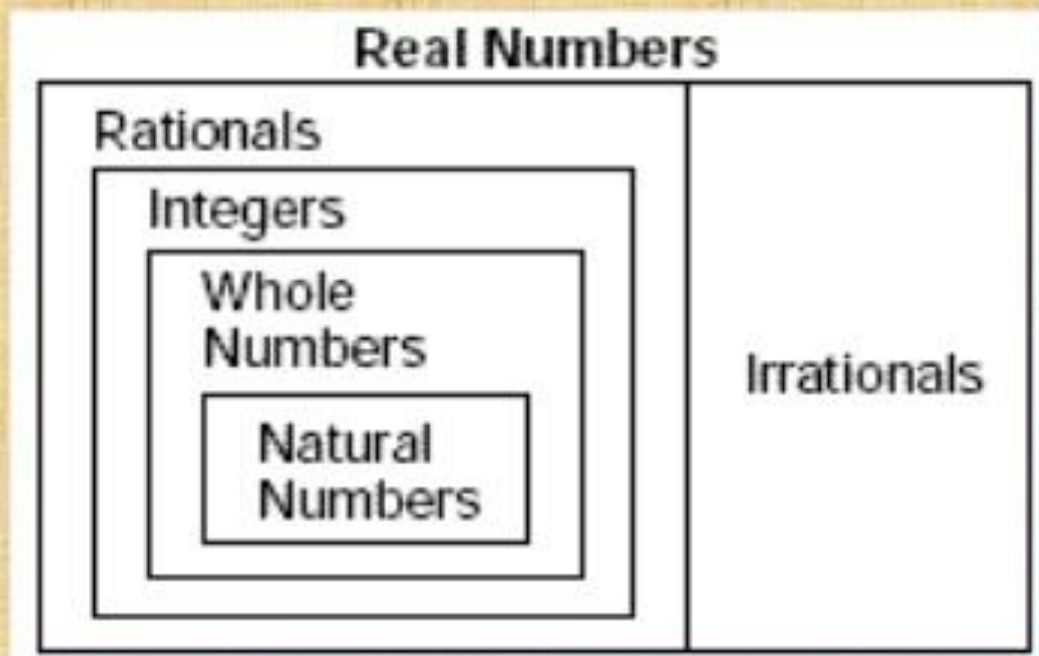


REAL NUMBERS

Real Numbers

- *Real numbers consist of all the rational and irrational numbers.*
- *The real number system has many subsets:*
 - ▣ *Natural Numbers*
 - ▣ *Whole Numbers*
 - ▣ *Integers*



Euclid's Division Lemma

Euclid's division lemma states that " For any two positive integers a and b , there exist integers q and r such that $a=bq+r$, $0 \leq r < b$



Lemma: A lemma is a proven statement used for proving another statement

Example . For $a=15, b=3$ it is observed that

*$15=3(5)+0$
where $q=5$ and $r=0$*

Euclid's Division Lemma

Let us show that *any positive integer is of the form $4q$, $4q + 1$, $4q + 2$, $4q + 3$*

Let ' a ' be any positive integer

On dividing ' a ' by 4, let ' q ' be the quotient and r be the remainder.

Then, by Euclid's Division Lemma, we have

$$(a = b \times q + r)$$

$$a = 4q + r, \text{ where } 0 \leq r < 4$$

$$\therefore a = 4q + r, \text{ where } r = 0, 1, 2 \text{ or } 3$$

$$\text{➤ } a = 4q \quad (\text{when } r = 0)$$

$$\text{➤ } a = 4q + 1 \quad (\text{when } r = 1)$$

$$\text{➤ } a = 4q + 2 \quad (\text{when } r = 2)$$

$$\text{➤ } a = 4q + 3 \quad (\text{when } r = 3)$$

$\therefore$ *any positive integer is of the form $4q$, $4q + 1$, $4q + 2$, $4q + 3$*

Euclid Division Algorithm

Algorithm: An algorithm is a series of well defined steps which gives a procedure for solving a type of problem.

Euclid division algorithm can be used to find the HCF of two numbers. It can also be used to find some common properties of numbers.

To obtain the HCF of two positive integers, say c and d , with $c > d$, we have to follow the steps below:

STEP 1: Apply euclid division lemma, to c and d . So, we find whole numbers, q and r such that $c = dq + r$

STEP 2: If $r = 0$, d is the HCF of c and d . If r does not equal to 0, apply the division lemma to d and r .

STEP 3: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

Use division algorithm to find HCF OF 225 AND 135

We start with larger number 225

By using Euclid's Division Algorithm

$$225 = 135 \times 1 + 90$$

Since remainder $90 \neq 0$, we apply division lemma to 135 and 90

$$135 = 90 \times 1 + 45$$

We consider new divisor 90 and new remainder 45, and apply division lemma to obtain

$$90 = 45 \times 2 + 0$$

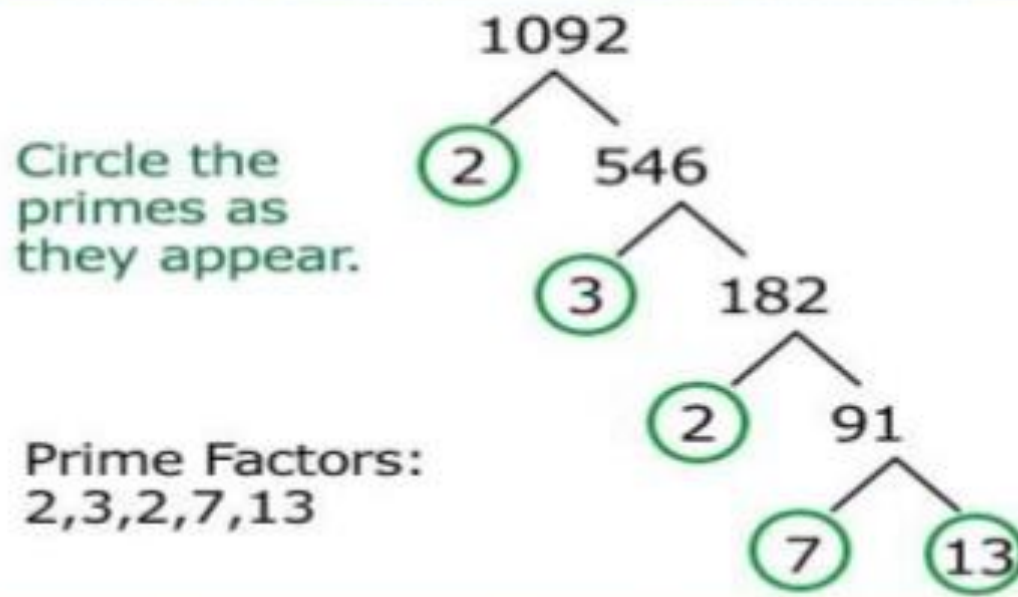
Since the remainder is 0, the process stops here.

Now divisor at this step is 45

Hence HCF of 225 and 135 is 45.

The Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes. This representation is called prime factorisation of the number. This factorisation is unique, apart from the order in which the prime factors occur.



Check whether 6^n can end with the digit 0 for any natural number n .

If any number to end with zero, it should be divisible by 10 or in other words it should be divisible by 2 and 5 as

$$10 = 2 \times 5$$

Prime factorization of $6^n = (2 \times 3)^n$

It can be observed that in prime factorization of 6 only contains 2 and 3.

According to uniqueness of Fundamental Theorem of Arithmetic prime factorization of 6^n will not contain 5.

So there is no natural number for which 6^n can end with the digit 0 for any natural number n .

Highest Common Factor(HCF):

The HCF of two numbers is equal to the product of the terms containing the least powers of common prime factors of the two numbers.

Lowest Common Multiple(LCM):

The LCM of two numbers is equal to the product of the terms containing the greatest powers of all prime factors of the two numbers.

Find the LCM and HCF of 6 and 20 by the prime factorisation method.

We have : $6 = 2^1 \times 3^1$ and $20 = 2 \times 2 \times 5 = 2^2 \times 5^1$

- $\text{HCF}(6, 20) = 2^1 =$ **Product of the smallest power of each common prime factor in the numbers.**
- $\text{LCM}(6, 20) = 2^2 \times 3^1 \times 5^1 =$ **Product of the greatest power of each prime factor, involved in the numbers.**

Relationship between HCF and LCM

For any two positive integers a and b ,

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

Example : if $a=3$ and $b=6$

$$\text{HCF}(3,6) \times \text{LCM}(3,6) = 3 \times 6$$

$$3 \times 6 = 18$$

$$18 = 18$$

Hence verified.....

By prime factorization

$$26 = 2 \times 13$$

$$91 = 7 \times 13$$

$$\text{HCF}(26, 91) = 13$$

$$\text{LCM}(26, 91) = 13 \times 2 \times 7 = 26 \times 7 = 182$$

$$\text{LCM} \times \text{HCF} = 13 \times 182 = 2366$$

$$\text{Product of two numbers} = 26 \times 91 = 2366..$$

$$\text{Hence, L.C.M.} \times \text{H.C.F.} = \text{Product of two numbers.}$$

Question: $\text{HCF}(306, 657) = 9$, FIND
 $\text{LCM}(306, 657)$

$$\text{HCF}(306, 657) \times \text{LCM}(306, 657) = 306 \times 657$$

$$9 \times \text{LCM}(306, 657) = 306 \times 657$$

$$\begin{aligned}\text{LCM}(306, 657) &= (306 \times 657) / 9 \\ &= 2338\end{aligned}$$

Irrational Numbers

- **Irrational numbers** are any numbers that cannot be expressed as a/b .
- They are expressed as **non-terminating, non-repeating decimals**; decimals that go on forever without repeating a pattern.
- Examples of irrational numbers:
 - 0.34334333433334...
 - 45.86745893...
 - Pi $\sqrt{2}$



Things to Remember

. Let p be a prime number. If p divides a , then p also divides a , where a is a positive integer.

. When prime factorisation of q is of the form $2^m 5^n$. Then x has a decimal expansion which terminates and when q is not of the form $2^m 5^n$, then x has a decimal expansion which is non-terminating repeating or recurring.



$\sqrt{2}$ is irrational

- Suppose $\sqrt{2}$ is rational, then it can be expressed in fraction form $\frac{a}{b}$. Let us assume that our fraction is in lowest term, i.e., their only common divisor is 1. Then, $\sqrt{2} = a/b$
- Squaring both sides, we have $2 = a^2/b^2$
- Or $2b^2 = a^2$
- Therefore 2 divides a^2 . By theorem 2 also divides a
- We can write $a = 2c$ for some integer c
- Substituting for a, we get $2b^2 = 4c^2$ i.e. $b^2 = 2c^2$
- Therefore 2 divides b^2 . By theorem 2 also divides b.
- Therefore a and b have at least 2 as a common factor.
- But this contradicts our assumption that a and b are co-prime.
- So our assumption is contradicted and $\sqrt{2}$ is irrational

. Is $17/8$ has a terminating decimal expansion?

Sol : We have $17/8$

$$= 17/2^3 \times 5^2$$

So , the denominator 8 of $17/8$ is of the form $2^m \times 5^n$
therefore it has a terminating decimal expansion.

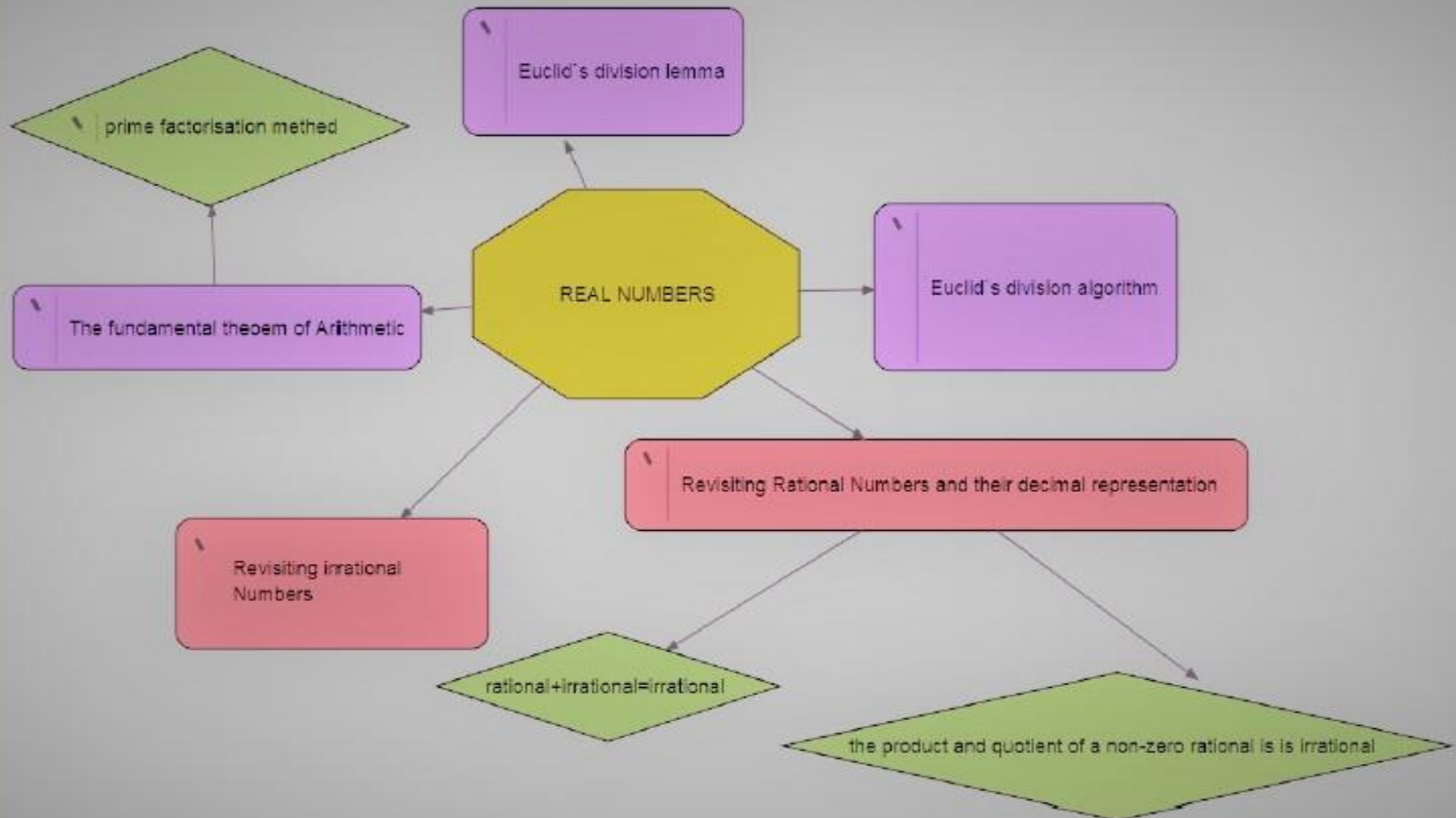
. Is $29/343$ has a terminating decimal expansion?

Sol : We have $29/343$

$$= 29/3^5$$

Clearly 343 is not of the form $2^m \times 5^n$
therefore it has a non terminating decimal
expansion.

CONCEPT MAP





THANK

YOU