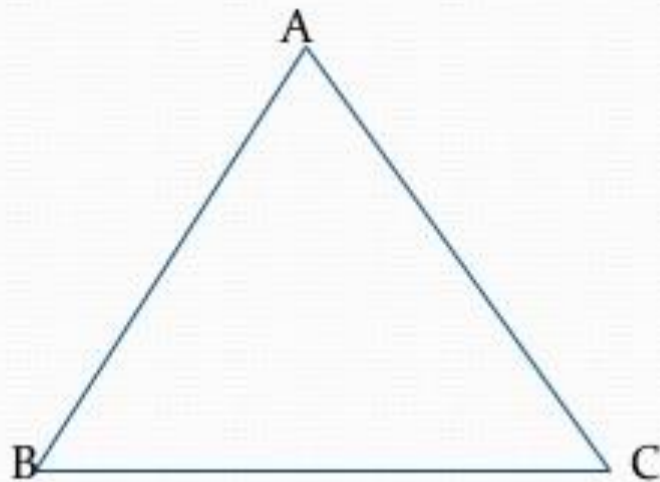


TRIANGLES

CLASS 9

Introduction

We know that a closed figure formed by three intersecting lines is called a triangle ('Tri' means 'three'). A triangle has three sides, three angles and three vertices. For e.g.-in Triangle ABC, denoted as $\triangle ABC$ AB, BC, CA are the three sides, $\angle A, \angle B, \angle C$ are three angles and A, B, C are three vertices.



OBJECTIVES IN THIS LESSON

1

- DEFINE THE CONGRUENCE OF TRIANGLE.

2

- STATE THE CRITERIA FOR THE CONGRUENCE OF TWO TRIANGLES.

3

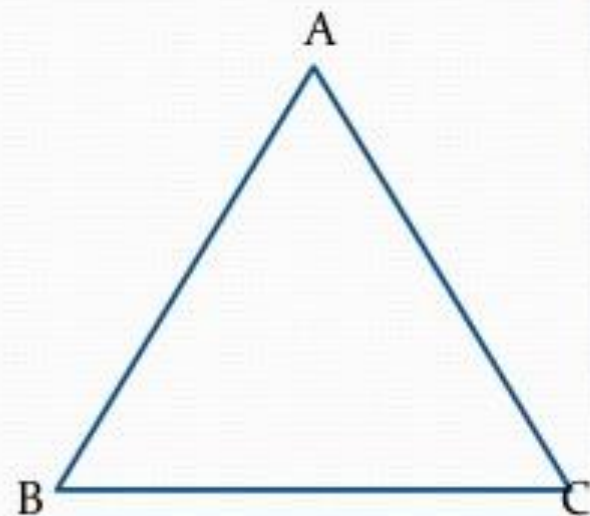
- SOME PROPERTIES OF A TRIANGLE.

4

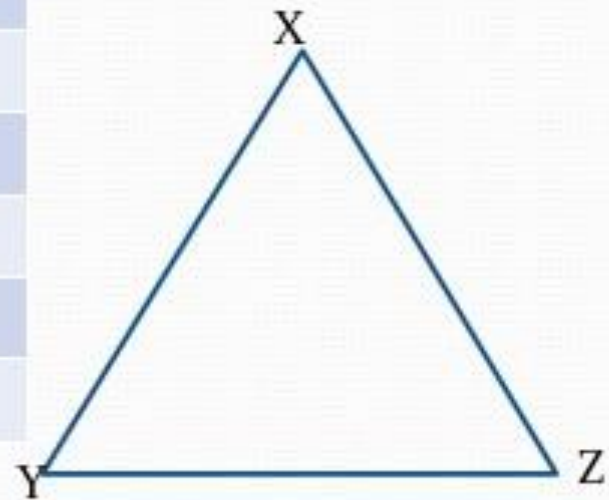
- INEQUALITIES IN A TRIANGLE.

DEFINING THE CONGRUENCE OF TRIANGLE:-

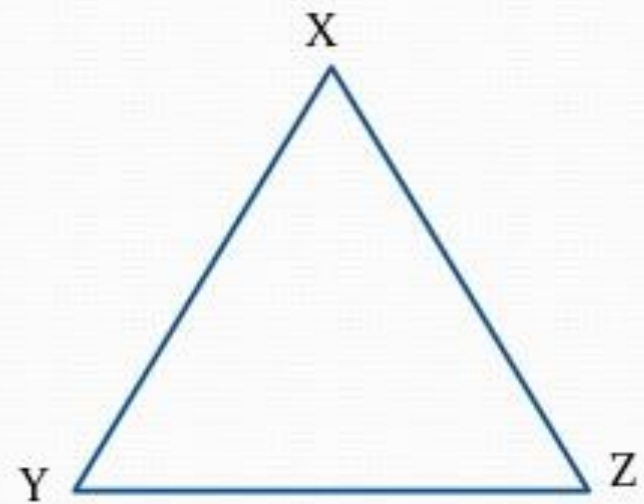
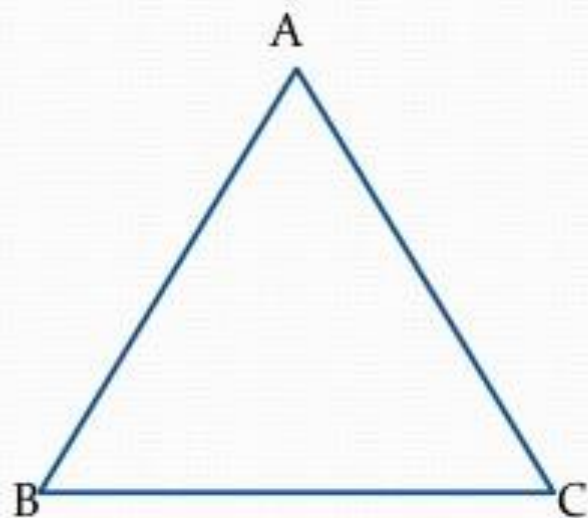
Let us take $\triangle ABC$ and $\triangle XYZ$ such that corresponding angles are equal and corresponding sides are equal :-



CORRESPONDING PARTS
$\angle A = \angle X$
$\angle B = \angle Y$
$\angle C = \angle Z$
$AB = XY$
$BC = YZ$
$AC = XZ$



Now we see that sides of $\triangle ABC$ coincides with sides of $\triangle XYZ$.



Here, $\triangle ABC \cong \triangle XYZ$

TWO TRIANGLES ARE CONGRUENT, IF ALL THE SIDES AND ALL THE ANGLES OF ONE TRIANGLE ARE EQUAL TO THE CORRESPONDING SIDES AND ANGLES OF THE OTHER TRIANGLE.

This also means that:-

A corresponds to X

B corresponds to Y

C corresponds to Z

For any two congruent triangles the corresponding parts are equal and are termed as:-

CPCT - Corresponding Parts of Congruent Triangles

CRITERIAS FOR CONGRUENCE OF TWO TRIANGLES

SAS(side-angle-side) congruence

- Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of other triangle.

ASA(angle-side-angle) congruence

- Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.

AAS(angle-angle-side) congruence

- Two triangles are congruent if any two pairs of angle and one pair of corresponding sides are equal.

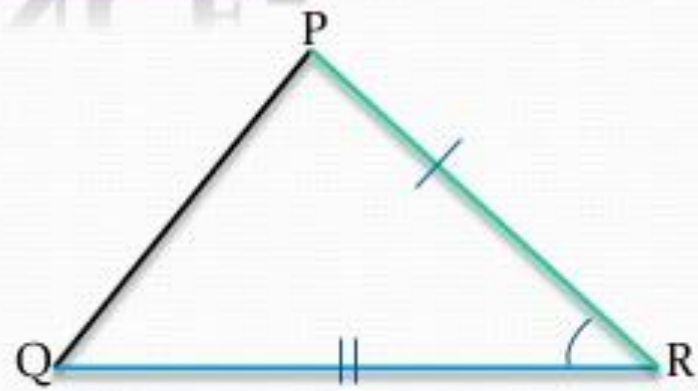
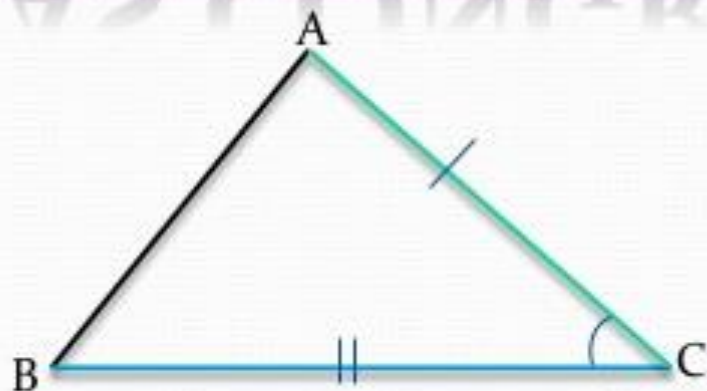
SSS(side-side-side) congruence

- If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

RHS(right angle-hypotenuse-side) congruence

- If in two right-angled triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

SAS CONGRUENCE-

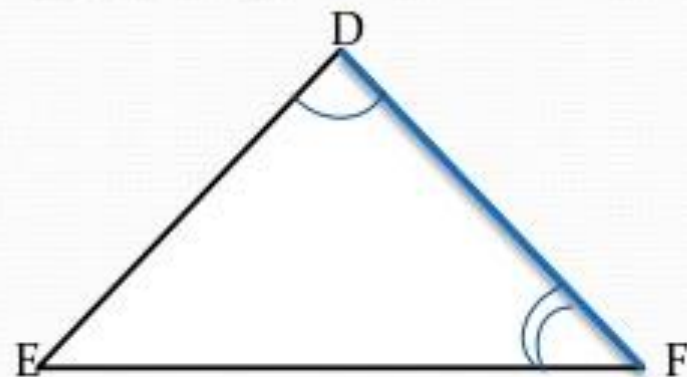
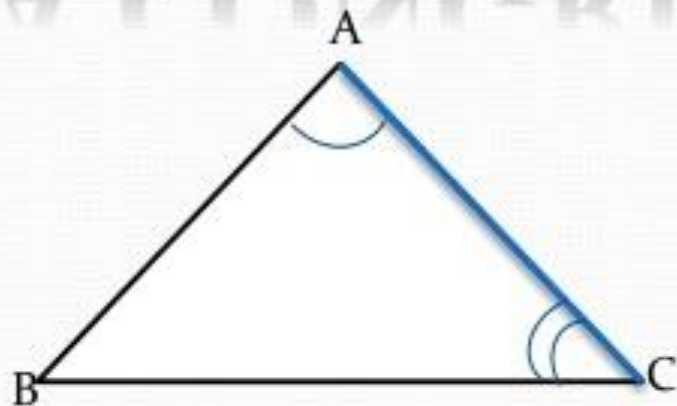


Now If,

S(1)	$AC = PQ$
A(2)	$\angle C = \angle R$
S(3)	$BC = QR$

Then $\triangle ABC \cong \triangle PQR$ (by SAS congruence)

ASA CONGRUENCE-



Now If,

A(1)	$\angle BAC = \angle EDF$
S(2)	$AC = DF$
A(3)	$\angle ACB = \angle DFE$

Then $\triangle ABC \cong \triangle DEF$ (by ASA congruence)

Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle

Proof : We are given two triangles ABC and DEF in which: $\angle B = \angle E$, $\angle C = \angle F$ and $BC = EF$

To prove that : $\Delta ABC \cong \Delta DEF$,

For proving the congruence of the two triangles see that three cases arise.

Case (i) : Let $AB = DE$ in figure

You may observe that

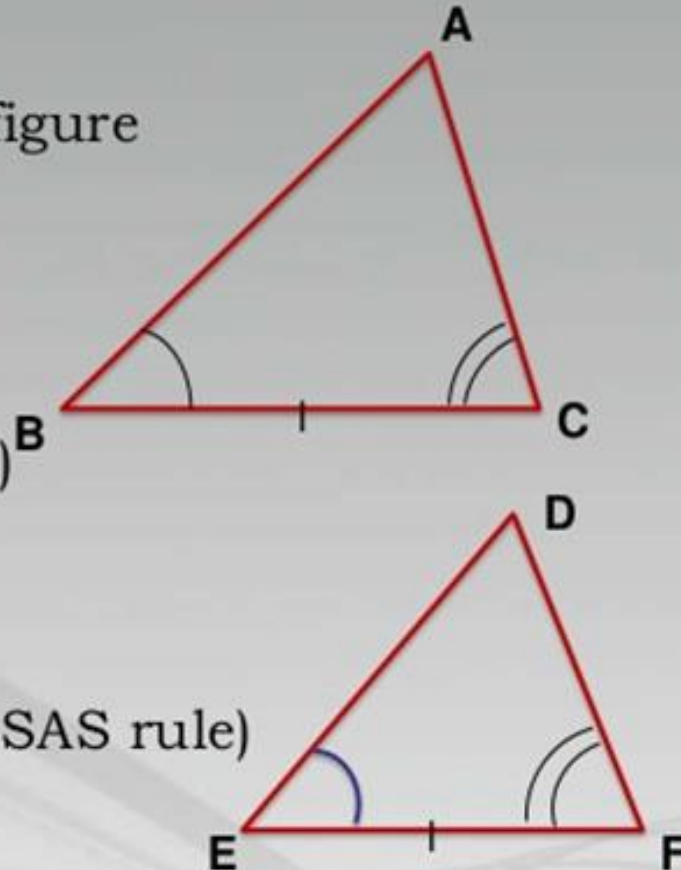
$AB = DE$ (Assumed)

$\angle B = \angle E$ (Given)

$BC = EF$ (Given)

So, $\triangle ABC \cong \triangle DEF$

.....(By SAS rule)



Case (ii) : Let if possible $AB > DE$.

So, we can take a point P on AB
such that $PB = DE$.

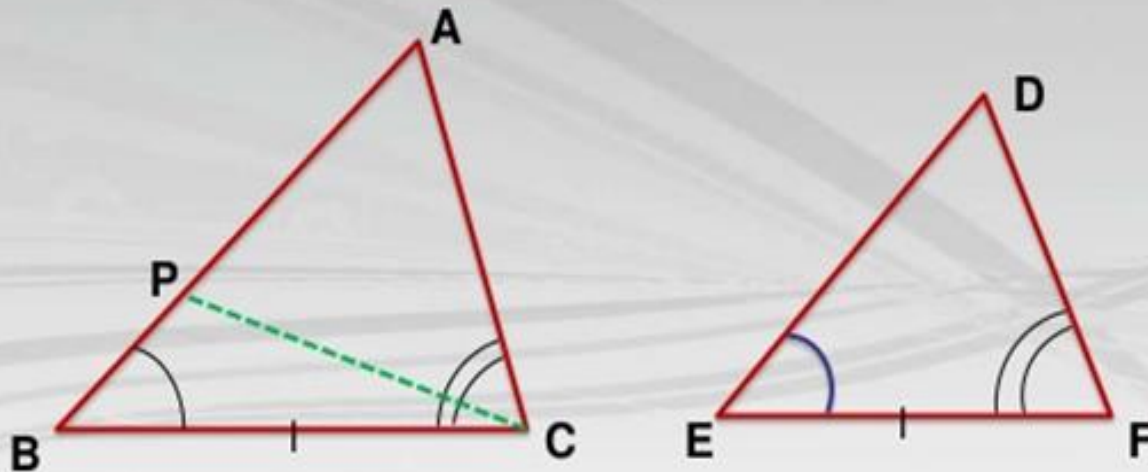
Now consider $\triangle PBC$ and $\triangle DEF$ (see Fig.)

In $\triangle PBC$ and $\triangle DEF$,

$PB = DE$ (By construction)

$\angle B = \angle E$, $BC = EF$ (Given)

So, $\triangle PBC \cong \triangle DEF$, by the SAS axiom for congruence.



Since the triangles are congruent, their corresponding parts will be equal.

So, $\angle PCB = \angle DFE$

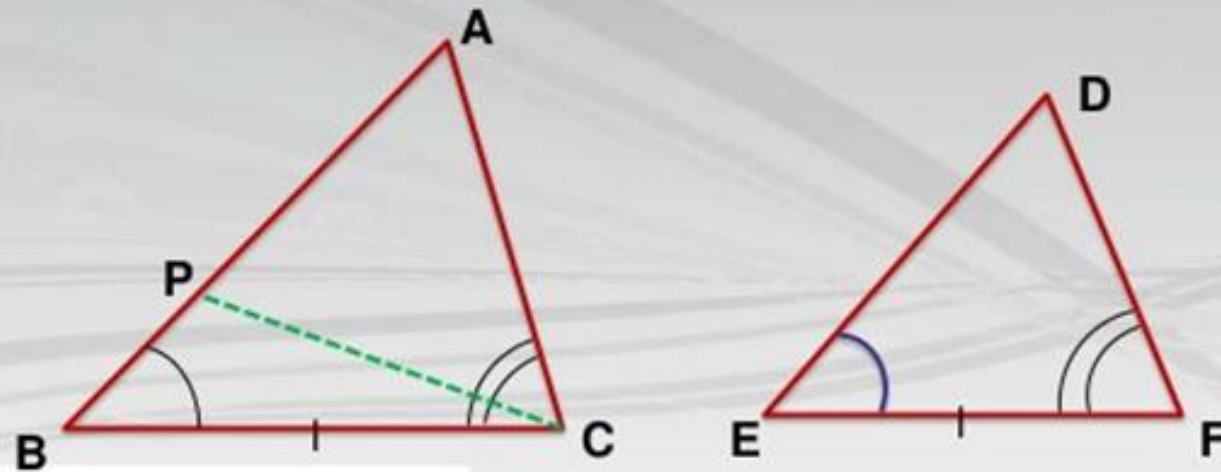
But, given that $\angle ACB = \angle DFE$

So, $\angle ACB = \angle PCB$

This is possible only if P coincides with A.

or, $BA = ED$

So, $\triangle ABC \cong \triangle DEF$ (by SAS axiom)



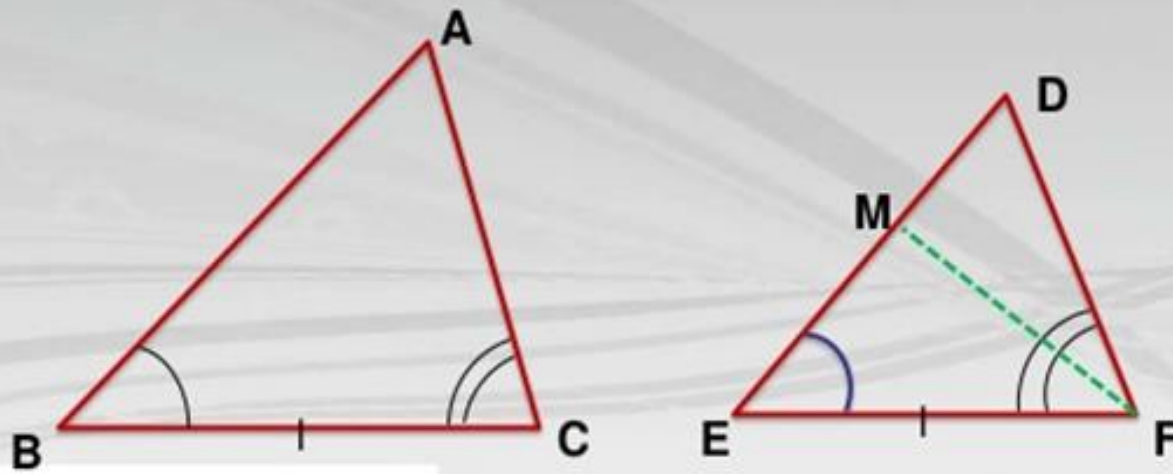
Case (iii) : If $AB < DE$,

we can choose a point M on DE such that $ME = AB$
 $\triangle ABC$ and $\triangle MEF$ (see Fig.)

$AB = ME$ (By construction)

$\angle B = \angle E$, $BC = EF$ (Given)

So, $\triangle ABC \cong \triangle MEF$, by the SAS axiom for congruence.



If $\triangle ABC \cong \triangle MEF$

then corresponding parts will be equal.

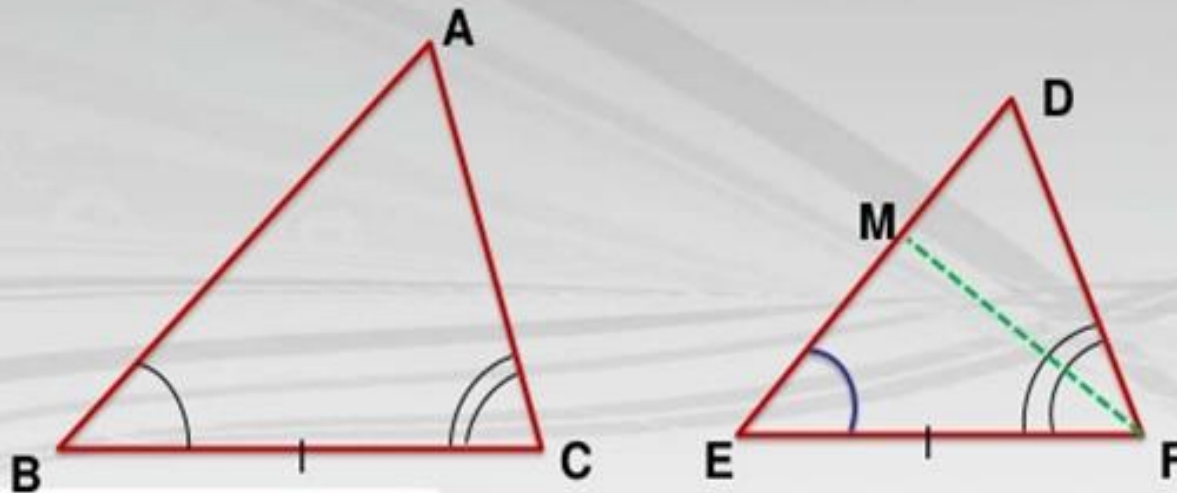
So, $\angle ACB = \angle MFE$, But $\angle ACB = \angle DFE$ (Given)

so, $\angle ACB = \angle MCB$

This is possible only if M coincides with D.

or, $BA = ED$

• So, $\triangle ABC \cong \triangle DEF$ (by SAS axiom)



• **So all the three cases:-**

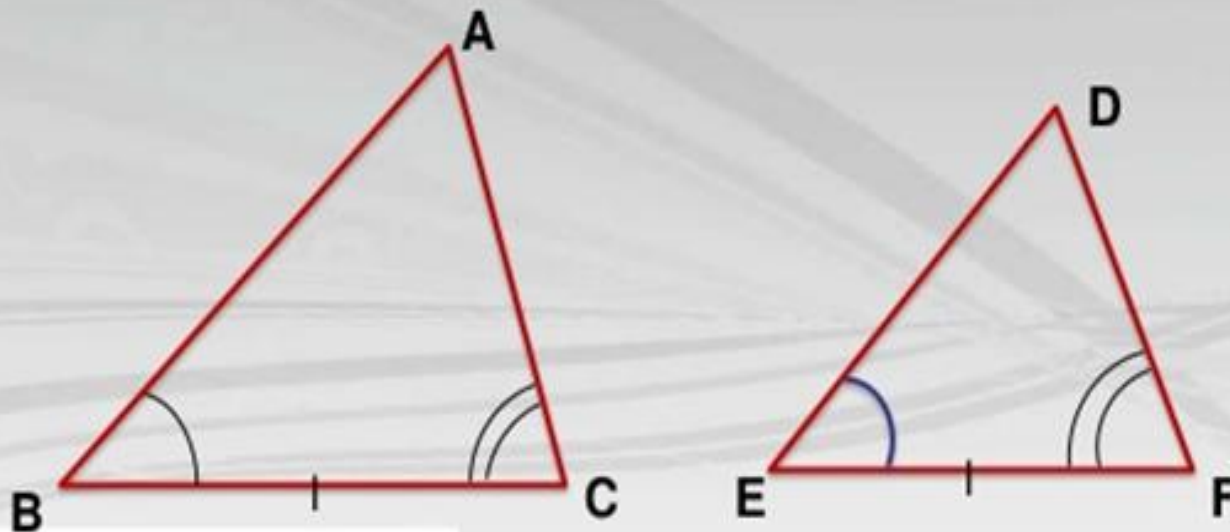
• Case (i) : $AB = DE$

• Case (ii) : $AB > DE$

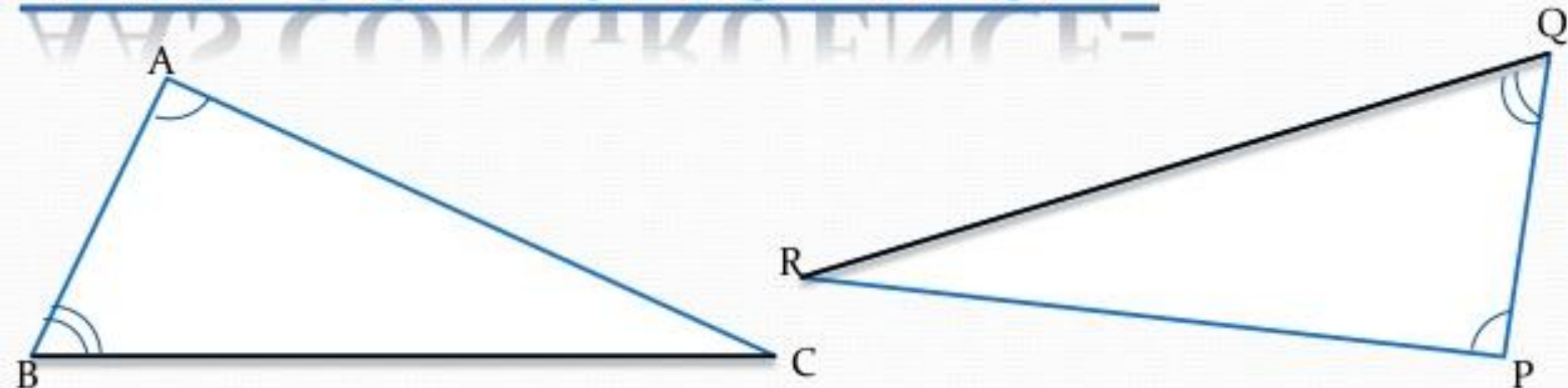
• Case (iii) : $AB < DE$,

We can see that $\triangle ABC \cong \triangle DEF$

Proved



AAS CONGRUENCE-

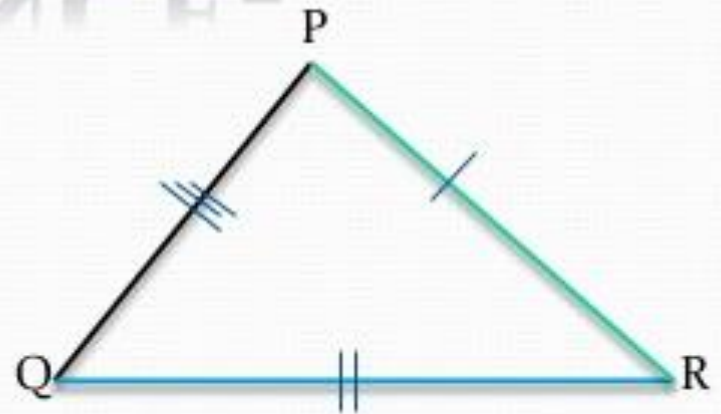
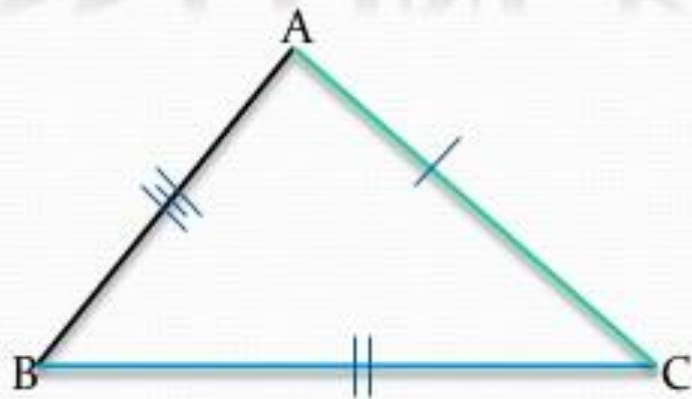


Now If,

A(1)	$\angle BAC = \angle QPR$
A(2)	$\angle CBA = \angle RQP$
S(3)	$BC = QR$

Then $\triangle ABC \cong \triangle PQR$ (by AAS congruence)

SSS CONGRUENCE-

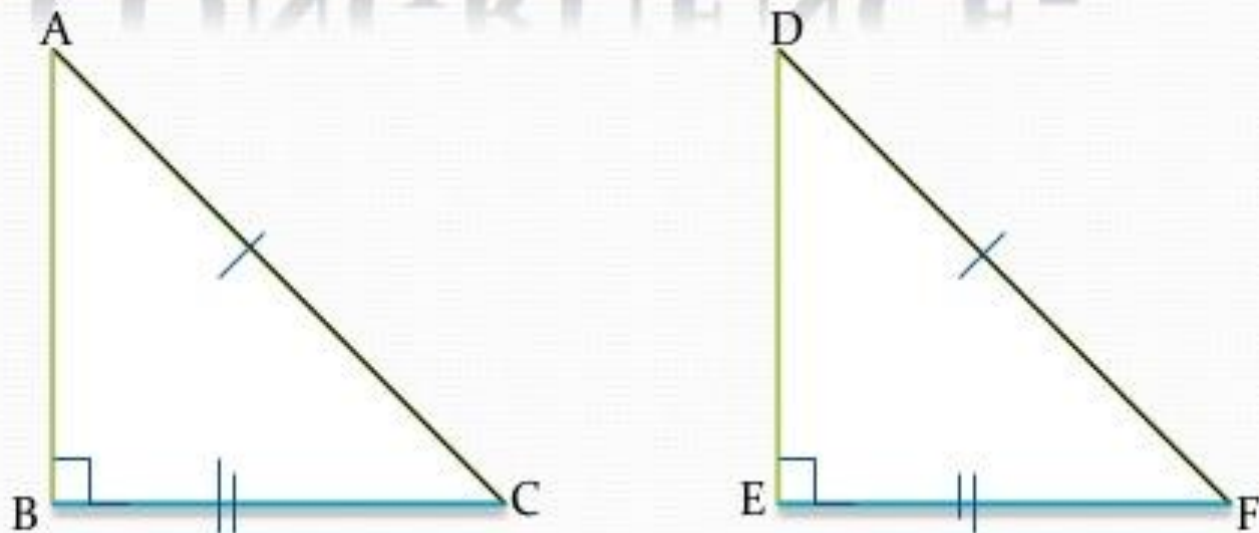


Now If,

S(1)	$AB = PQ$
S(2)	$BC = QR$
S(3)	$CA = RP$

Then $\triangle ABC \cong \triangle PQR$ (by SSS congruence)

RHS CONGRUENCE-

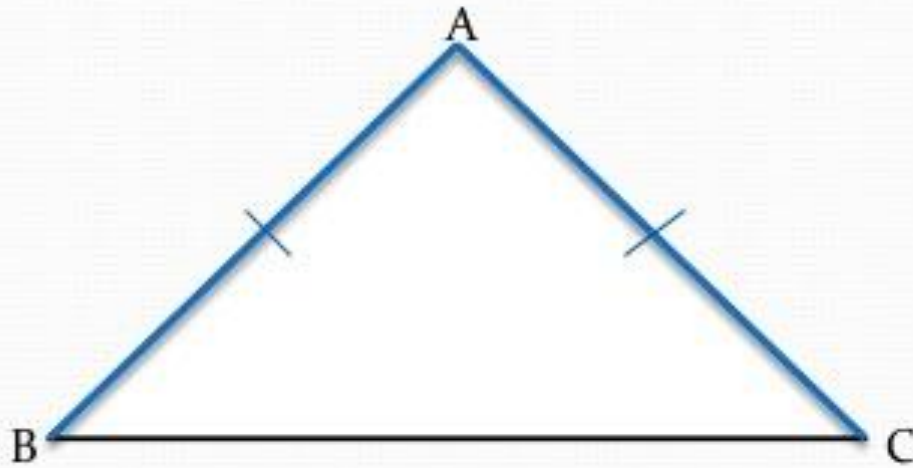


Now If,

R(1)	$\angle ABC = \angle DEF = 90^\circ$
H(2)	$AC = DF$
S(3)	$BC = EF$

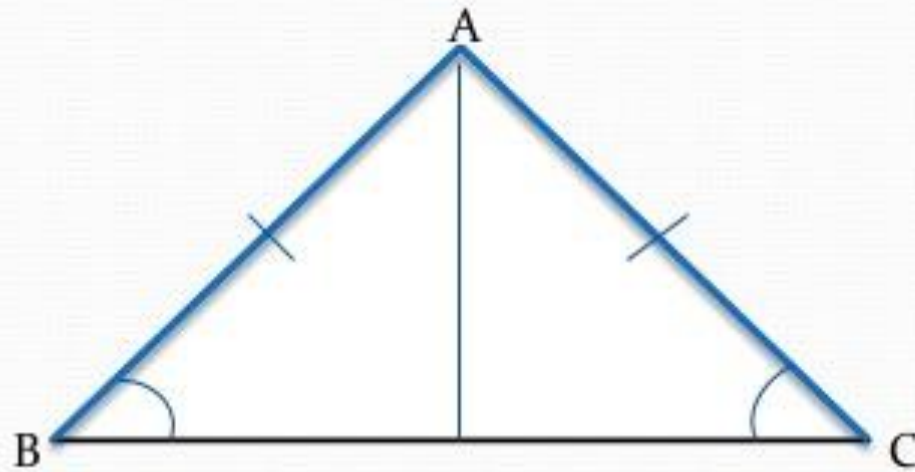
Then $\triangle ABC \cong \triangle DEF$ (by RHS congruence)

PROPERTIES OF TRIANGLE



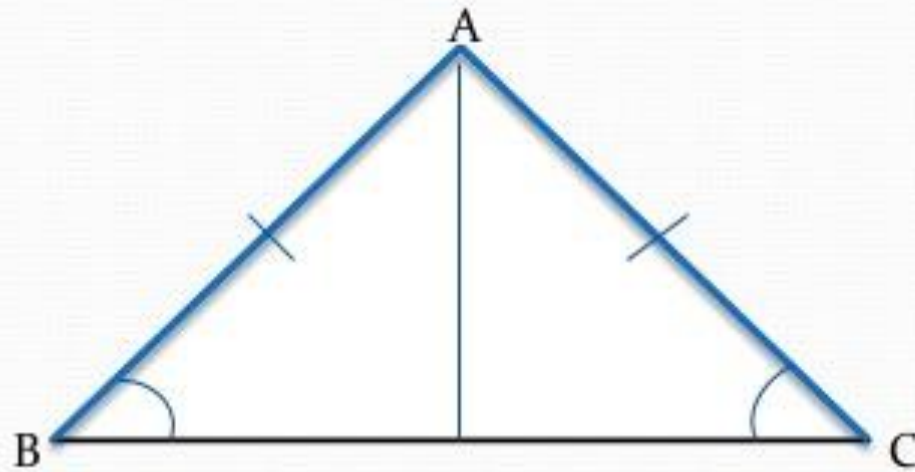
A Triangle in which two sides are equal in length is called **ISOSCELES TRIANGLE**. So, $\triangle ABC$ is a isosceles triangle with $AB = BC$.

Angles opposite to equal sides of an isosceles triangle are equal.



Here, $\angle ABC = \angle ACB$

The sides opposite to equal angles of a triangle are equal.

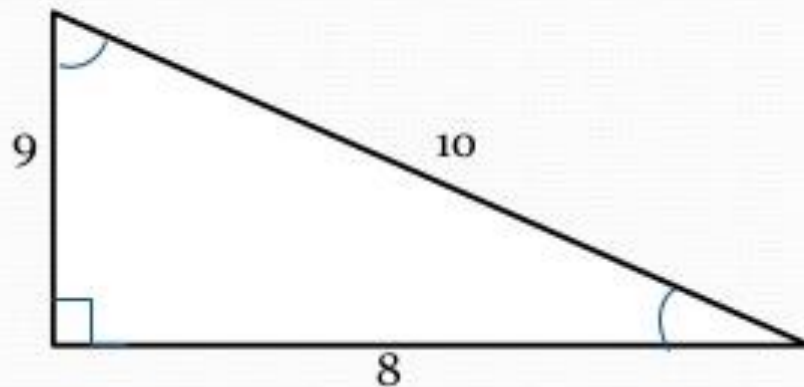


Here, $AB = AC$

INEQUALITIES IN A TRIANGLE

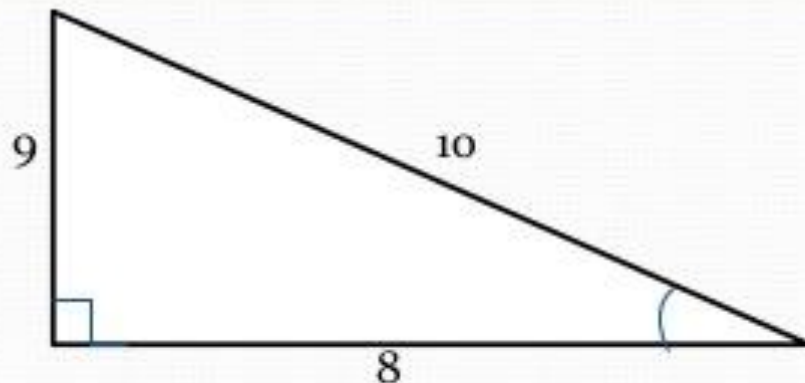
Theorem on inequalities in a triangle

If two sides of a triangle are unequal, the angle opposite to the longer side is larger (or greater)



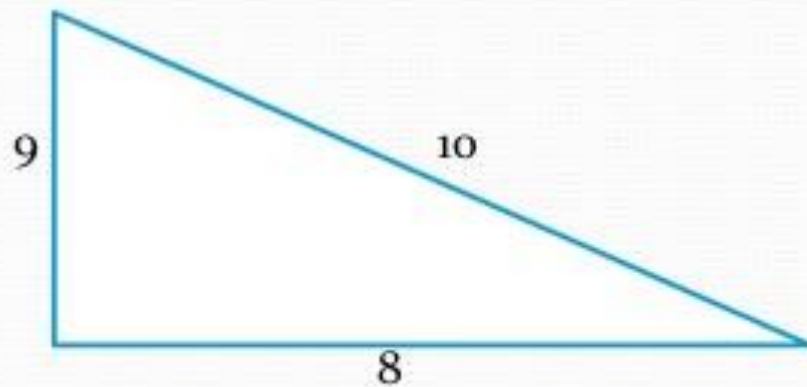
Here, by comparing we will get that-
Angle opposite to the longer side(10) is greater(i.e. 90°)

In any triangle, the side opposite to the longer angle is longer.



Here, by comparing we will get that-
Side(i.e. 10) opposite to longer angle (90°) is longer.

The sum of any two side of a triangle is greater than the third side.



Here by comparing we get-

$$9+8>10$$

$$8+10>9$$

$$10+9>8$$

So, sum of any two sides is greater than the third side.

SUMMARY

1. Two figures are congruent, if they are of the same shape and size.
2. If two sides and the included angle of one triangle is equal to the two sides and the included angle then the two triangles are congruent (by SAS).
3. If two angles and the included side of one triangle are equal to the two angles and the included side of other triangle then the two triangles are congruent (by ASA).
4. If two angles and the one side of one triangle is equal to the two angles and the corresponding side of other triangle then the two triangles are congruent (by AAS).
5. If three sides of a triangle is equal to the three sides of other triangle then the two triangles are congruent (by SSS).
6. If in two right-angled triangle, hypotenuse one side of the triangle are equal to the hypotenuse and one side of the other triangle then the two triangle are congruent. (by RHS)
7. Angles opposite to equal sides of a triangle are equal.
8. Sides opposite to equal angles of a triangle are equal.
9. Each angle of equilateral triangle are 60°
10. In a triangle, angles opposite to the longer side is larger
11. In a triangle, side opposite to the larger angle is longer.
12. Sum of any two sides of triangle is greater than the third side.



THANKYOU