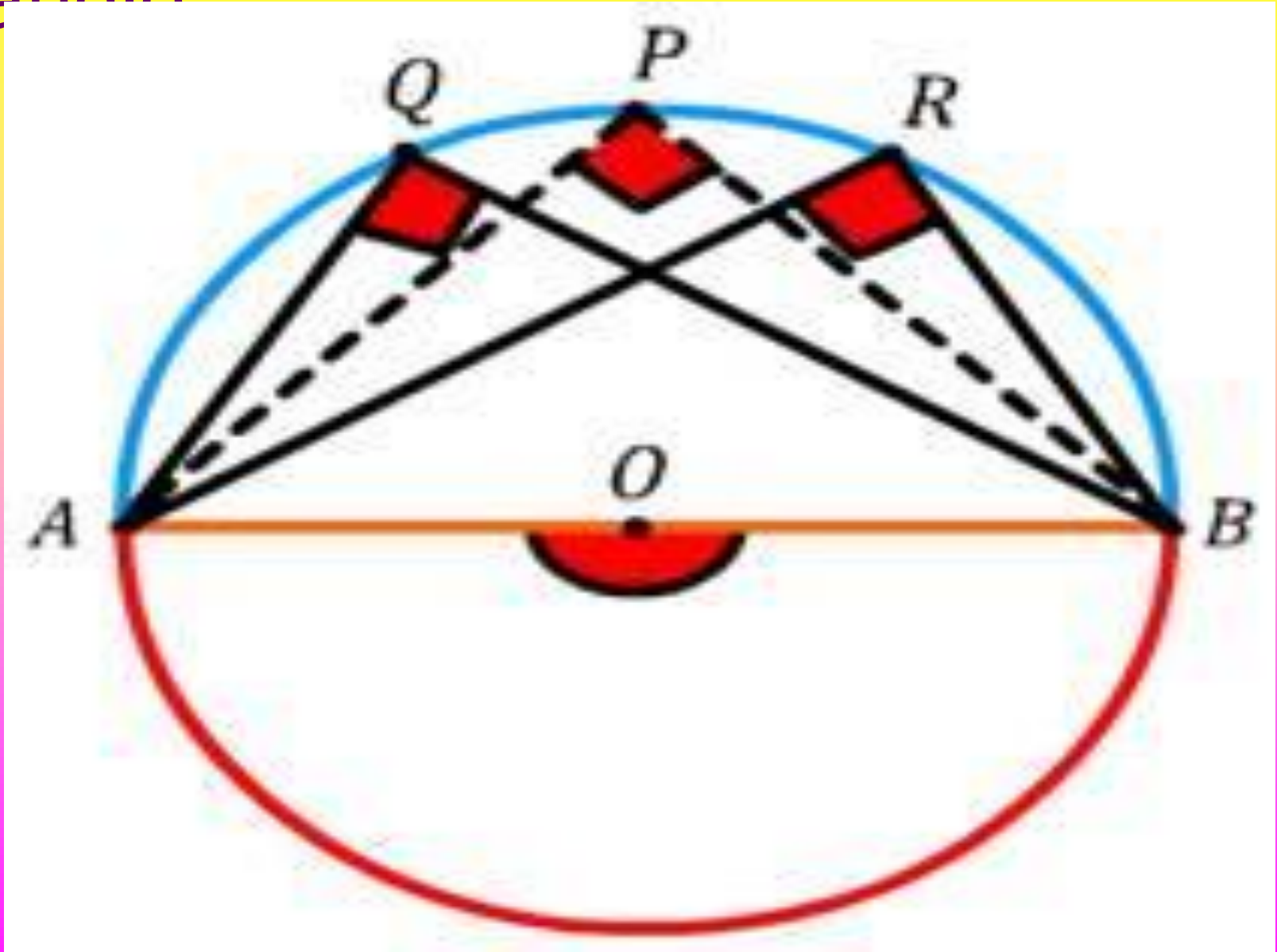
From (1) and (2), $\angle BOP + \angle BOQ = 2(\angle OAP + \angle OAQ)$

This is the same as $\angle POQ = 2 \angle PAQ$ (3)

For the case (iii), where PQ is the major arc, (3) is replaced by reflex angle $POQ = 2 \angle PAQ$.



Angle in a Semi-Circle is a right angle



THEOREM 7

TO PROVE: Angles in the same segment of a circle are equal.

PROOF: According to Theorem 6, $\angle PAQ = \frac{1}{2} \angle POQ = \frac{1}{2} \times 180^\circ = 90^\circ$.

If we take any other point **C** on the semicircle, again we get that $\angle PCQ = 90^\circ$ (Fig 1).

Suppose we join points **P** and **Q** and form a chord **PQ** in the figures used in Theorem 6. Then $\angle PAQ$ is also called the angle formed in the segment **PAQP**.

A can be any point on the remaining part of the circle. So if we take any other point **C** on the remaining part of the circle,

$$\angle POQ = 2 \angle PCQ = 2 \angle PAQ$$

Therefore, $\angle PCQ = \angle PAQ$.



CYCLIC QUADRILATERALS

A quadrilateral **ABCD** is called **cyclic** if all the four vertices of it lie on a circle.

If we observe these cyclic quadrilateral, we find that $\angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ$, neglecting the error in measurements.

We conclude that the sum of either pair of opposite angles of a cyclic quadrilateral is 180° .

In converse of this,

We conclude that if the sum of a pair of opposite angles of a quadrilateral is 180° , the quadrilateral is cyclic.

WHAT WE CONCLUDE?

We Conclude that,

A circle is the collection of all points in a plane, which are equidistant from a fixed point in the plane.

Equal chords of a circle (or of congruent circles) subtend equal angles at the centre.

If the angles subtended by two chords of a circle (or of congruent circles) at the centre (corresponding centers) are equal, the chords are equal.

The perpendicular from the centre of a circle to a chord bisects the chord.

The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

There is one and only one circle passing through three non-collinear points.

Equal chords of a circle (or of congruent circles) are equidistant from the centre (or corresponding centers).

Chords equidistant from the centre (or corresponding centers) of a circle (or of congruent circles) are equal.

If two arcs of a circle are congruent, then their corresponding chords are equal and conversely if two chords of a circle are equal, then their corresponding arcs (minor, major) are congruent.

Congruent arcs of a circle subtend equal angles at the centre.

The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Angles in the same segment of a circle are equal.

Angle in a semicircle is a right angle.

If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.

The sum of either pair of opposite angles of a cyclic quadrilateral is 180° .

If sum of a pair of opposite angles of a quadrilateral is 180° , the quadrilateral is cyclic.