

**Title of my
Course**

Surface Areas and Volumes

Cube

Sphere

Cuboid

Hemisphere

Cylinder

Cone

Target Audience

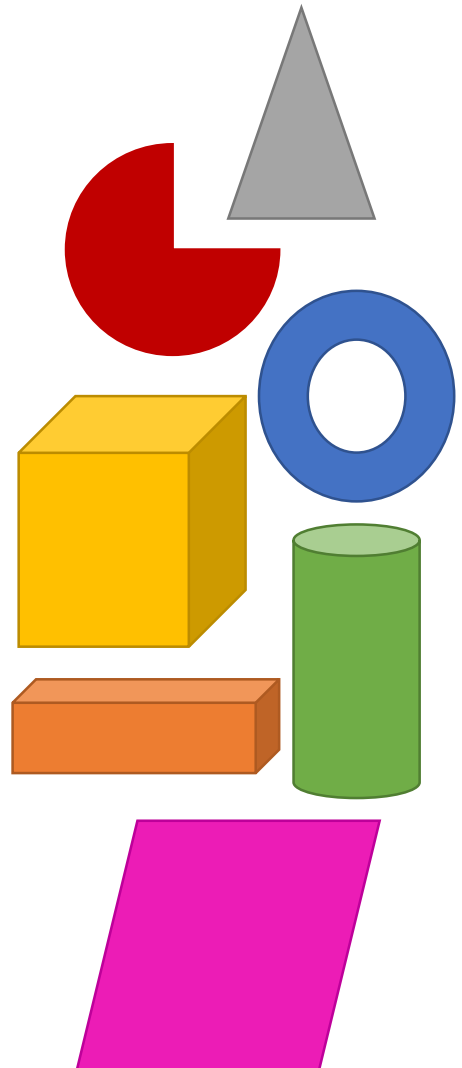
- Students who are currently in class 9th and 10th.
- Students who wish to have a recap of the topic.

Reference

- NCERT Mathematics textbook for class 9th.

Learning Objective

- Students shall be able to distinguish between plane figures and solid figures.
- With the help of 3D nets, students will be able to understand different kinds of solids with high proficiency.
- Taking examples from our day to day lives, students would learn how to derive formulae for calculating surface areas and volumes for different solid figures.



Introduction

Figures

Plane Figures

- Includes concepts like perimeters, areas and methods to find them.
- These are 2-dimensional (have only length and breath or radius).
- Squares, rectangles circles, triangles etc.

Solid Figures

- Includes concepts like total surface area , curved surface area, volume of solids.
- These are 3-dimensional.
- These are obtained when plane figures are piled up thereby giving the third dimension i.e. height.
- Cubes, cuboids, cylinders, pyramids, spheres etc.

Cuboid

Objective- To understand the concept of cuboid including its various components.

Basics

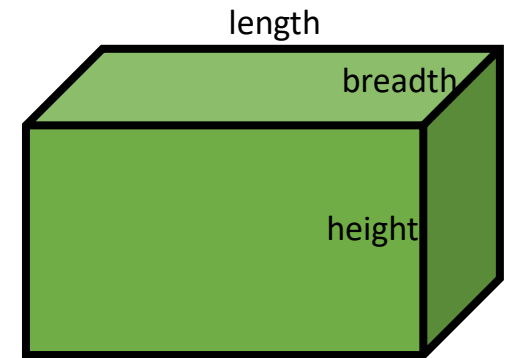
Area of Rectangle = length of rectangle $\times$ breadth of rectangle

How to obtain Cuboid?

Stacking up identical rectangles vertically up, we get a 3D figure called Cuboid which has 3 components- length, breadth and height.

What are Faces of a cuboid?

The outer surface of a cuboid is made of 6 rectangular regions called faces of cuboid.



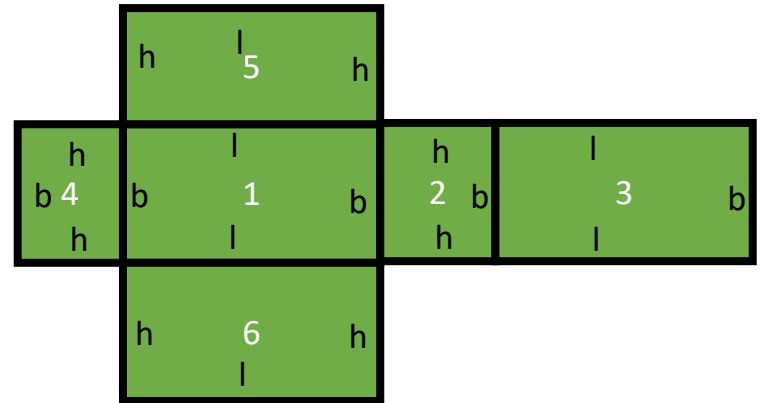
Surface area of Cuboid

Total Surface area

- Sum of area of 6 faces of the cuboid.
- Area of each face is calculated using the formula of area of rectangle.
- Formula is $2 \times (l \times b + b \times h + h \times l)$ where l = length, b = breadth, h = height.

Lateral Surface area

- Sum of the areas of 4 faces, leaving the top and bottom of the cuboid.
- Formula is $2 \times (b \times h + h \times l)$.



Cube and its Surface area

Objective- To understand the concept of a cube and its properties.

What is a cube?

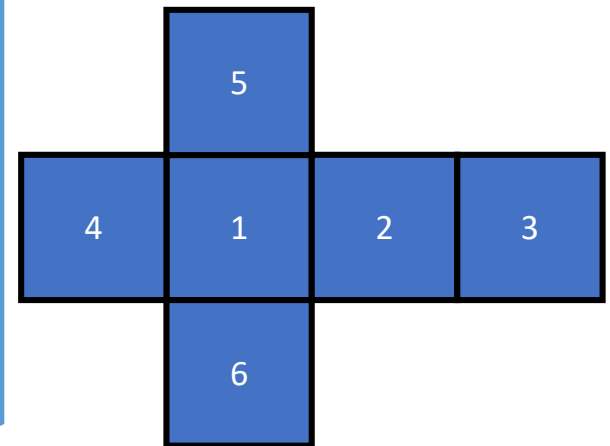
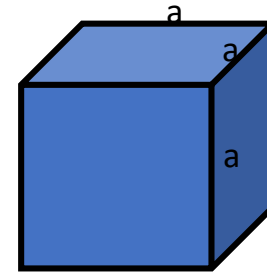
- If length, breadth and height of a cuboid are all equal in measurement, then the solid figure obtained is a cube.

Total Surface area

- Let $l = b = h = a$ units.
- Using the formula of total surface area of a cuboid, the formula for cube is given by $2 \times (a^2 + a^2 + a^2) = 2 \times 3 a^2 = 6 a^2$

Lateral Surface area

- Again, using the formula for lateral surface area of cuboid, the formula for cube is given by $2 \times (a^2 + a^2) = 4 a^2$



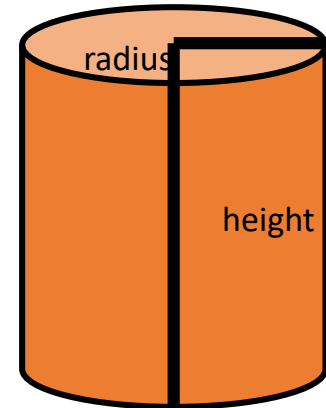
Cylinder

Objective- To learn how to obtain a cylinder and calculate its surface area.

Right Circular Cylinders

Stacking up a number of identical circular sheets vertically, we get a solid figure called right circular cylinder.

In this solid, there is a circular base of a given radius and its height is perpendicular to the base i.e. height and radius forms a right angle.



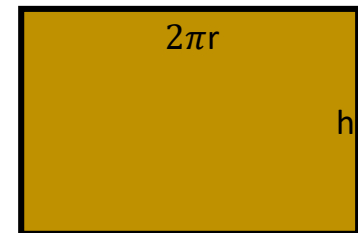
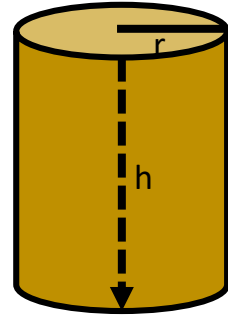
Surface area of Cylinder

**Curved
Surface area**

- On opening a cylinder of a given radius r and height h , we get a rectangle whose breadth = height of the cylinder and length = circumference of the circular base of cylinder i.e. $2\pi r$
- So, curved surface area = area of this rectangle = length $\times$ breadth = $2\pi r \times h = 2\pi rh$

**Total
Surface area**

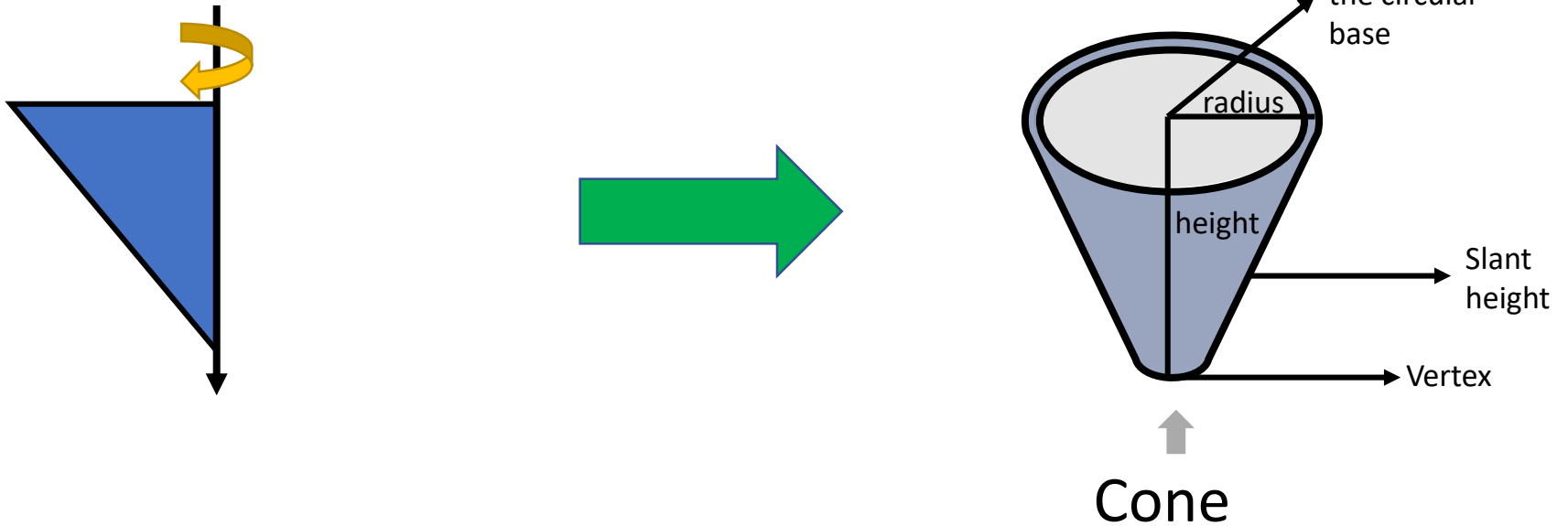
- Total surface area = Curved surface area + Area of the top and bottom circular base of the cylinder = $2\pi rh + \pi r^2 + \pi r^2 = 2\pi r (h + r)$



Cone

Objective- To learn the various aspects of a cone and derive its surface area.

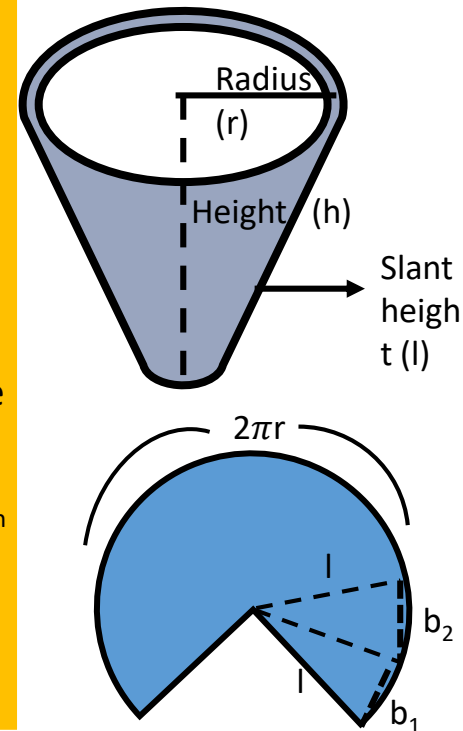
What is a cone?



Surface area of a Cone

**Curved
Surface area**

- On opening a cone along its slant height, we get a part of a circle.
- In this circle, the radius = slant height of the cone and the curved surface area of the cone = area of this part of the circle. Also, we observe that the boundary of this part = circumference of the circular base of cone = $2\pi r$
- Now, dividing this part into n small triangles so that the height of each of these triangle is = slant height of the cone (which is the radius of this part of circle).
- Area of each of these triangles is given by $\frac{1}{2} \times b_n \times h_n$ and the total area of this part of circle = sum of all these n triangles
- Curved Surface area = $\frac{1}{2} \times b_1 \times h_1 + \dots + \frac{1}{2} \times b_n \times h_n$ where $h_n = l$ for all n
 $= \frac{1}{2} \times l \times (b_1 + \dots + b_n)$ where this sum of all b_n = boundary of this part of circle = $2\pi r$
- Therefore, Curved surface area = $\frac{1}{2} \times l \times 2\pi r = \pi r l$



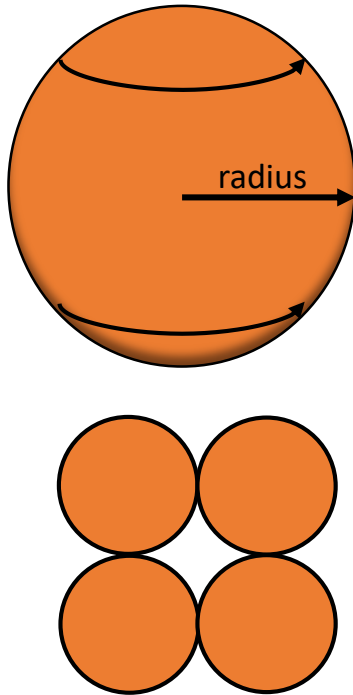
**Total Surface
area**

- Total surface area of the cone = Curved surface area + Area of the circular base = $\pi r l + \pi r^2 = \pi r (l + r)$

Relationship between l , r and h is given by: Using Pythagoras theorem, $l^2 = r^2 + h^2$

Sphere

Objective- To understand the concept of a sphere and its properties.



- A sphere is a solid figure which is made up of all points in the space, which lie at a constant distance called the radius, from a fixed point called the centre of the sphere.
- Sphere is like the surface of a ball.
- There is only one face in the surface of the sphere which is curved. Hence, the curved surface area and total surface area of the sphere are same.
- Consider, winding a string on the surface of a ball of radius r , starting from north pole of this ball by taking support of a nail at the top and continuing winding till the south pole of this ball. Then on a sheet of paper, draw 4 circles of radius r and fill these circles with the thread wound around the ball. On doing so we observe that the string which had completely covered the surface of the ball has also completely filled the 4 circles where area of each of these circles = πr^2
- **Surface area of sphere = $4 \pi r^2$ where r is the radius of the sphere.**

Hemisphere

Objective- To learn about hemisphere, its surface area and its relations with a sphere.

What is a hemisphere?

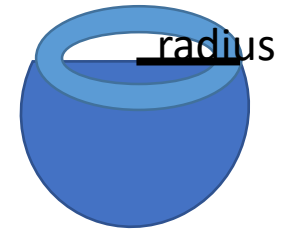
Half of a sphere is called a hemisphere

Curved Surface area

$$\frac{\text{Surface area of sphere}}{2} = \frac{4\pi r^2}{2} = 2\pi r^2$$

Total Surface area

Curved surface area + area of the circular base
 $= 2\pi r^2 + \pi r^2 = 3\pi r^2$



Volume v/s Capacity

Objective- To understand what do we mean by volume and capacity of different shapes and figures.

**Volume
of
container**

- It is the measure of the space the container occupies.

**Capacity
of
container**

- It is the volume of substance its interior can accommodate.

Points to remember-

- When a number of identical plane figures (i.e. 2D figures) are stacked up vertically to a height say h , then measure of the space occupied by the resultant 3D figure = the area of each plane region $\times$ height .
- Volume or capacity is measured in cubic units.

Volume

Formulae Sheet

- Volume of Cuboid = $l \times b \times h$
- Volume of Cube = $a \times a \times a = a^3$
- Volume of Cylinder = $\pi r^2 \times h = \pi r^2 h$
- Volume of Cone = $\frac{1}{3} \pi r^2 h$
- Volume of Sphere = $\frac{4}{3} \pi r^3$
- Volume of Hemisphere = $\frac{2}{3} \pi r^3$

Question

Question: The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of \$7.50 per m².

Answer: Calculating the total area to be white washed-

$$\begin{aligned} & \text{Total Surface area of the cuboid (room) – Area of the floor} \\ &= 2 \times (lb + bh + hl) - lb \quad \text{where } l = 5 \text{ m, } b = 4 \text{ m, } c = 3 \text{ m} \\ &= 2 \times (5 \times 4 + 4 \times 3 + 3 \times 5) - (5 \times 4) \\ &= 2 \times (20 + 12 + 15) - 20 \\ &= 2 \times (47) - 20 \\ &= 94 - 20 \\ &= 74 \text{ m}^2 \end{aligned}$$

Therefore, total area to be painted is 74 m².

$$\begin{aligned} \text{Now, total cost of white washing} &= \text{Total area to be white washed} \times \text{Rate} \\ &= 74 \times 7.50 \\ &= \$555 \end{aligned}$$

Therefore, the total cost for white washing is \$555.

Question

Question: If the curved surface area of a cylindrical tank is 88 cm^2 and height is 14 cm then find out the diameter of the base of this tank.

Answer: Let radius be r and height be h .

Curved surface area of cylinder = $2\pi rh$

Here, $h = 14$, Curved surface area = 88

Then, $2\pi r \times 14 = 88$

So, $r = \frac{88}{14 \times 2 \pi}$ taking $\pi = \frac{22}{7}$ we get, $r = 1$

Therefore, **radius = 1 cm**

And, diameter = $2 \times \text{radius} = 2 \times 1 = 2 \text{ cm}$

So, the **diameter is 2 cm.**

Question Source: NCERT

Question

Question: If diameter = 10.5 cm and slant height = 10 cm then find out the curved surface area of the cone. Also, find the cost of painting this area at the rate of \$10 per cm² .

Answer: Curved Surface area = $\pi r l$ where r = radius and l = slant height

Here, radius = $\frac{\text{diameter}}{2} = \frac{10.5}{2} = 5.25$ cm and $l = 10$ cm taking $\pi = \frac{22}{7}$

So, Curved surface area = $\frac{22}{7} \times 5.25 \times 10 = 165$ cm² .

Therefore, **curved surface area is 165 cm² .**

Now, the cost of painting = area to be painted x rate
= $165 \times 10 = 1650$

Therefore, **total cost to paint is \$1650.**

Question Source: NCERT

Question

Question: The hollow sphere, in which the circus motorcyclist performs his stunts, has a diameter of 7 m. Find the area available to him for riding.

Answer: Surface area of a sphere = $4\pi r^2$

Here, radius = $\frac{\text{diameter}}{2} = \frac{7}{2} = 3.5 \text{ m}$ taking $\pi = \frac{22}{7}$

Now, surface area = $4 \times \frac{22}{7} \times 3.5 \times 3.5 = 154 \text{ m}^2$.

Question Source: NCERT

Questions

Question: A wall of length 10 m was to be built across an open ground. The height of the wall is 4 m and thickness of the wall is 24 cm. If this wall is to be built up with bricks whose dimensions are 24 cm × 12 cm × 8 cm, how many bricks would be required?

Answer: Since the wall with all its bricks makes up the space occupied by it, we need to find the volume of the wall, which is nothing but a cuboid.

Here, Length = 10 m = 1000 cm Thickness = 24 cm Height = 4 m = 400 cm Therefore,

Volume of the wall = length × thickness × height = $1000 \times 24 \times 400 \text{ cm}^3$

Now, each brick is a cuboid with length = 24 cm, breadth = 12 cm and height = 8 cm

So, volume of each brick = length × breadth × height = $24 \times 12 \times 8 \text{ cm}^3$

So, number of bricks required = $\frac{\text{volume of the wall}}{\text{volume of each brick}} = \frac{1000 \times 24 \times 400}{24 \times 12 \times 8} = 4166.6$

So, **the wall requires 4167 bricks.**

Question: At a Ramzan Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small cylindrical glasses of radius 3 cm up to a height of 8 cm, and sold for \$3 each. How much money does the stall keeper receive by selling the juice completely?

Answer: The volume of juice in the vessel = volume of the cylindrical vessel = $\pi R^2 H$
(where R and H are taken as the radius and height respectively of the vessel) = $\pi \times 15 \times 15 \times 32$
 cm^3

Similarly, the volume of juice each glass can hold = $\pi r^2 h$
(where r and h are taken as the radius and height respectively of each glass) = $\pi \times 3 \times 3 \times 8 \text{ cm}^3$

So, number of glasses of juice that are sold = $\frac{\text{volume of the vessel}}{\text{volume of each glass}} = \frac{\pi \times 15 \times 15 \times 32}{\pi \times 3 \times 3 \times 8} = 100$

Therefore, **amount received by the stall keeper = \$3 × 100 = \$ 300**

Question Source: NCERT

Question: The height and the slant height of a cone are 21 cm and 28 cm respectively. Find the volume of the cone.

Answer: Using the relationship between l, r and h, we have $l^2 = r^2 + h^2$

$$\text{Here, } 28^2 = r^2 + 21^2$$

$$\text{So, } r = 7\sqrt{7} \text{ cm}$$

$$\begin{aligned} \text{Volume of Cone} &= \frac{1}{3} \pi r^2 h && \text{taking } \pi = \frac{22}{7} \\ &= \frac{1}{3} \pi \times 7\sqrt{7}^2 \times 21 \\ &= 7456 \text{ cm}^3 \end{aligned}$$

Therefore, **volume is 7456 cm³** .

Question: Find the volume of a sphere of radius 11.2 cm.

$$\text{Answer: Required volume} = \frac{4}{3} \pi r^3 = \frac{4 \times 22 \times 11.2 \times 11.2 \times 11.2}{3 \times 7} = \mathbf{5887.32 \text{ cm}^3}.$$

Question: A hemispherical bowl has a radius of 3.5 cm. What would be the volume of water it would contain?

$$\text{Answer: The volume of water the bowl can contain} = \frac{2}{3} \pi r^3 = \frac{2 \times 22 \times 3.5 \times 3.5 \times 3.5}{3 \times 7} = \mathbf{89.8 \text{ cm}^3}.$$

Thank You!