

LINEAR EQUATION IN TWO VARIABLES

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Linear Equations

Definition of a Linear Equation

A linear equation in two variable x is an equation that can be written in the form $ax + by + c = 0$, where a , b and c are real numbers and a and b is not equal to 0.

An example of a linear equation in x is $2x - 3y + 4 = 0$.

equation which can be put in the form $ax + by + c = 0$, where a , b and c are real numbers, and a and b are not both zero, is called a linear equation in two variables x and y . (We often denote the condition a and b are not both zero by $a^2 + b^2 \neq 0$). You have also studied that a solution of such an equation is a pair of values, one for x and the other for y , which makes the two sides of the equation equal.

A pair of linear equations in two variables can be solved by:

1. Graphically Method

2. Algebraic Method

GRAPHICAL SOLUTIONS OF A LINEAR EQUATION

Let us consider the following system of two simultaneous linear equations in two variable.

$$2x - y = -1$$

$$3x + 2y = 9$$

Here we assign any value to one of the two variables and then determine the value of the other variable from the given equation.

FOR THE EQUATION

$$2X - Y = -1 \text{ ---(1)}$$

$$2X + 1 = Y$$

$$Y = 2X + 1$$

$$3X + 2Y = 9 \text{ ---}$$

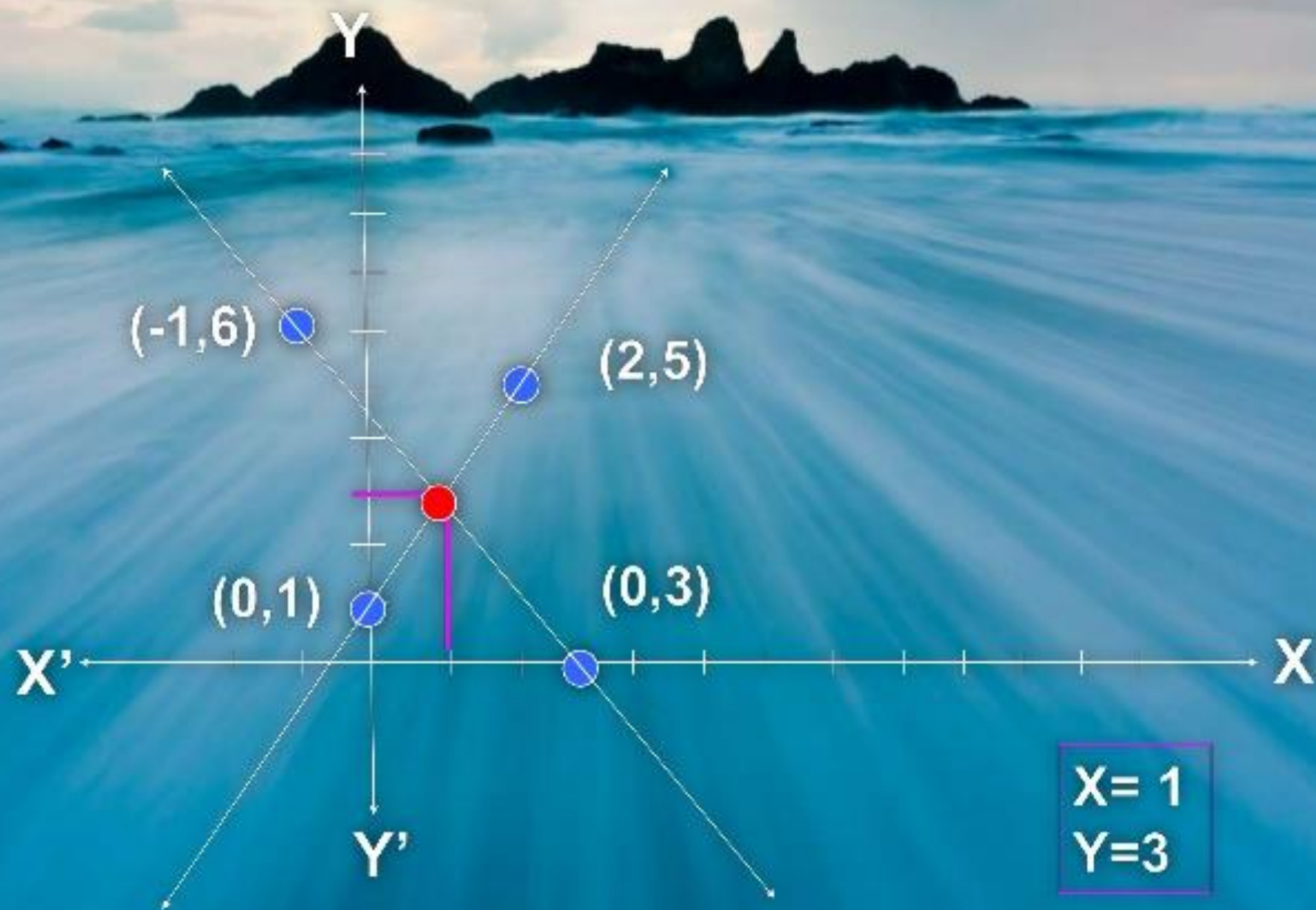
(2)

$$2Y = 9 - 3X$$

$$2Y = 9 - 3X$$

X	0	2
Y	1	5

X	3	-1
Y	0	6



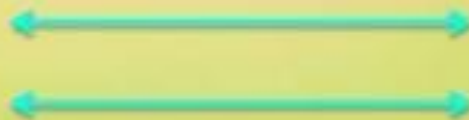
$$\begin{aligned} X &= 1 \\ Y &= 3 \end{aligned}$$

Types of Solutions of Systems of Equations

- One solution – the lines cross at one point



- No solution – the lines do not cross

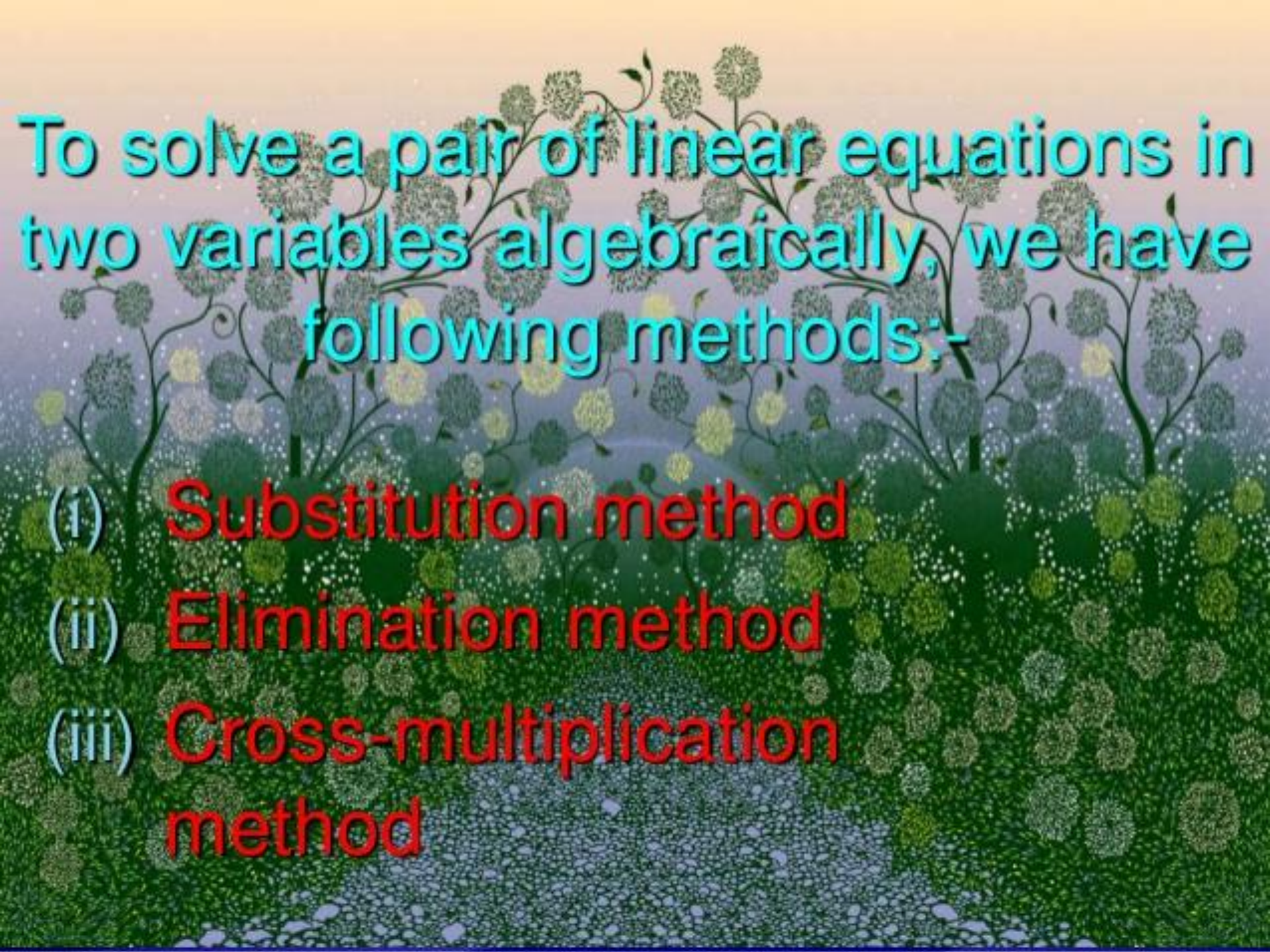


- Infinitely many solutions – the lines coincide



Algebraic Method to Solve a Pair of Linear Equations in Two Variables

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To solve a pair of linear equations in two variables algebraically, we have following methods:-

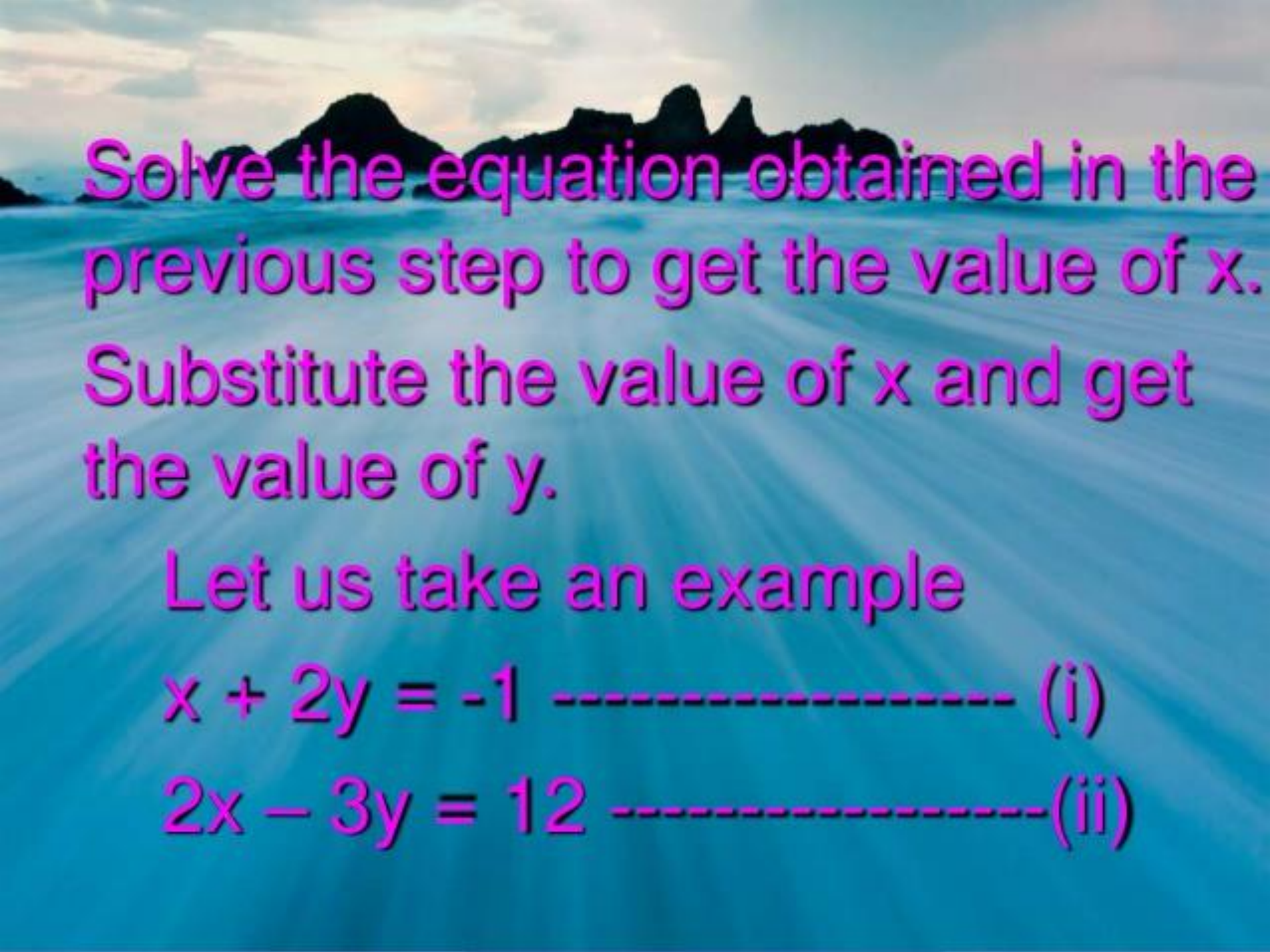
- (i) Substitution method
- (ii) Elimination method
- (iii) Cross-multiplication method

Substitution Method

Step 1 : Find the value of one variable, say y in terms of the other variable, i.e., x from either equation, whichever is convenient.

Step 2 : Substitute this value of y in the other equation, and reduce it to an equation in one variable, i.e., in terms of x , which can be solved. Sometimes, as in Examples 9 and 10 below, you can get statements with no variable. If this statement is true, you can conclude that the pair of linear equations has infinitely many solutions. If the statement is false, then the pair of linear equations is inconsistent.

Step 3 : Substitute the value of x (or y) obtained in Step 2 in the equation used in Step 1 to obtain the value of the other variable.



Solve the equation obtained in the previous step to get the value of x .
Substitute the value of x and get the value of y .

Let us take an example

$$x + 2y = -1 \text{ ----- (i)}$$

$$2x - 3y = 12 \text{ -----(ii)}$$

$$x + 2y = -1$$

$$x = -2y - 1 \text{ ----- (iii)}$$

Substituting the value of x in equation (ii), we get

$$2x - 3y = 12$$

$$2(-2y - 1) - 3y = 12$$

$$-4y - 2 - 3y = 12$$

$$-7y = 14 ; y = -2 ,$$

Putting the value of y in eq. (iii), we get

$$x = -2y - 1$$

$$x = -2 \times (-2) - 1$$

$$= 4 - 1$$

$$= 3$$

Hence the solution of the equation
is

$$(3, -2)$$

Elimination Method

Step 1 : First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.

Step 2 : Then add or subtract one equation from the other so that one variable gets eliminated. If you get an equation in one variable, go to Step 3.

If in Step 2, we obtain a true statement involving no variable, then the original pair of equations has infinitely many solutions.

If in Step 2, we obtain a false statement involving no variable, then the original pair of equations has no solution, i.e., it is inconsistent.

Step 3 : Solve the equation in one variable (x or y) so obtained to get its value.

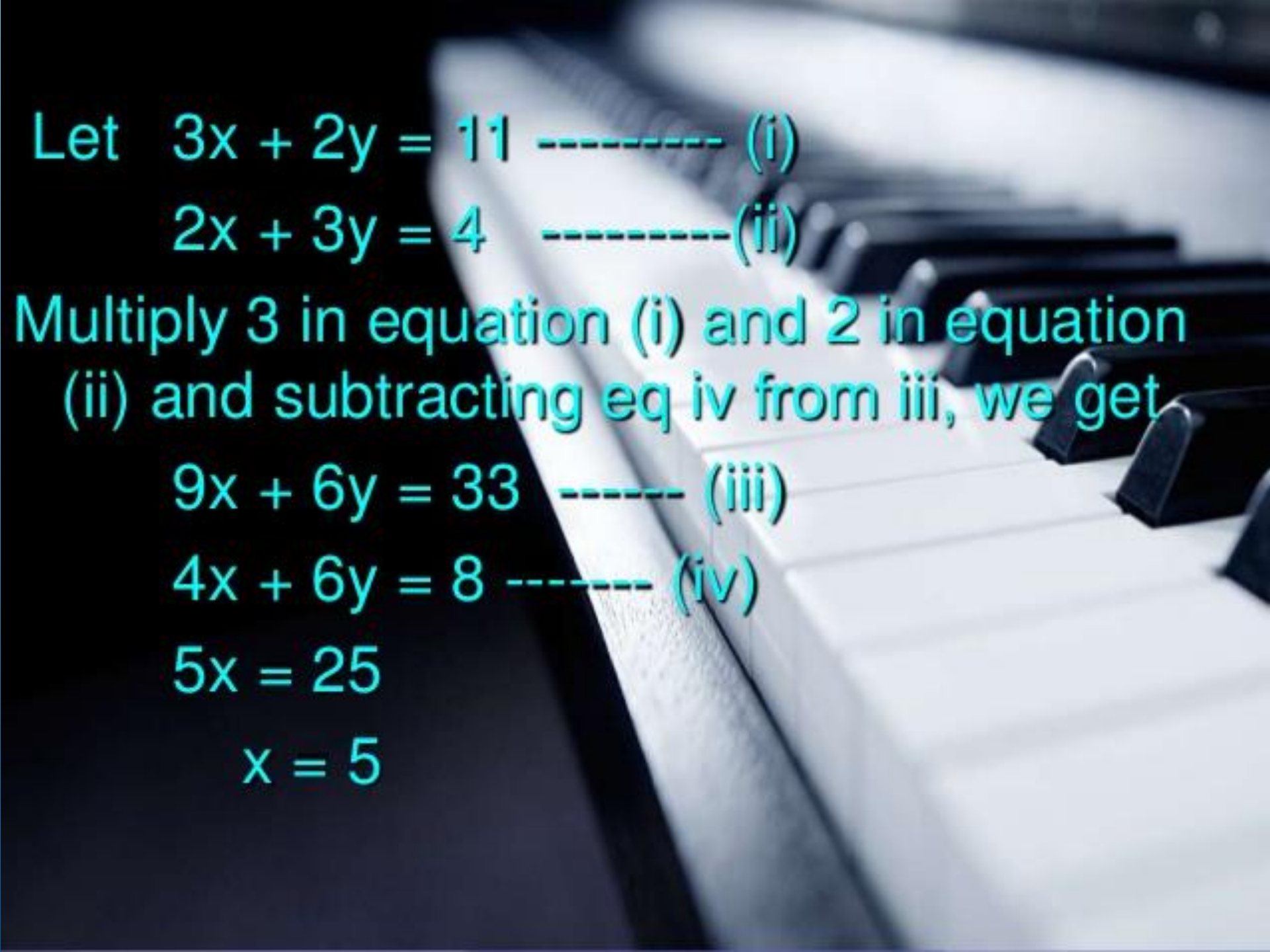
Step 4 : Substitute this value of x (or y) in either of the original equations to get the value of the other variable.

Elimination Method

- For example: we want to solve,

$$3x + 2y = 11$$

$$2x + 3y = 4$$



Let $3x + 2y = 11$ ----- (i)

$2x + 3y = 4$ ----- (ii)

Multiply 3 in equation (i) and 2 in equation (ii) and subtracting eq iv from iii, we get

$9x + 6y = 33$ ----- (iii)

$4x + 6y = 8$ ----- (iv)

$5x = 25$

$x = 5$

- putting the value of X in equation (ii) we get,

$$2x + 3y = 4$$

$$2 \times 5 + 3y = 4$$

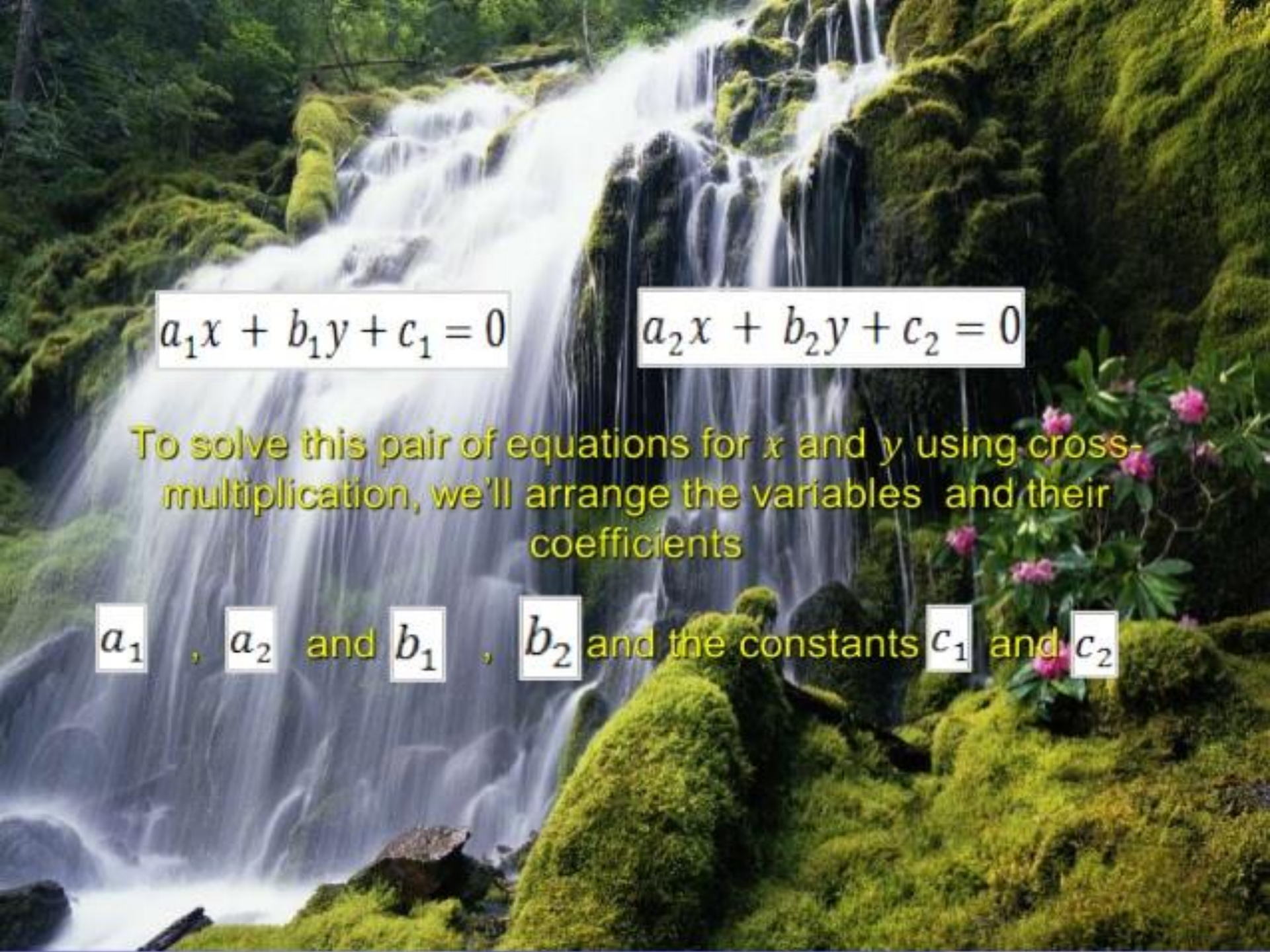
$$10 + 3y = 4$$

$$3y = 4 - 10$$

$$3y = -6$$

$$y = -2$$

Hence, $x = 5$ and $y = -2$

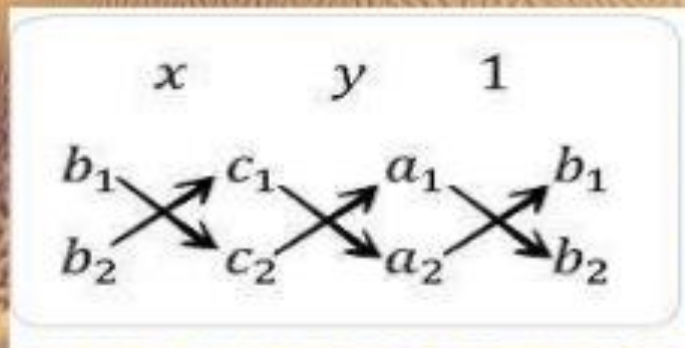

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

To solve this pair of equations for x and y using cross-multiplication, we'll arrange the variables and their coefficients

a_1 , a_2 and b_1 , b_2 and the constants c_1 and c_2

CROSS MULTIPLICATION METHOD



$$\Rightarrow x = \frac{b_1 c_2 - b_2 c_1}{a_1 b_2 - a_2 b_1}$$

$$\Rightarrow y = \frac{c_1 a_2 - c_2 a_1}{a_1 b_2 - a_2 b_1}$$

Thank
You