

A decorative graphic on the left side of the slide consisting of white lines and circles on a dark blue background, resembling a circuit board or a stylized tree structure.

QUADRATIC EQUATIONS

DEFINITION

- In Mathematics Quadratic Equation is equation of second degree.
- A quadratic equation in the variable x is an equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers, $a \neq 0$

- A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.

CHECK WHETHER THE FOLLOWING ARE QUADRATIC EQUATIONS:

Example:

$$(x - 2)^2 + 1 = 2x - 3$$

- $\text{LHS} = (x - 2)^2 + 1 = x^2 - 4x + 4 + 1 = x^2 - 4x + 5$
- Therefore, $(x - 2)^2 + 1 = 2x - 3$ can be rewritten as
- $x^2 - 4x + 5 = 2x - 3$
- i.e., $x^2 - 6x + 8 = 0$
- It is of the form $ax^2 + bx + c = 0$.
- Therefore, the given equation is a quadratic equation.

Another Example

$$x(x + 1) + 8 = (x + 2)(x - 2)$$

- Since $x(x + 1) + 8 = x^2 + x + 8$ and $(x + 2)(x - 2) = x^2 - 4$

Therefore, $x^2 + x + 8 = x^2 - 4$

- i.e., $x + 12 = 0$
- It is not of the form $ax^2 + bx + c = 0$.
- Therefore, the given equation is not a quadratic equation.

HOW TO SOLVE IT?

There are 3 ways to solve a quadratic equation

1. We can factor a quadratic equation.
2. We can complete the square
3. We can use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus $ax^2 + bx + c = 0$ has two roots α and β , given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

SOLVING BY FACTORISATION

Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

$$6x^2 - x - 2 = 6x^2 + 3x - 4x - 2$$

$$= 3x(2x + 1) - 2(2x + 1)$$

$$= (3x - 2)(2x + 1)$$

The roots of $6x^2 - x - 2 = 0$ are the values of x for which $(3x - 2)(2x + 1) = 0$. Therefore, $3x - 2 = 0$ or $2x + 1 = 0$,

$$x = 2/3 \text{ or } x = -1/2$$

COMPLETING THE SQUARE METHOD

We can use following steps

- Step 1 Divide all terms by a (coefficient of x^2)
- Step 2 Transpose the constant term on RHS
- Step 3 Add $(b/2a)^2$ on both the sides
- Step 4 Complete the square on both sides
- Step 5 Take the square root on both sides
- Step 6 Add or subtract the number that remains on the LHS of the equation to find x .

NATURE OF ROOTS

we find the discriminant of the given quadratic equation as follows:

$$D=b^2-4ac,$$

(a) If $D > 0$, then the given equation will have real and distinct roots

and we can find the roots of the given equation.

(b) If $D = 0$, then the given equation will have real and equal roots.

(c) If $D < 0$, then the given equation will have no real roots.

In this case roots will be imaginary. Here we find the imaginary roots only.

In case of real roots we can find the roots by using the formula.

$$X = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

EXAMPLE SOLVE $x^2 + 4x + 1 = 0$.

- Step 1 can be skipped as coefficient of x^2 is already 1
- Step 2 Transpose 1 on RHS
- $x^2 + 4x = -1$
- Completing the square by adding 4 on both sides
- $x^2 + 4x + 4 = -1 + 4$
- Making perfect squares on both sides
- $(x + 2)^2 = (\sqrt{3})^2$
- Taking square root on both sides and solving for x
- $x + 2 = \pm \sqrt{3}$
- $x = -2 \pm \sqrt{3}$

FIND TWO CONSECUTIVE ODD POSITIVE INTEGERS, SUM OF WHOSE SQUARES IS 290.

Let the smaller of the two consecutive odd positive integers be x .
Then, the second integer will be $x + 2$. Given

$$x^2 + (x + 2)^2 = 290$$

$$\text{i.e., } x^2 + x^2 + 4x + 4 = 290$$

$$\text{i.e., } 2x^2 + 4x - 286 = 0$$

$$\text{i.e., } x^2 + 2x - 143 = 0$$

Using the quadratic formula, we get

$$x = \frac{-2 \pm \sqrt{576}}{2}$$

Either $x = 11$ or $x = -13$

But as numbers are positive so 11 and 13