

Angle Sum Property of a Quadrilateral

➤ *The sum of the angles of a quadrilateral is 360° .*

Given: ABCD is a quadrilateral,

To Prove:

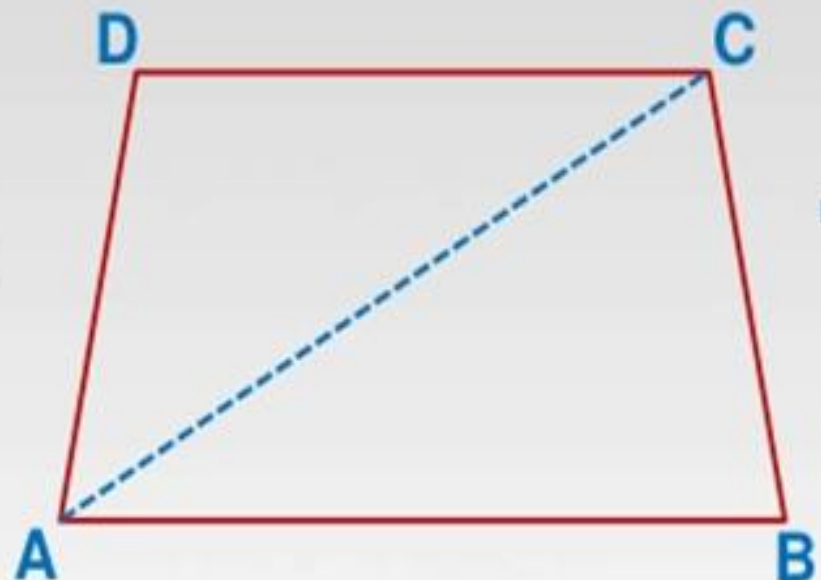
$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

Construction: Draw AC be a diagonal (see Fig.)

Proof:

In $\triangle ADC$ we know that

$$\angle DAC + \angle ACD + \angle D = 180^\circ \dots\dots\dots(i)$$



Angle Sum Property of a Quadrilateral

Similarly, in $\triangle ABC$,

$$\angle CAB + \angle ACB + \angle B = 180^\circ \dots\dots\dots (ii)$$

Adding (i) and (ii), we get

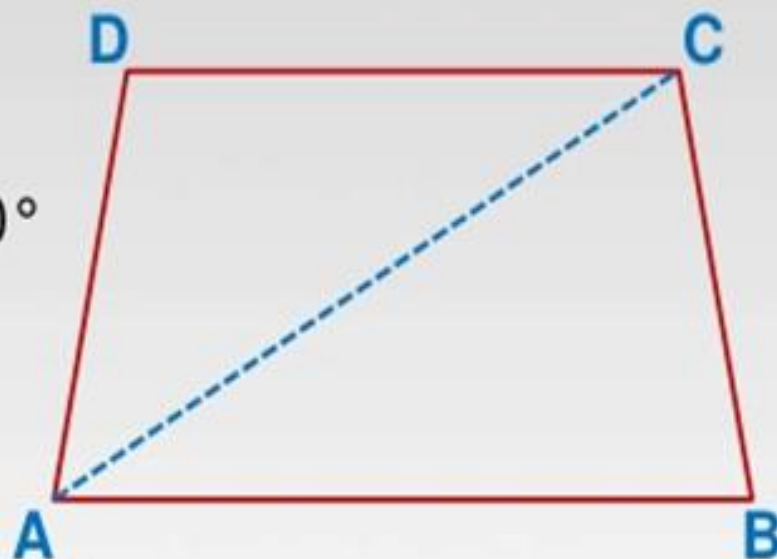
$$\begin{aligned} &\angle DAC + \angle ACD + \angle D + \\ &\angle CAB + \angle ACB + \angle B \\ &= 180^\circ + 180^\circ = 360^\circ \end{aligned}$$

Also, $\angle DAC + \angle CAB = \angle A$

and $\angle ACD + \angle ACB = \angle C$

So, $\angle A + \angle D + \angle B + \angle C = 360^\circ$

i.e., **the sum of the angles of a quadrilateral is 360° .**



Properties of a Parallelogram

➤ *A diagonal of a parallelogram divides it into two congruent triangles.*

Given: ABCD be a parallelogram and AC be a diagonal
AC divides into two
triangles, $\triangle ABC$ & $\triangle CDA$.

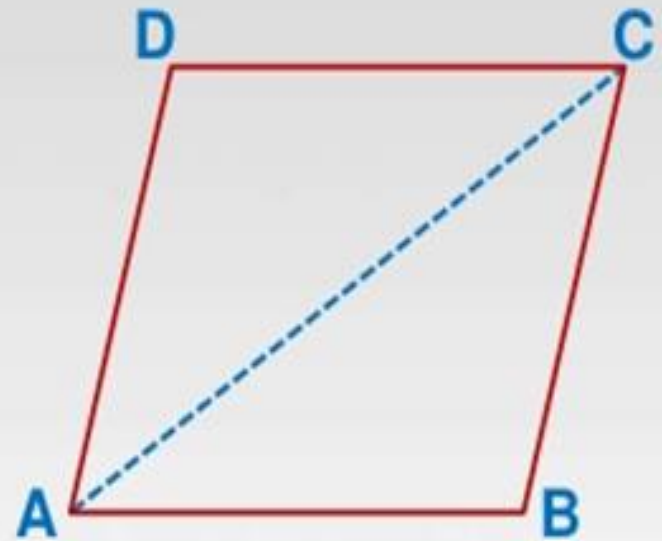
To prove: $\triangle ABC \cong \triangle CDA$

Proof: In $\triangle ABC$ and $\triangle CDA$

We know that $BC \parallel AD$ and

AC is a transversal

So, $\angle BCA = \angle DAC$ (Pair of alternate angles)



Properties of a Parallelogram

Also, $AB \parallel DC$ and AC is a transversal

So,

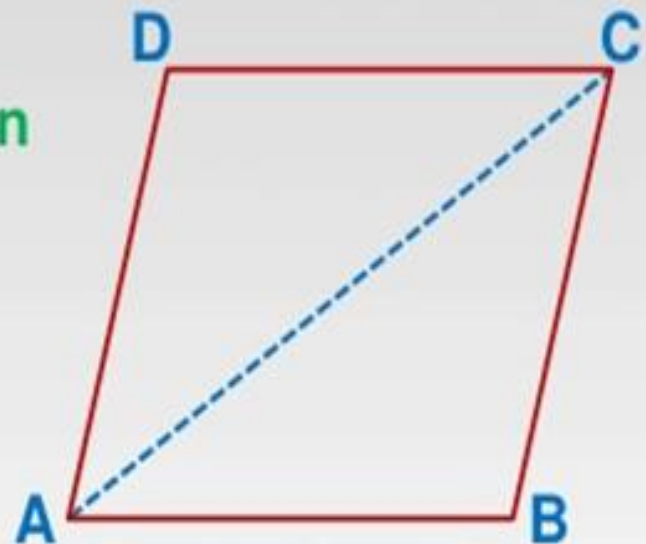
$\angle BAC = \angle DCA$ Pair of alternate angles

And

$AC = CA$ Common

So, by the Rule of ASA

$\triangle ABC \cong \triangle CDA$



Diagonal AC divides
parallelogram ABCD into two congruent triangles

Properties of a Parallelogram

➤ *If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.*

Given: Quadrilateral ABCD, sides AB and CD is equal and also $AD = BC$

To Prove: ABCD is a Parallelogram

Construction: Draw diagonal AC

Proof: In $\triangle ABC$ & $\triangle CDA$

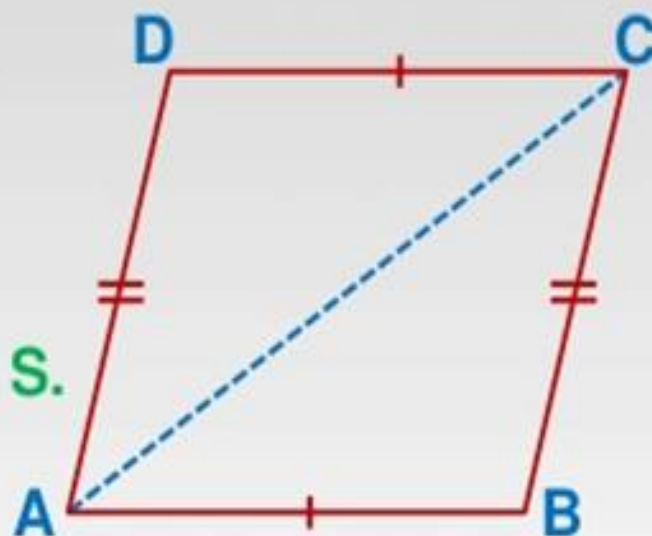
$\triangle ABC \cong \triangle CDA$ **S. S. S.**

So, $\angle BAC = \angle DCA$

and $\angle BCA = \angle DAC$,

So that the line $AB \parallel DC$ and $AD \parallel BC$,

So that ABCD is a parallelogram



Properties of a Parallelogram

➤ *If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.*

Given: Quadrilateral ABCD, diagonal AC and BD

Where $OA = OC$ and $OB = OD$

To Prove: ABCD is a parallelogram

So, $\triangle AOB \cong \triangle COD$ **SAS**

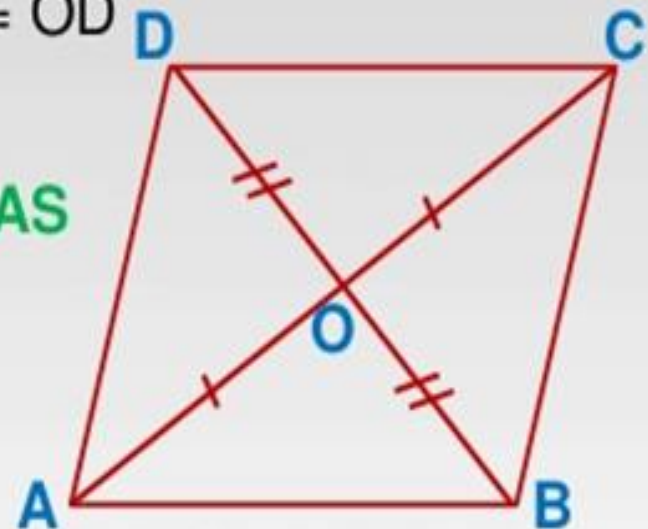
Therefore,

$$\angle ABO = \angle CDO$$

From this, we get $AB \parallel CD$

Similarly, $BC \parallel AD$

Therefore ABCD is a parallelogram.



Properties of a Parallelogram

- A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.

Given: ABCD be a quadrilateral in which

$$AB = CD \text{ \& } AB \parallel CD.$$

Construction: Draw diagonal AC

Proof: In $\triangle ABC$ & $\triangle CDA$

$$AC = AC \quad \dots\dots\dots \text{same}$$

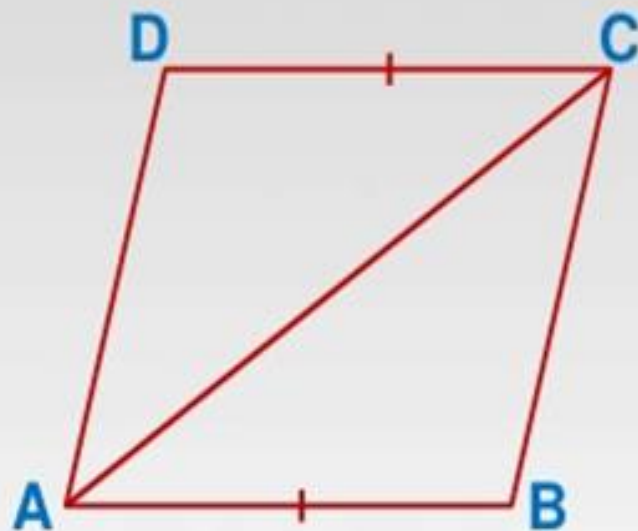
$$AB = CD \quad \dots\dots\dots \text{Given}$$

$$\angle BAC = \angle ACD \quad \dots\dots\dots \text{Given}$$

$$\triangle ABC \cong \triangle CDA \quad \dots\dots\dots \text{by SAS congruence rule.}$$

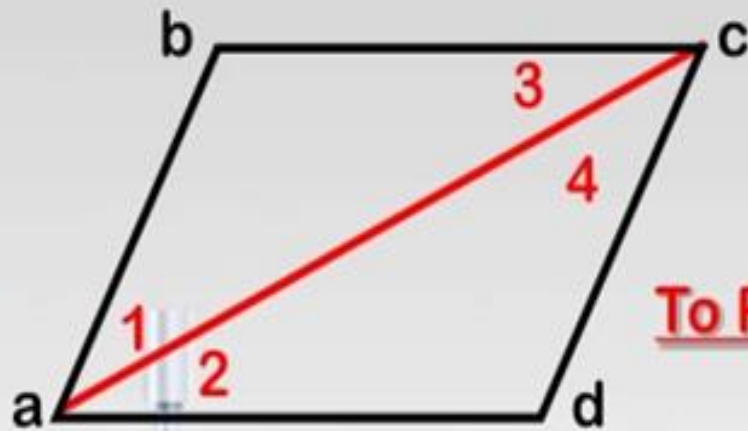
So,

$$BC \parallel AD$$



Properties of a Parallelogram

The opposite sides and opposite sides of a parallelogram are respectively equal in measure.



Given: Parallelogram abcd

To Prove: $|ab| = |cd|$ and $|ad| = |bc|$
and $\angle abc = \angle adc$

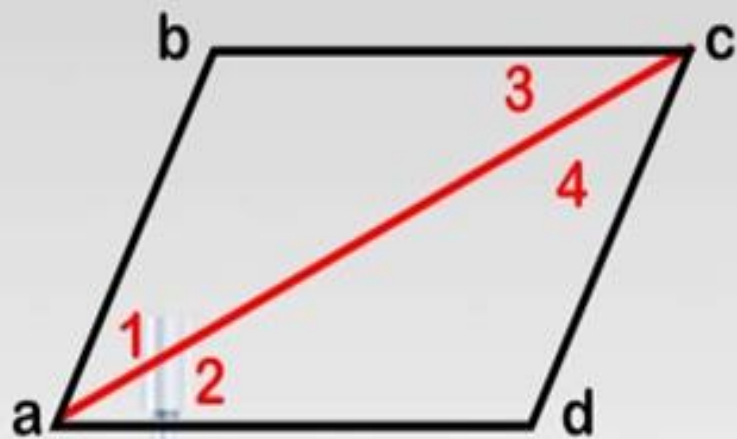
Construction:

Draw the diagonal $|ac|$



Properties of a Parallelogram

The opposite sides and opposite sides of a parallelogram are respectively equal in measure.



Proof: In the triangle abc and the triangle adc

$\angle 1 = \angle 4$ Alternate angles

$\angle 2 = \angle 3$ Alternate angles

$|ac| = |ac|$ Common

$\Rightarrow$ The triangle abc is congruent to the triangle adc **ASA = ASA.**

$\Rightarrow |ab| = |cd|$ and $|ad| = |bc|$

and $\angle abc = \angle adc$

Mid-point Theorem

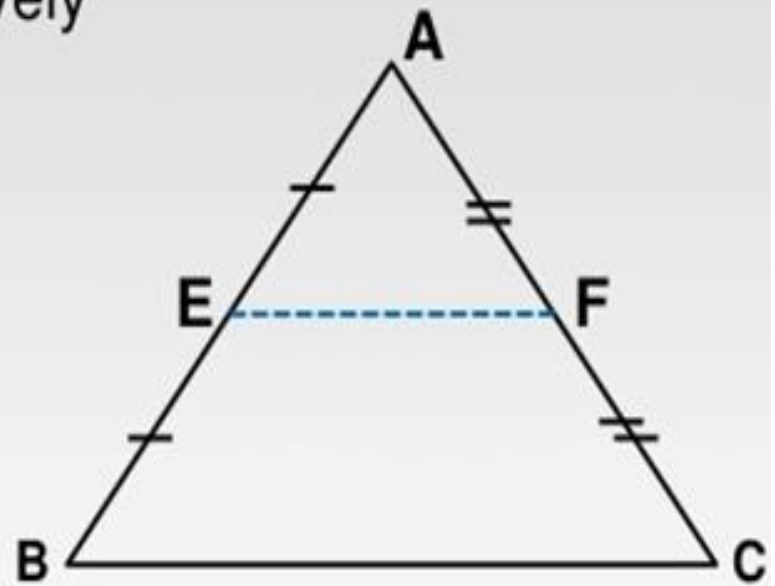
➤ *The line which joins the midpoints of two sides of a triangle is parallel to the third side and is equal to half of the length of the third side*

Given: In $\triangle ABC$ where E and F are mid-points of AB and AC respectively

To Prove:

(i) $EF \parallel BC$

(ii) $EF = \frac{1}{2} BC$



Mid-point Theorem

Construction: Draw $DC \parallel AB$ to meet EF produced at D .

Proof: In $\triangle AEF$ & $\triangle CDF$

$$\angle EFA = \angle CFD \quad \dots\dots\dots \text{V. A. A.}$$

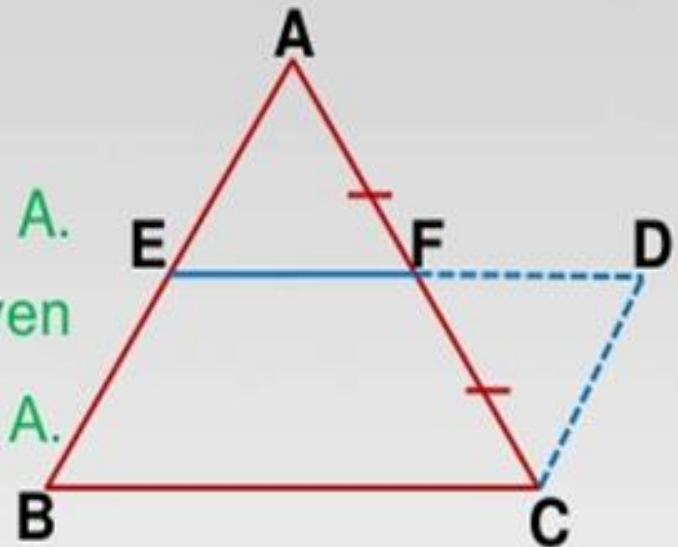
$$AF = CF \quad \dots\dots\dots \text{Given}$$

$$\angle AEF = \angle CDF \quad \dots\dots\dots \text{A. I. A.}$$

$$\triangle AEF \cong \triangle CDF \quad \dots\dots\dots \text{A. S. A. Rule}$$

So, $EF = DF$ and $BE = AE = DC$

And $DC \parallel BE$, the quadrilateral $BCDE$



Mid-point Theorem

Therefore, BCDE is a parallelogram
from the Properties of parallelogram
gives $EF \parallel BC$.

----- (i) Proved

$$BC = DE = EF + FD$$

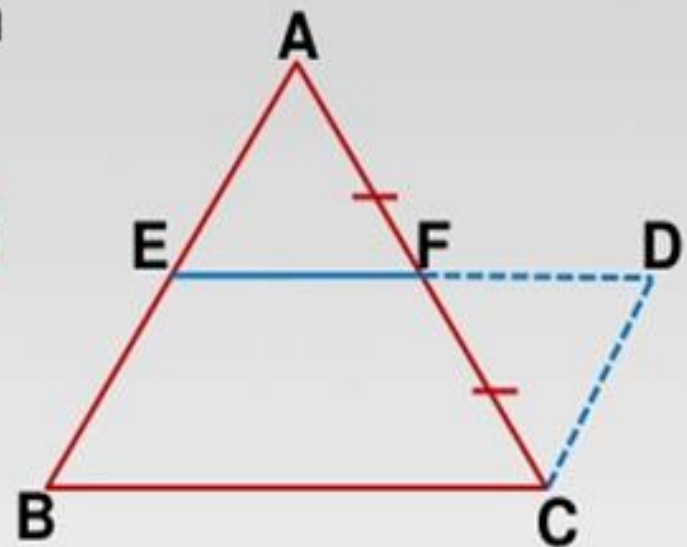
We Know that $EF = DE$

So, $BC = EF + EF$

Or, $BC = 2 EF$

Or ,

$$EF = \frac{1}{2} BC \text{ ----- (ii) Proved}$$

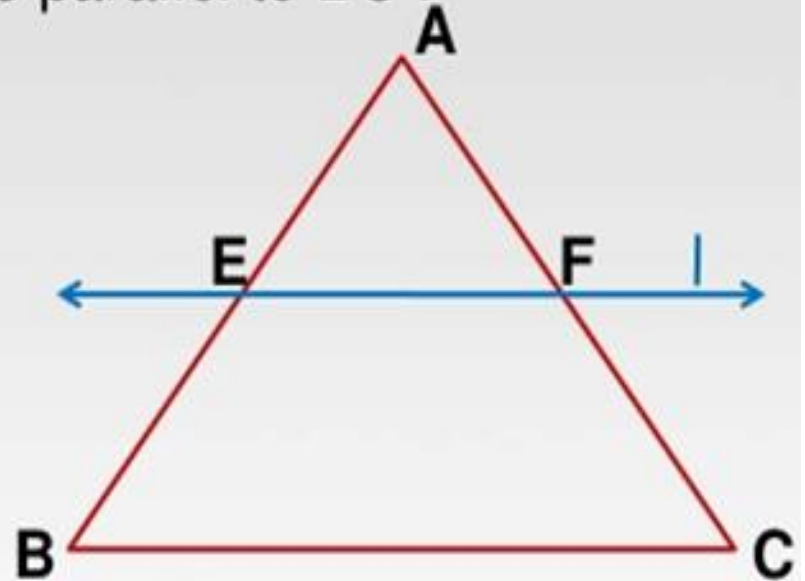


Mid-point Theorem

- *The line through the midpoint of one side of a triangle when drawn parallel to a second side bisects the third side.*

Given: In $\triangle ABC$ where E is the mid-point of AB , line ℓ is passing through E and is parallel to BC .

To Prove: $AF = FC$



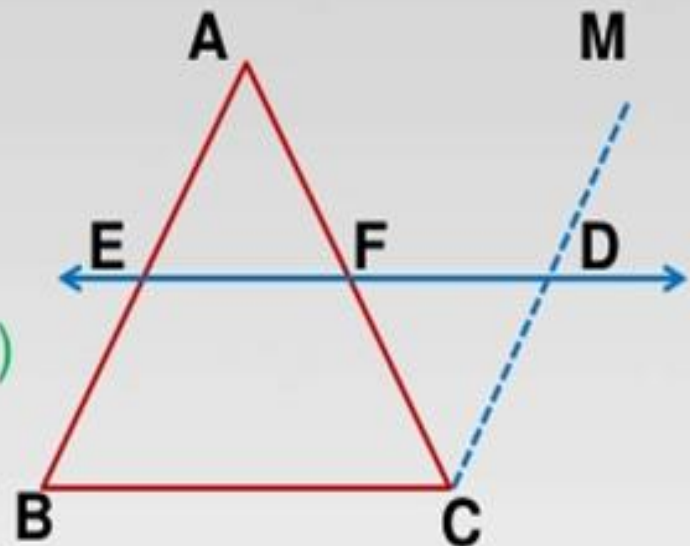
Mid-point Theorem

Construction: Draw a line $CM \parallel AB$ to meet EF produced at D .

Proof:

$CM \parallel AB$ (Const.)

$EF \parallel BC$ (Given)



So,
Quadrilateral BCDE is a parallelogram, then
 $BE = CD$

Now In $\triangle AEF$ and $\triangle CDF$.

$$CD = BE = AE ,$$

Mid-point Theorem

$$\angle CFD = \angle EFA \quad \dots\dots\dots(\text{Vertically opposite})$$

$$\angle DCF = \angle EAF \quad \dots\dots(\text{Alternate})$$

So,

$$\triangle AEF \cong \triangle CDF \quad \dots(\text{ASA Rule})$$

$$AF = CF$$

Proved .

