

POLYNOMIALS

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Objective: To study

- Polynomials of degree 1, 2 & 3
- The zeroes of polynomial $p(x)$
- Relationship between zeroes and coefficients of a polynomial
- The division algorithm of a polynomial

Do we remember this???

VARIABLES

**(one, two, three,
many)**

DEGREE

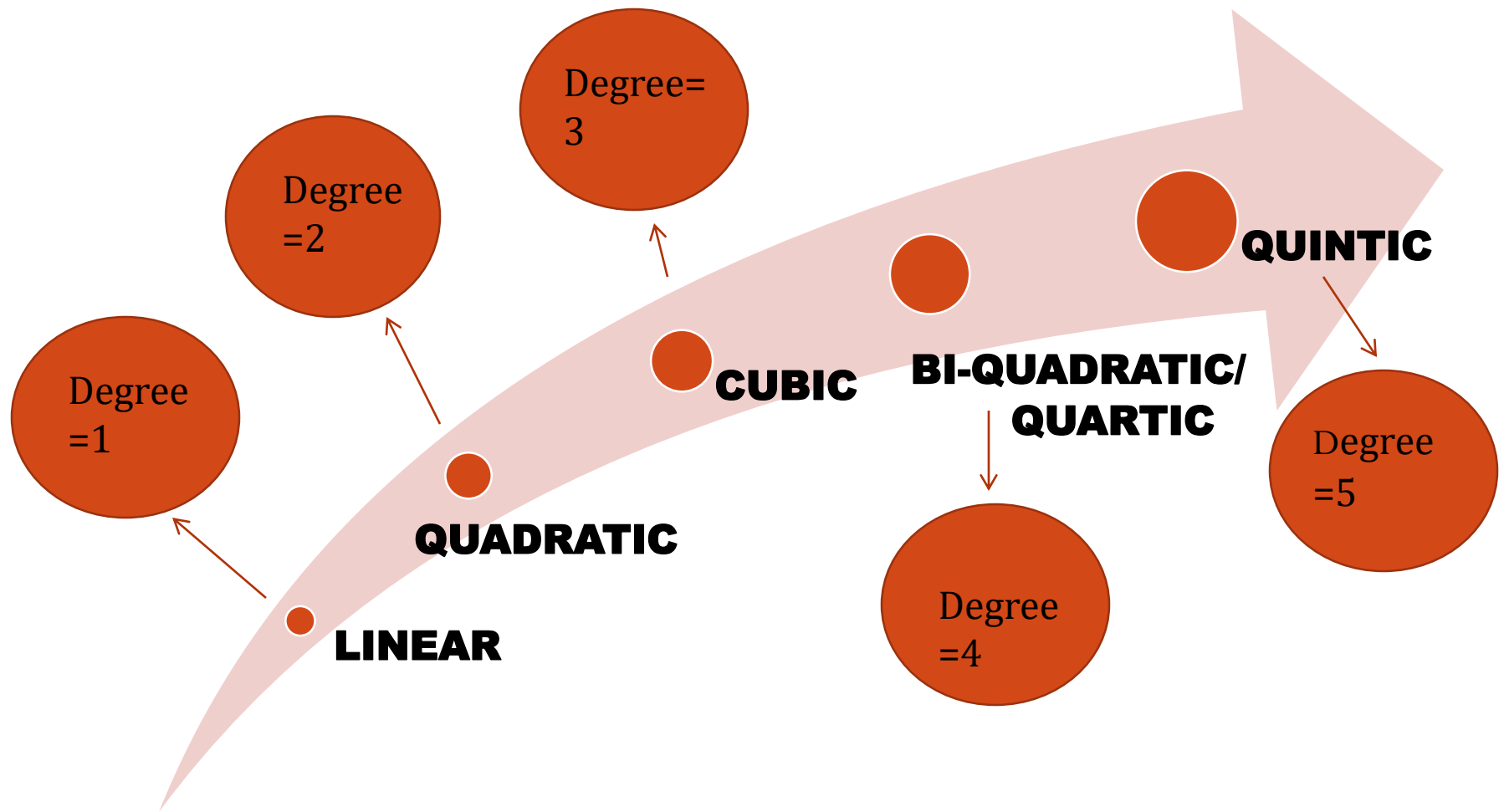
(1, 2, 3, 4, 5)

POLYNOMIAL

**VALUE OF A
POLYNOMIAL**

**ZERO OF A
POLYNOMIAL**

DEGREE OF A POLYNOMIAL



VALUE OF A POLYNOMIAL

If $p(x)$ is a polynomial in x , and if k is any real number, then the value obtained by replacing x by k in $p(x)$ is called the **value of $p(x)$ at $x=k$** .

Example: $p(x) = x^2 + 2$

$$\begin{aligned}\text{Then, } p(-4) &= (-4)^2 + 2 \\ &= 16 + 2 \\ &= 18\end{aligned}$$

ZERO OF A POLYNOMIAL

- A real number k , is said to be a zero of the polynomial $p(x)$, if $p(k)=0$
- **Example:** For a linear polynomial $p(x)= 2x+3$
Then , $p(k)= 0$ gives us
 $2k+3 = 0$. i.e. $k= - 3/2$
- In general, if k is a zero of $p(x)= ax+b$, then,
 $k= - b/a$

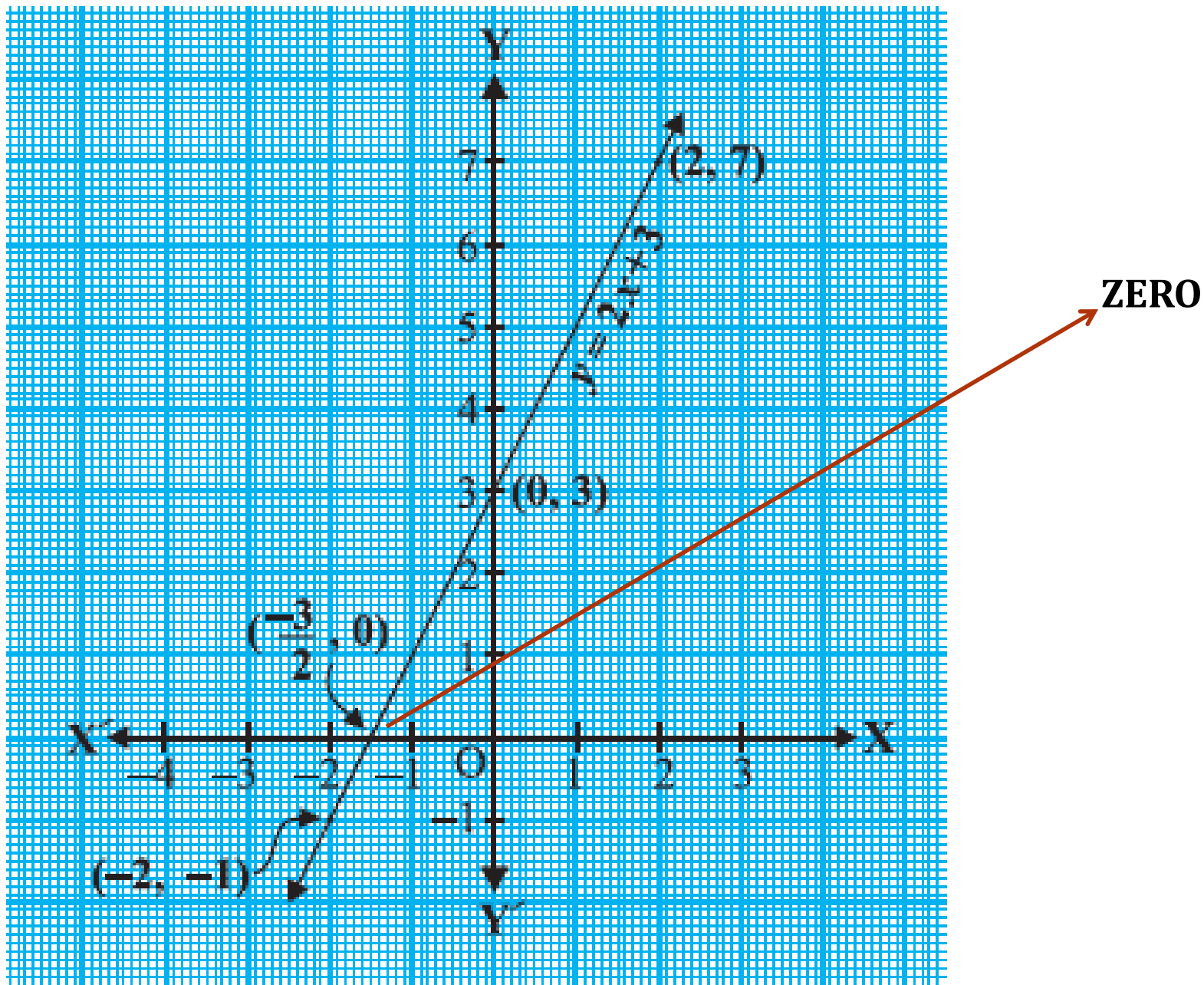
WHAT'S NEW THIS TIME?

- Geometrical meaning of the zeroes of a polynomial
- Relationship between the zeroes and coefficients of a polynomial
- Division Algorithm for Polynomials

Geometrical meaning of the zeroes of a polynomial

LINEAR POLYNOMIAL

Plot the graph of the polynomial $y = 2x + 3$

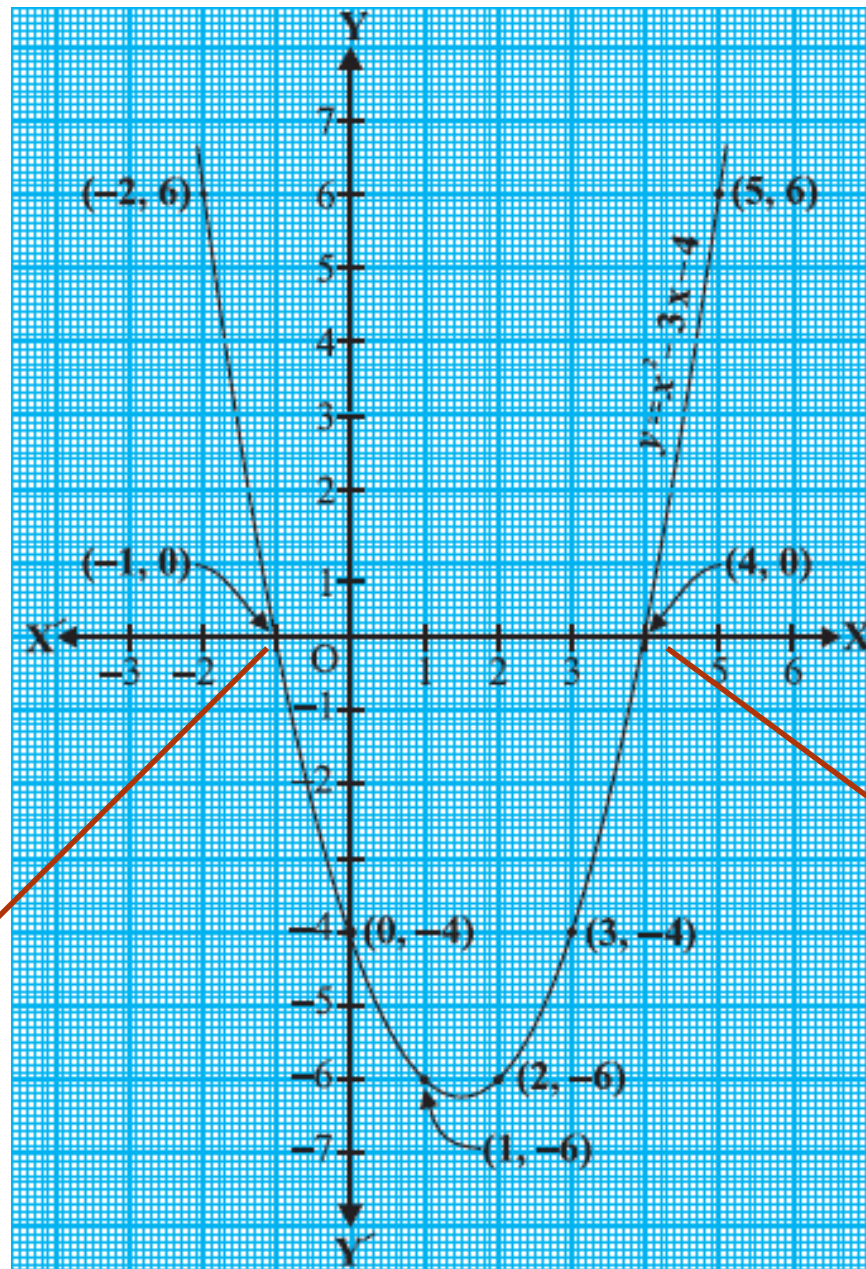


Geometrical meaning of the zeroes of a polynomial

QUADRATIC POLYNOMIAL

Plot the graph of the polynomial $y = x^2 - 3x + 4$

FIRST ZERO



SECOND ZERO

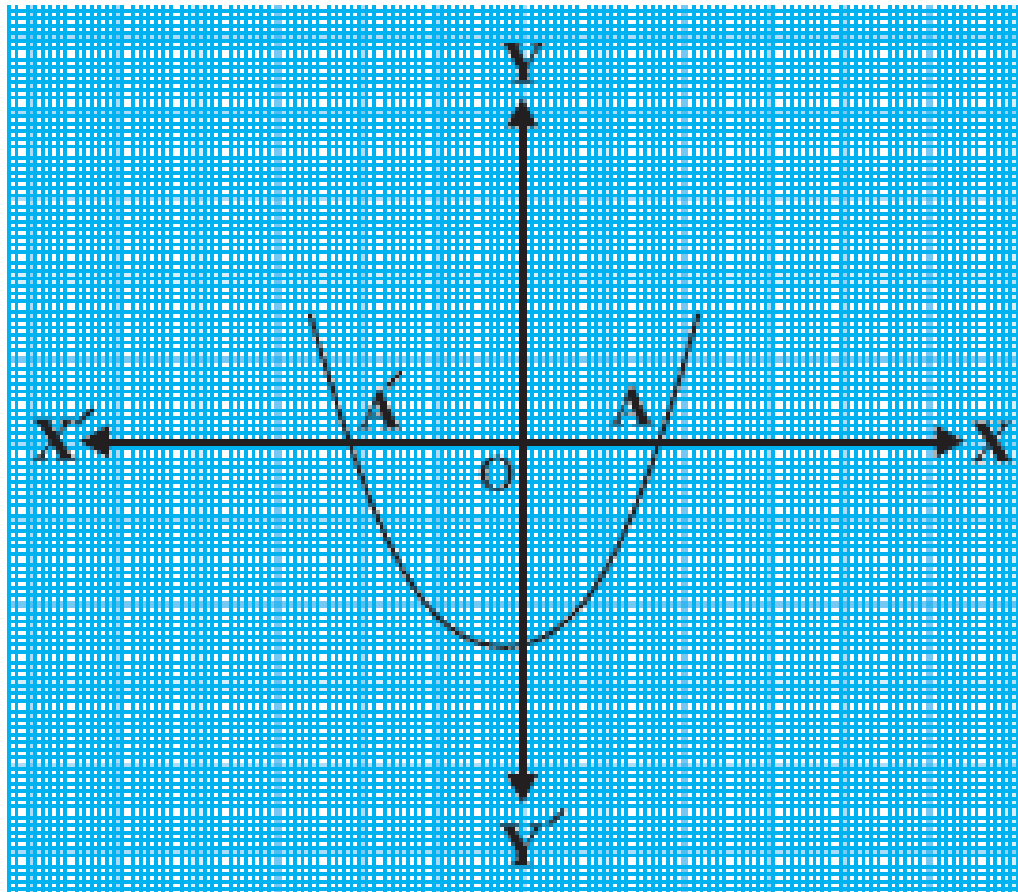
Special cases in quadratic polynomials

Question:

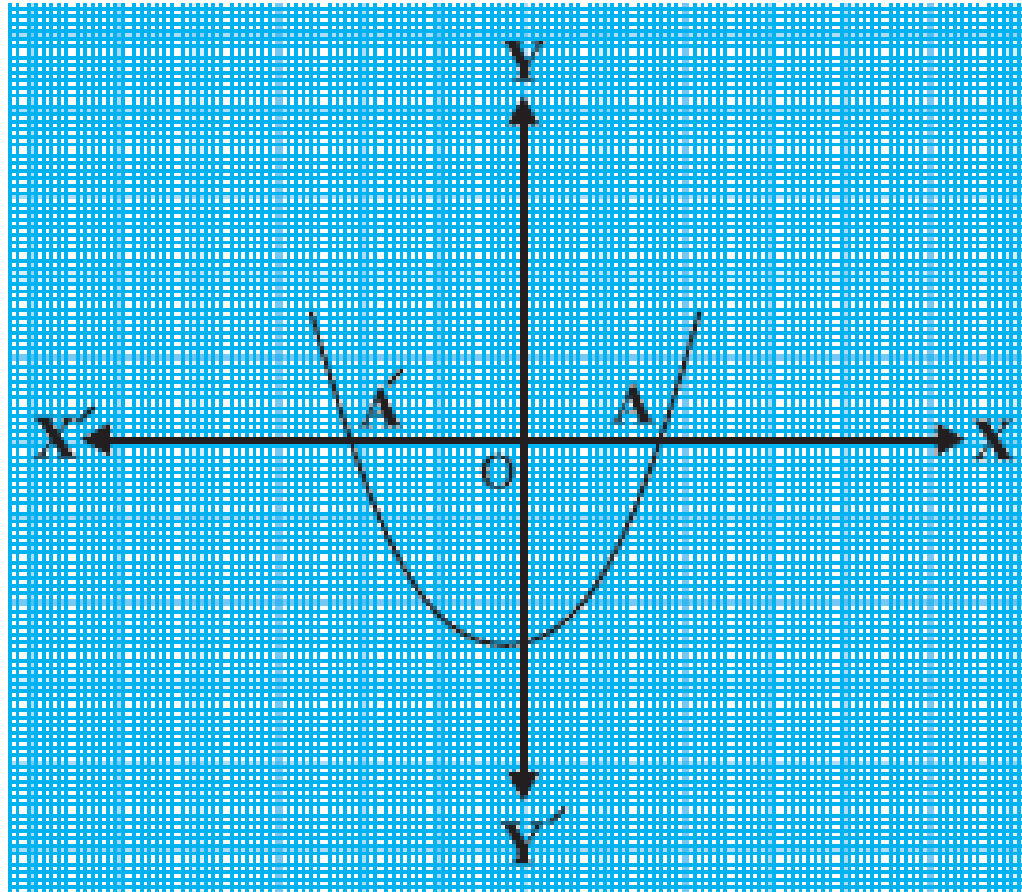
How many zeroes do the following polynomials have?

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(a)



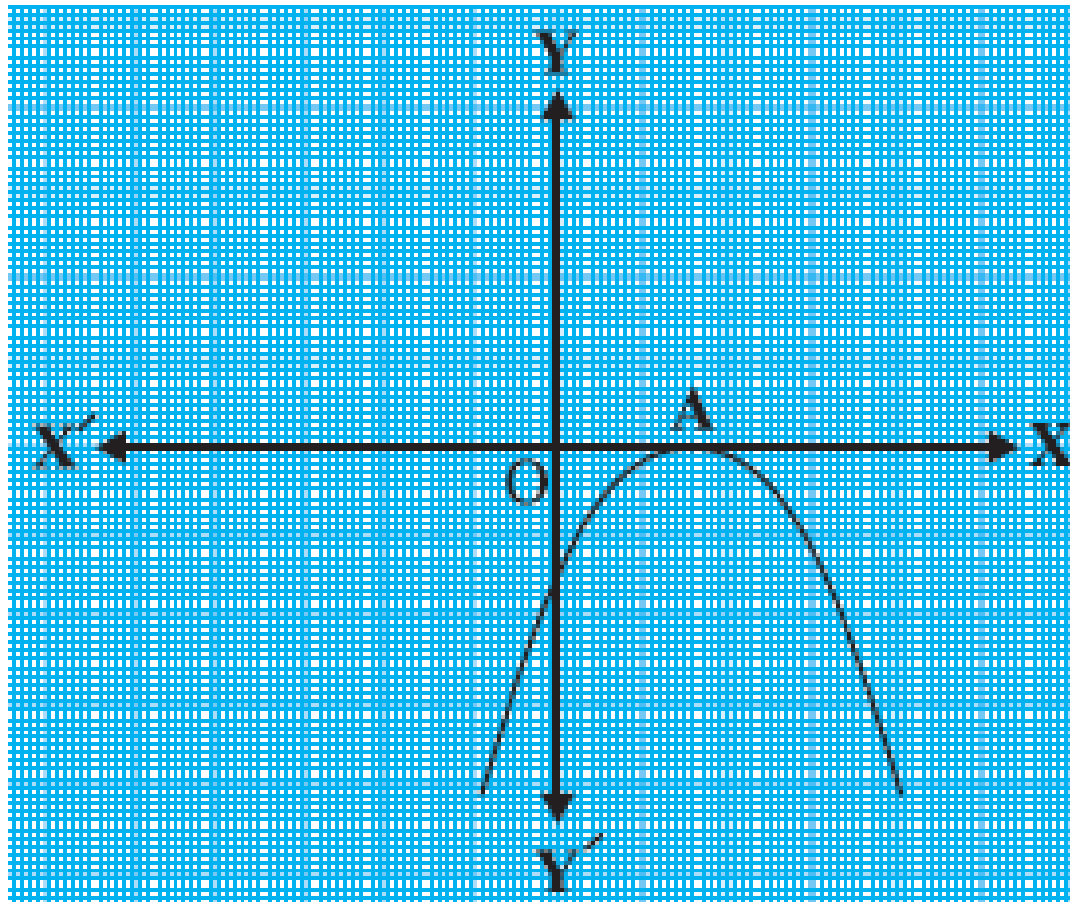
(a)



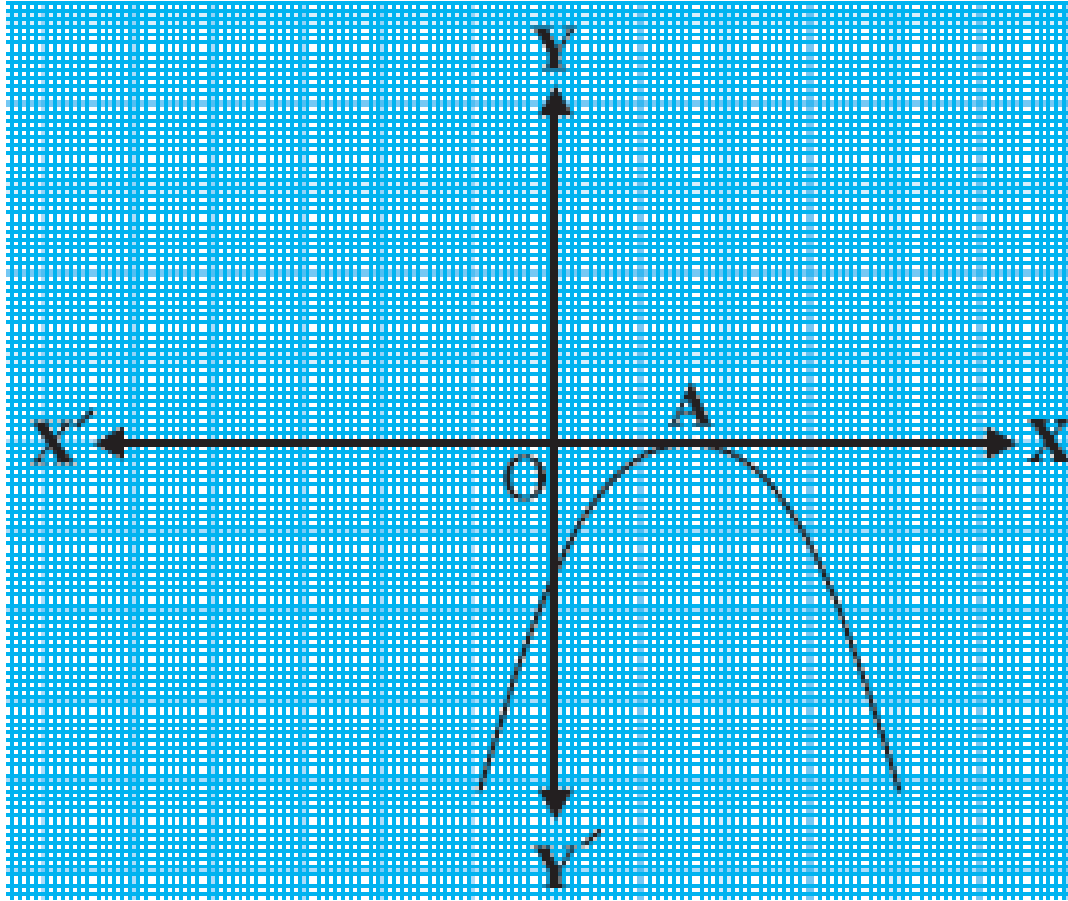
Answer: 2

How many zeroes do the following polynomials have ?

(b)



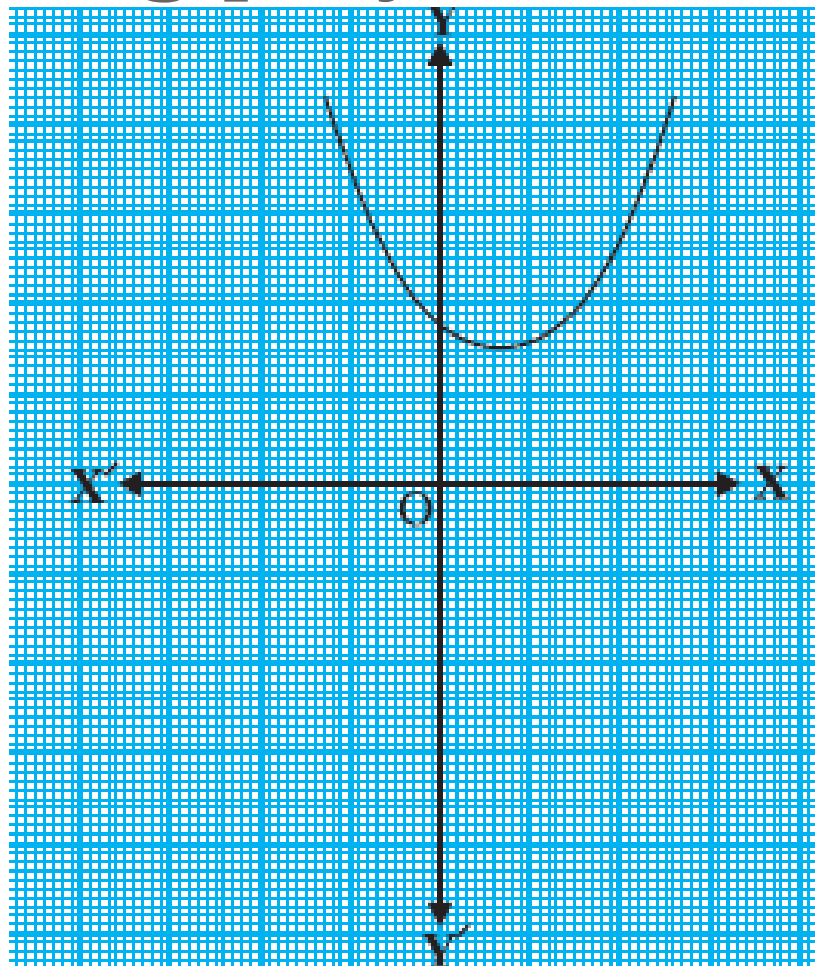
(b)



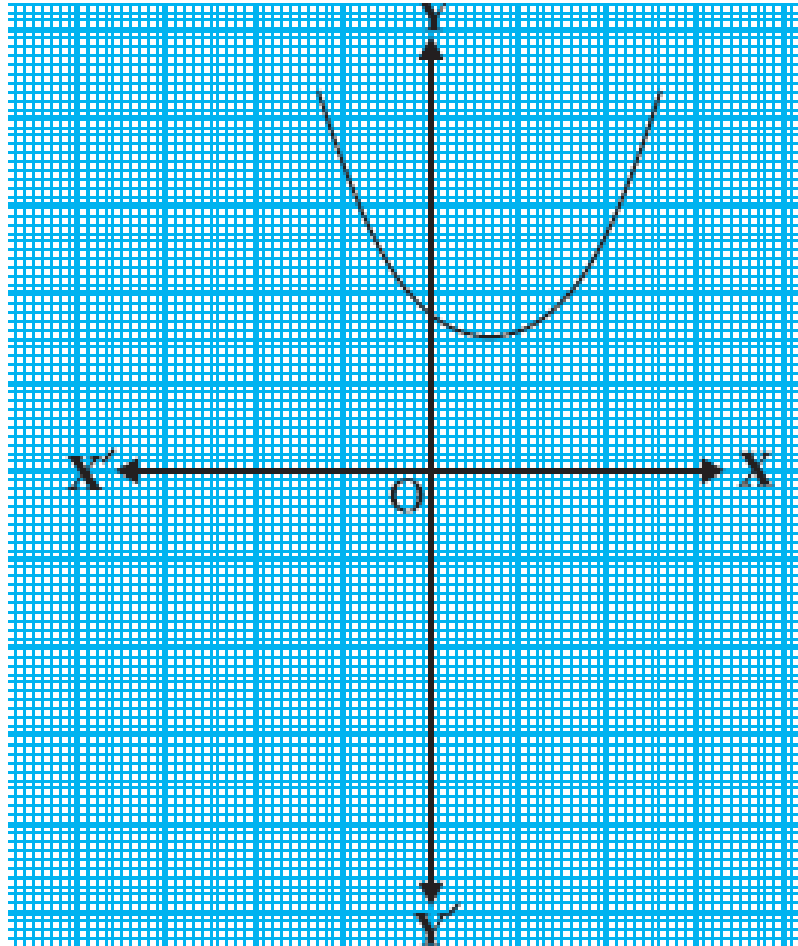
Answer: 1

How many zeroes do the following polynomials have ?

(c)



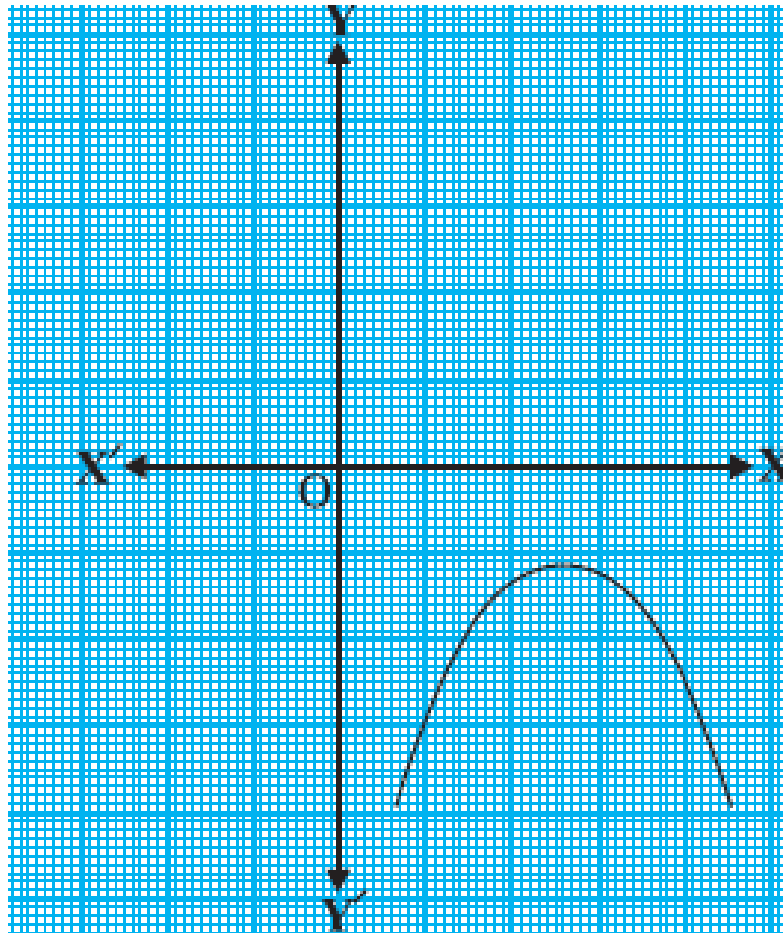
(c)



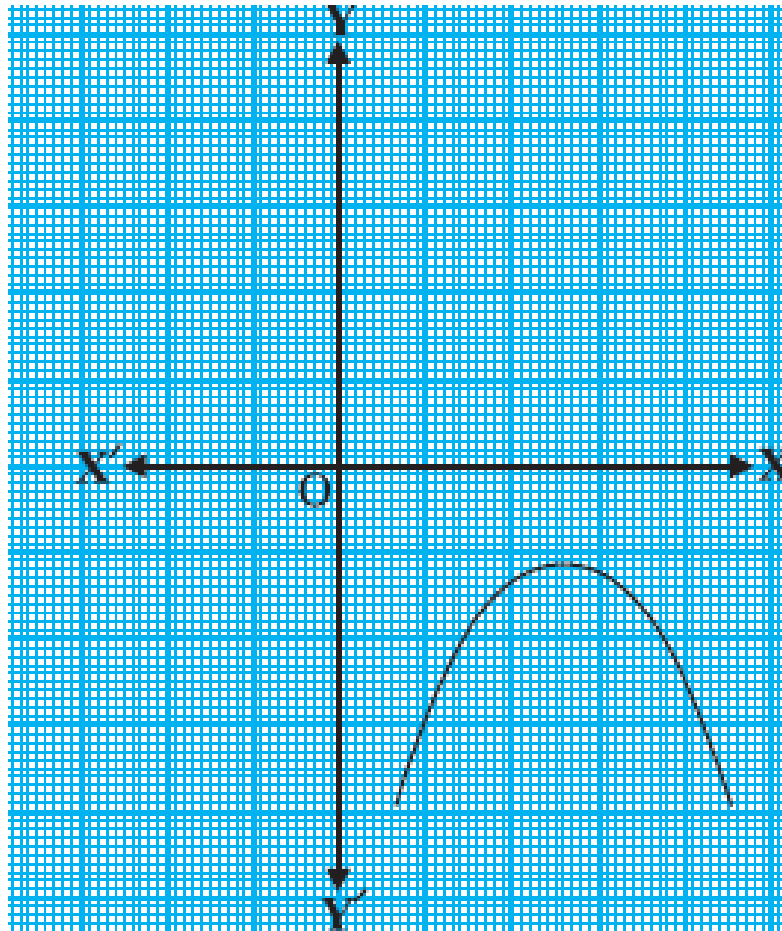
Answer: no zero

How many zeroes do the following polynomials have ?

(d)

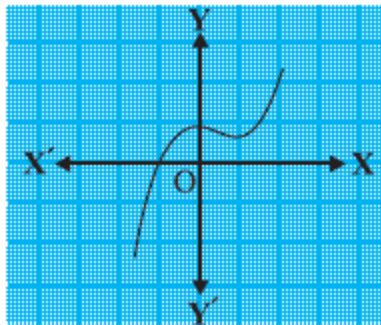


(d)

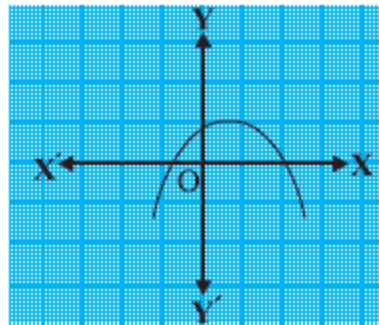


Answer: no zero

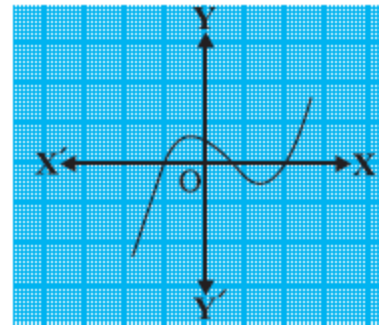
Some more.....



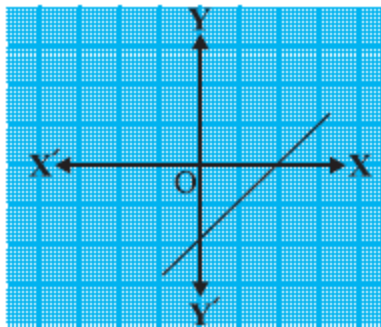
(i)



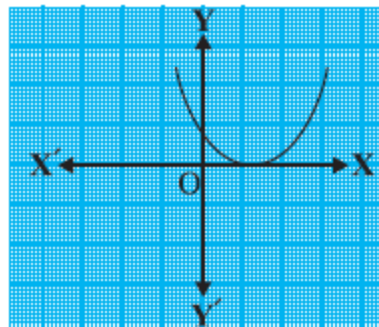
(ii)



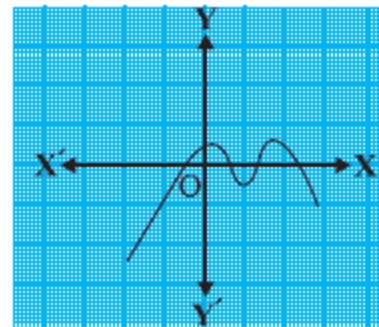
(iii)



(iv)



(v)



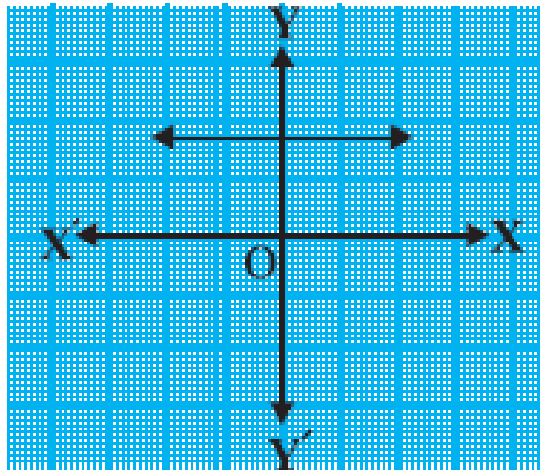
(vi)

TESTING TIME

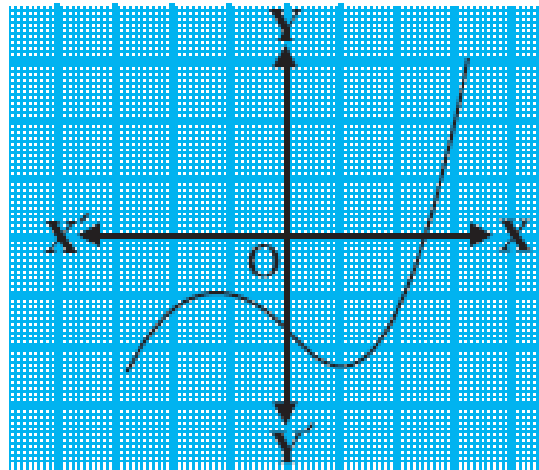
Solve Ex 2.1 and see if you can get the answers.

EXERCISE 2.1

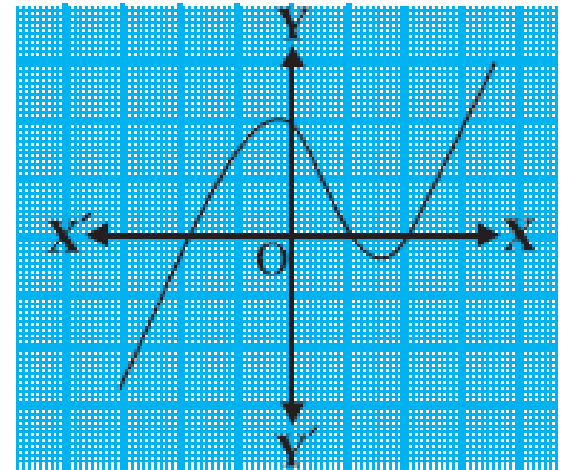
1. The graphs of $y = p(x)$ are given in Fig. 2.10 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



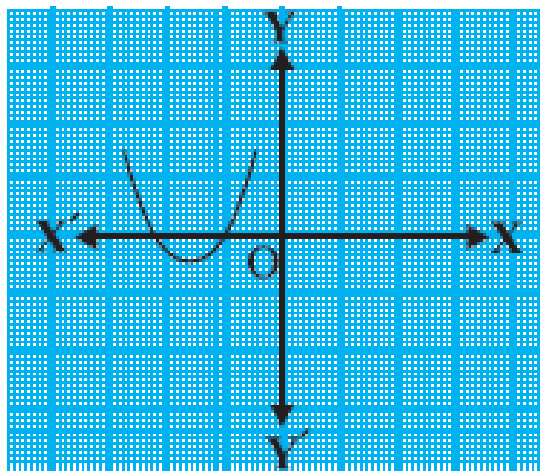
(i)



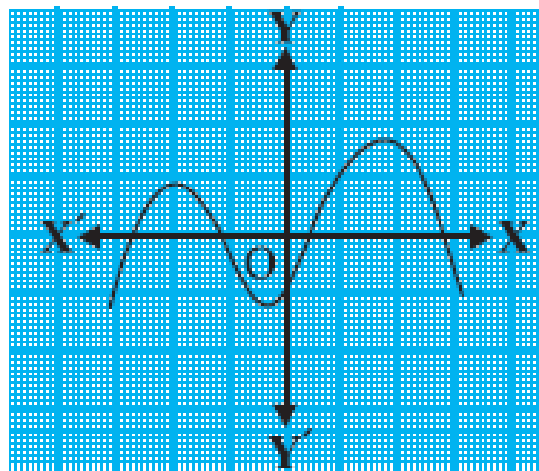
(ii)



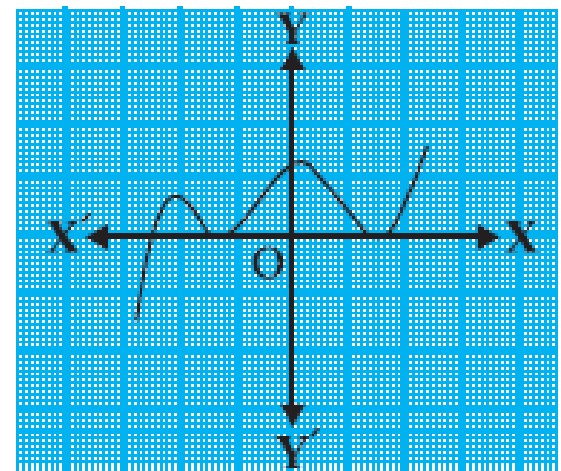
(iii)



(iv)



(v)



(vi)

WHAT'S NEW THIS TIME?

- Geometrical meaning of the zeroes of a polynomial
- Relationship between the zeroes and coefficients of a polynomial
- Division Algorithm for Polynomials

Objectives

- To find sum/product of roots *without* knowing the actual value of x .
- Use the sum/product of roots to solve for other results

Derivation of Formula

Given α and β are roots of a quadratic equation, we can infer that

$x_1 = \alpha$ and $x_2 = \beta$, where α and β are just some numbers.

For example, $x^2 - 5x + 6 = 0$ has roots α and $\beta \rightarrow$ means that $\alpha = 2$ and $\beta = 3$

However...

Sometimes, we do not need the values of x to help us solve the problem.

Knowing the relationship between the quadratic equation and its roots helps us save time

Derivation of Formula

$ax^2 + bx + c = 0$ has roots α and β -----(1)

This tells us that:

$$(x - \alpha)(x - \beta) = 0$$

Expanding, we have:

$$x^2 - \alpha x - \beta x + \alpha\beta = 0$$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

So, if we compare the above with (1):

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Result: $\alpha\beta = \frac{c}{a}$, $\alpha + \beta = -\frac{b}{a}$

In other words...

$ax^2 + bx + c = 0$ has roots α and β

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$$

Let us try

Q1. Find a quadratic polynomial , the sum and product of whose zeroes are 3 and 2 respectively.

Q2. Find the zeroes of the polynomial $x^2 - 3x + 4$ and verify the relationship between the zeroes and the coefficients..

Q3. Find a quadratic polynomial , whose zeroes are 1,1 respectively.

Application – Example 1

$2x^2 + 6x - 3 = 0$ has roots α and β , find the value of

(a) $\frac{1}{\alpha} + \frac{1}{\beta}$

Without finding α and β , we know the value for $\alpha + \beta$ and $\alpha\beta$

$$\alpha + \beta = -3 \text{ and } \alpha\beta = -3/2$$

Hence,

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-3}{-\frac{3}{2}} = 2$$

Application – Example 1—contd.

$2x^2 + 6x - 3 = 0$ has roots α and β , find the value of

(b)

We already know that: $\alpha + \beta = -3$ and $\alpha\beta = -3/2$

Expanding (b), we have

$$\begin{aligned}(2\alpha + 1)(2\beta + 1) &= 4\alpha\beta + 2\alpha + 2\beta + 1 \\&= 4\alpha\beta + 2(\alpha + \beta) + 1 \\&= 4\left(-\frac{3}{2}\right) + 2(-3) + 1 \\&= -11\end{aligned}$$

Application – Example 2

$x^2 - 2x - 4 = 0$ has roots α and β , find the value of

Step 1: $\alpha + \beta = 2$ and $\alpha\beta = -4$

To find this, we use the algebraic property

$$(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$$

Rearranging:

$$\begin{aligned}\alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= (2)^2 - 2(-4) \\ &= 12\end{aligned}$$

Independent Practice

$2x^2 + 4x - 1 = 0$ has roots α and β , form the equation with roots α^2 and β^2

Remember!

$$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$$

$$x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0$$

So in order to get the equation, find $\alpha^2 + \beta^2$ and $\alpha^2\beta^2$

DIVISION ALGORITHM

Ways to divide a polynomial

Factor Theorem

- Use zero of a polynomial to find the other factor
- Factorize it to get the zeroes

Long Division

- Actual division of two polynomials
- Factorize the quotient to get the remaining zeroes

DIVISION ALGORITHM

$$\text{Dividend} = (\text{Quotient} \times \text{Divisor}) + \text{Remainder}$$

Example 1:

Divide $2x^2 + 3x + 1$ by $x + 2$.

$$\begin{array}{r} 2x - 1 \\ x + 2 \overline{) 2x^2 + 3x + 1} \\ \underline{2x^2 + 4x} \\ -x + 1 \\ \underline{-x - 2} \\ 3 \end{array}$$

Example 2:

Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$, and verify the division algorithm.

$$\begin{array}{r}
 x - 2 \\
 \hline
 -x^2 + x - 1 \overline{) -x^3 + 3x^2 - 3x + 5} \\
 \underline{-x^3 + x^2 - x} \\
 2x^2 - 2x + 5 \\
 \underline{2x^2 - 2x + 2} \\
 3
 \end{array}$$

Divisor $\times$ Quotient + Remainder

$$= (-x^2 + x - 1)(x - 2) + 3$$

$$= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$$

$$= -x^3 + 3x^2 - 3x + 5$$

$$= \text{Dividend}$$

Example: 3

Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

$$\begin{array}{r} 2x^2 - 3x + 1 \\ x^2 - 2 \overline{) 2x^4 - 3x^3 - 3x^2 + 6x - 2} \\ \underline{- 2x^4 \qquad + 4x^2} \\ -3x^3 + x^2 + 6x - 2 \\ \underline{- 3x^3 \qquad + 6x} \\ x^2 - 2 \\ \underline{- x^2 \qquad + 2} \\ 0 \end{array}$$

Example 3...contd..

So, $2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x^2 - 2)(2x^2 - 3x + 1)$.

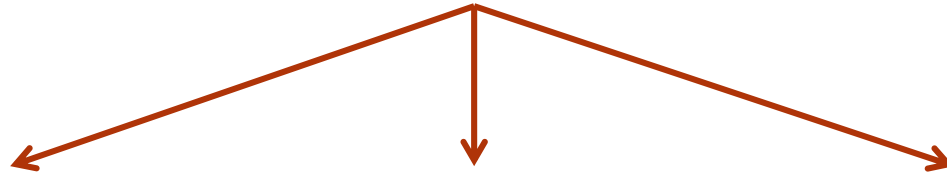
Now, by splitting $-3x$, we factorise $2x^2 - 3x + 1$ as $(2x - 1)(x - 1)$. So, its zeroes are given by $x = \frac{1}{2}$ and $x = 1$. Therefore, the zeroes of the given polynomial are

$\sqrt{2}$, $-\sqrt{2}$, $\frac{1}{2}$, and 1 .

Let us try.....

- Obtain all other zeroes of the polynomial $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{5/3}$ and $-\sqrt{5/3}$

What have we learnt so far.....



Zeroes of a polynomial

- Geometrical representation

Relationship between zeroes and coefficients

- Sum and product of zeroes
- Forming a polynomial

Division Algorithm

- Long Division
- Finding all zeroes of the given polynomial

Just a closing thought....

“Do not worry about your difficulties in Mathematics. I assure you, mine are still greater...”

---- Albert Einstein