

Planning Ahead :-

1. Cube

- i. Lateral surface area*
- ii. Total surface area*
- iii. Volume*
- iv. Examples*

2. Cuboid

- i. Lateral Surface Area*
- ii. Total surface area*
- iii. Volume*





SURFACE AREA AND VOLUME



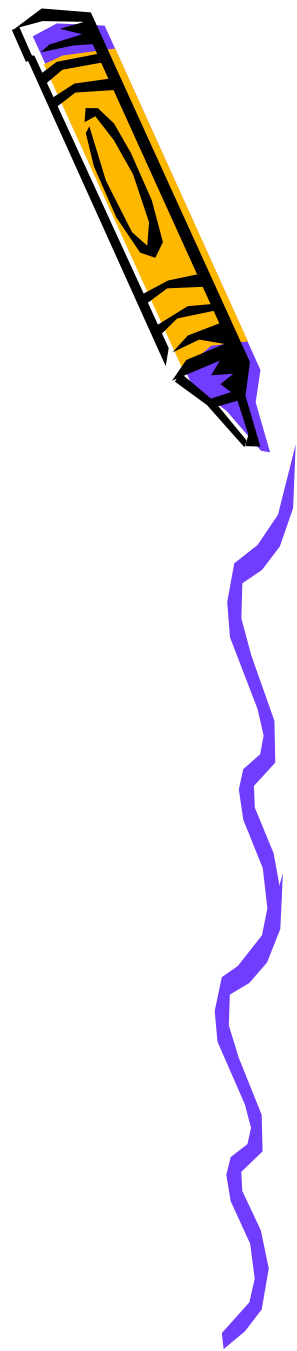
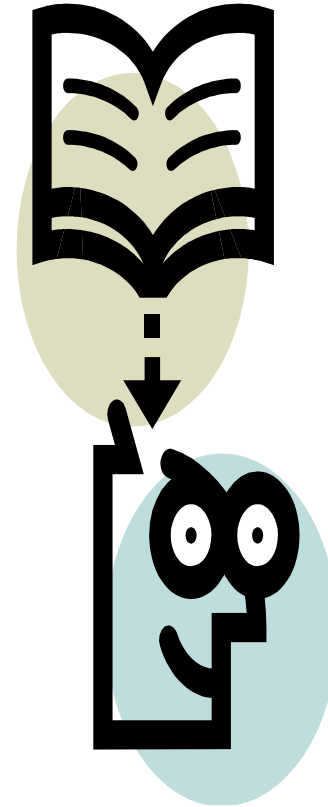
Planning Ahead :-

3. Cylinder

- i. Curved surface area*
- ii. Total surface area*
- iii. Volume*

4. Cone

- i. Curved surface area*
- ii. Total surface area*
- iii. Volume*



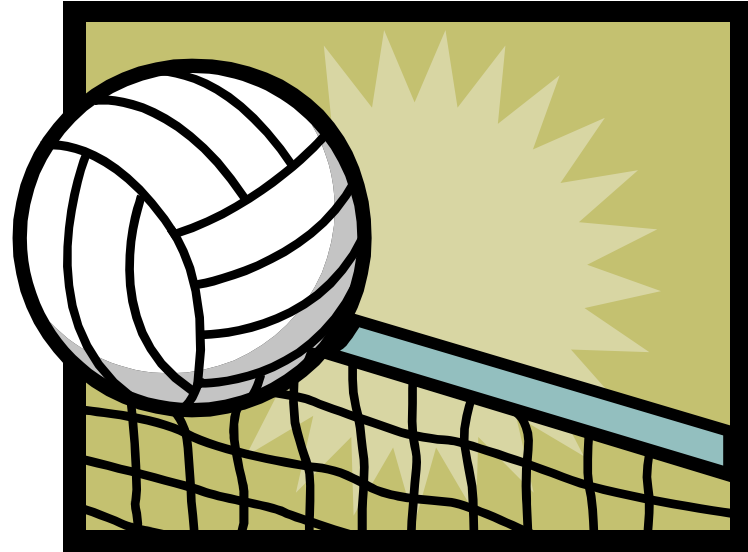
Planning Ahead :-

5. Sphere

- i. Surface area*
- ii. Volume*

6. Hemisphere

- i. Curved surface area*
- ii. Total surface area*
- iii. Volume*



Planning Ahead :-

7. Frustum

- i. Curved surface area*
- ii. Total surface area*
- iii. Volume*

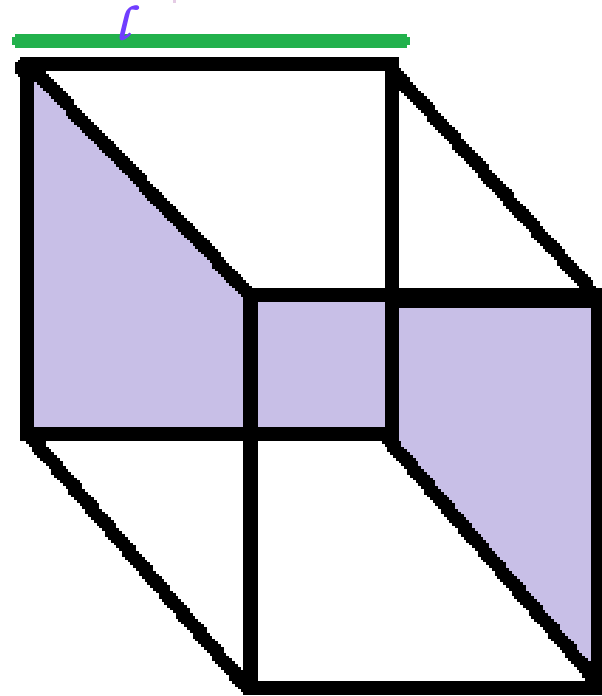
8. Surface area of combination of solids

9. Conversion of solids



CUBE :-

A cube is a three dimensional figure, with six sides- all Sides in shape of Square.



Length of side is denoted by the letter 'l'.



Lateral Surface Area :-

Lateral surface area refers to the area of only the walls (it does not include the area of the floor and roof).

Formula :- $4 l^2$

Derivation :- Since all the sides of cube are in the shape of square.

area of the square = l^2

no. of sides = 4

area = $4l^2$



EXAMPLES :-

1. Find the lateral surface area of the cube with side of 15cm.

Sol.- We are given-

$$l = 15\text{cm}$$

$$\text{lateral surface area} = 4l^2$$

$$= 4(15\text{ cm})^2$$

$$= 4 \star 225\text{cm}^2$$

$$= 900\text{cm}^2$$



EXAMPLES :-

2. Find the lateral surface area of the cube with area of one face 81cm^2 . Also find the length of the side.

Sol. – Area of one face = 81cm^2

$$l^2 = 81\text{cm}^2$$

$$l = \sqrt{81\text{cm}^2}$$

$$l = 9\text{cm}$$

Lateral surface area of cube = $4l^2$

$$= 4(9\text{cm})^2$$

$$= 4 \times 81\text{cm}^2$$

$$= 324\text{cm}^2$$



EXAMPLES :-

3. Find the side of the cube whose lateral surface area is 196cm^2 .

Sol.- Lateral surface area of cube = 196cm^2

$$4l^2 = 196\text{cm}^2$$

$$l^2 = 196\text{cm}^2/4$$

$$l^2 = 49\text{cm}^2$$

$$l = \sqrt{49\text{cm}^2}$$

$$l = 7\text{ cm}$$



Total Surface Area Of Cube :-

Formula :- $6l^2$

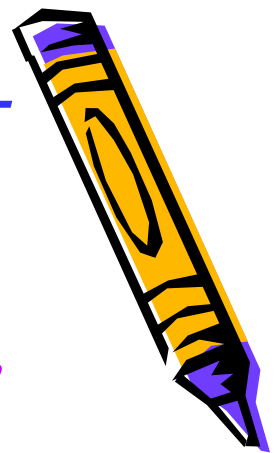
Derivation :- Since all the faces of a cube are squares ,

Area of square = l^2

No. of square = 6

Area of 6 square = Total surface area of cube
= $6l^2$

Therefore , total surface area of the cube is $6l^2$.



EXAMPLE :-

1. Find the total surface area of the cube with side of 7.2cm.

Sol. - We are given,

$$l = 7.2\text{cm}$$

$$\text{Total surface area} = 6l^2$$

$$= 6(7.2\text{cm})^2$$

$$= 6 \times 51.84 \text{ cm}^2$$

$$= 311.04 \text{ cm}^2$$



EXAMPLE :-

2. A gift in a shape of cube is to be wrapped in a gift paper.
Find the total cost of the wrapper need to cover the gift
whose side is 6.8cm, at the cost of Rs.5 per m².

Sol. – We are given ,

$$\text{side of the cube (l)} = 6.8 \text{ cm}$$

$$\begin{aligned}\text{Total surface area} &= 6 (l)^2 \\ &= 6 (6.8 \text{ cm})^2 \\ &= 6 \star 46.24 \text{ cm}^2 \\ &= 277.44 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{cost of the wrapper} &= \text{Rs. } 5/\text{m}^2 \\ &= 5 \star 2.7744 \text{ m}^2 \\ &= \text{Rs. } 13.87\end{aligned}$$

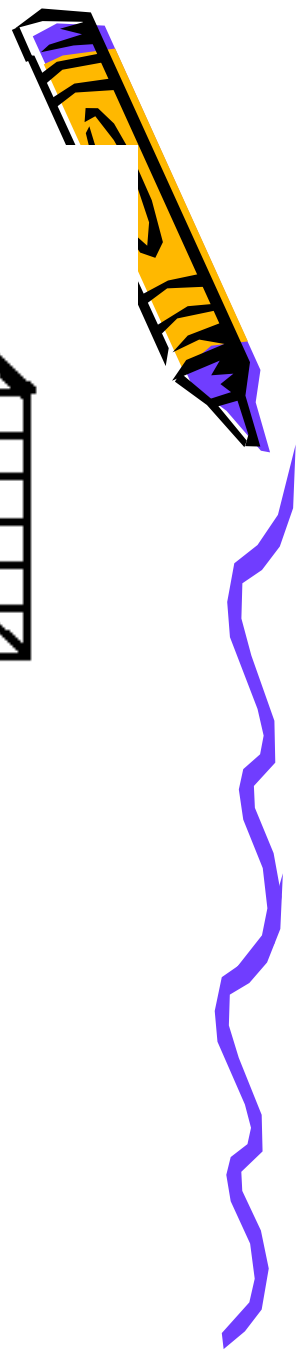
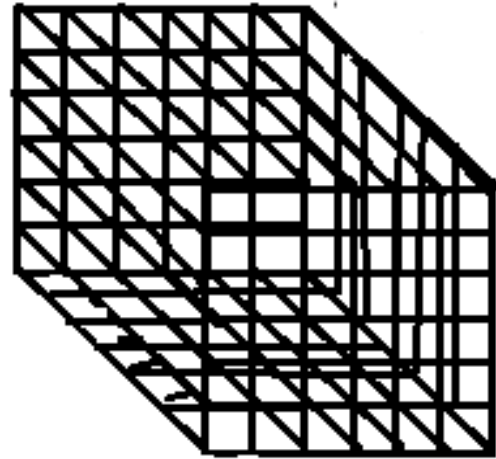


Volume Of Cube : -

Volume of the cube refers to the space inside the six walls.

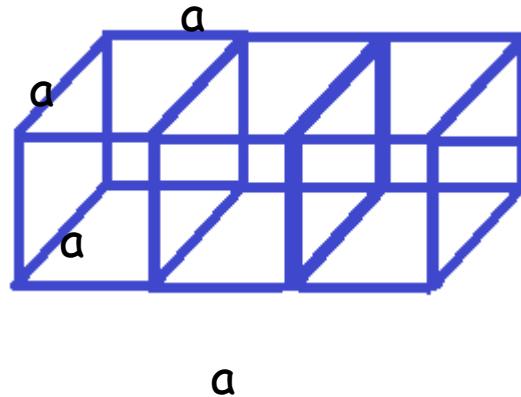
$$\begin{aligned}\text{Formula :- } l \times l \times l \\ = l^3\end{aligned}$$

Unit :- unit³



EXAMPLE :-

1. Three equal cubes are placed side by side in a row. Find the volume of the new figure formed, Also find its ratio in respect to the single cube



Sol.- Let 'a' be the edge of each cube.

$$\text{Volume of the single cube} = a^3$$

$$\begin{aligned}\text{Sum of the volume of three cubes} &= 3 \times a^3 \\ &= 3a^3\end{aligned}$$

$$\text{Ratio of the volume of two figures} = \frac{\text{Volume of the cube}}{\text{Volume of the new figure}}$$

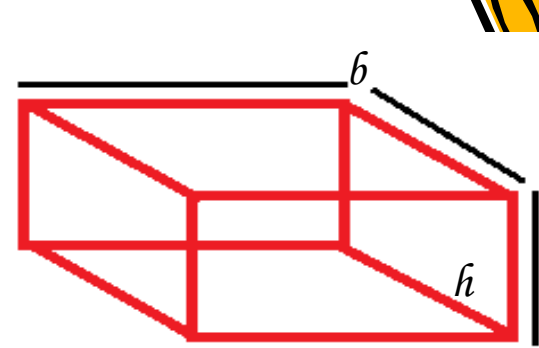
$$= \frac{a^3}{3a^3}$$

$$= 1:3$$



CUBOID :-

Cuboid is a three dimensional figure, with six sides and all sides of equal length. In Cuboid opposite rectangles are equal.



It's three dimensions are :-

- 1. Length (l)*
- 2. Breadth (b)*
- 3. Height (h)*



LATERAL SURFACE AREA:-

Lateral surface area of the cuboid refer to the area of the four walls of it

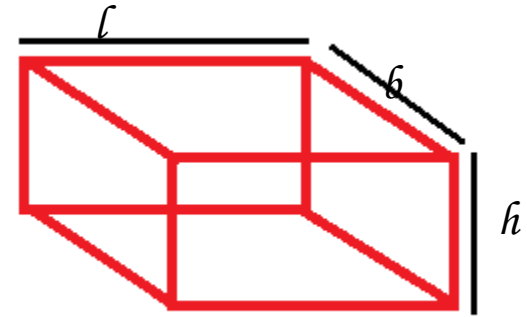
Formula :- $2(l+b)h$

Derivation :- Area of rectangle 1 = $l \times h$

Area of rectangle 2 = $b \times h$

Area of rectangle 3 = $l \times h$

Area of rectangle 4 = $b \times h$



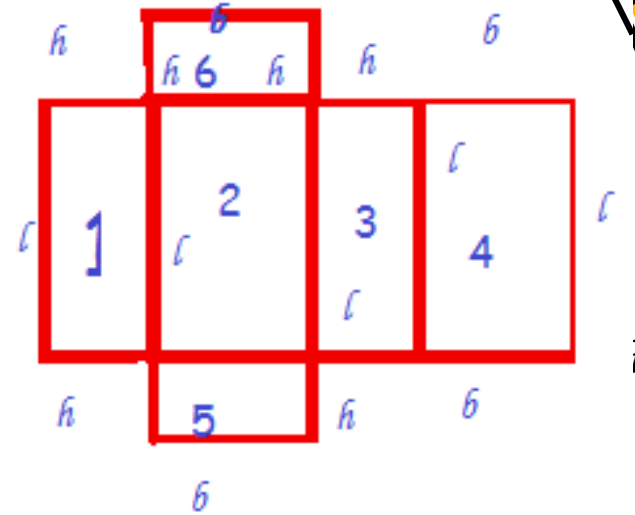
$$\begin{aligned}\text{Total area} &= 2lh + 2bh \\ &= 2(l+b)h\end{aligned}$$



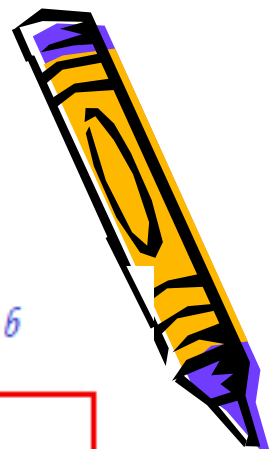
TOTAL SURFACE AREA:-

Formula :- $2(lb + bh + hl)$

*Derivation :- Area of rectangle 1 ($=lh$) +
Area of rectangle 2 ($=lb$) +
Area of rectangle 3 ($=lh$) +
Area of rectangle 4 ($=lb$) +
Area of rectangle 5 ($=bh$) +
Area of rectangle 6 ($=bh$)*



$$\begin{aligned} &= 2(l \times b) + 2(b \times h) + 2(l \times h) \\ &= 2(lb + bh + hl) \end{aligned}$$



EXAMPLE :-

1. Marry wants to decorate her Christmas tree. She wants to place her tree on a wooden box covered with coloured paper with picture of Santa clause on it . She must know the exact quantity of paper to buy it. If the dimensions of the box are : $80\text{cm} \times 40\text{cm} \times 20\text{cm}$, how many square sheets of paper of side 40cm would she require?

Sol. — *The surface area of the box* $= 2(lb + bh + hl)$

$$= 2[(80 \times 40) + (40 \times 20) + (20 \times 80)]$$
$$= 2 (3200 + 800 + 1600)$$
$$= 2 \times 5600 \text{ cm}^2 = 11200 \text{ cm}^2$$

The area of each sheet of paper $= 40 \times 40 \text{ cm}^2 = 1600 \text{ cm}^2$

Therefore no. of sheets require $= \text{Surface area of the box} / \text{Area of one sheet of paper}$

$$= 11200 / 1600 = 7$$

Therefore , she would require 7 sheets.



EXAMPLE :-

1. A wall of length 10m was to be built across an open ground. the height of wall is 4m and thickness of the wall is 24cm. If this wall is to be built up with bricks whose dimensions are $24\text{cm} \times 12\text{cm} \times 8\text{cm}$, how many bricks would be required?

Sol. – We are given,

$$\text{Length} = 10\text{m} = 1000\text{cm}$$

$$\text{Breadth} = 24\text{cm}$$

$$\text{Height} = 4\text{m} = 400\text{cm}$$

$$\text{So, volume of wall} = \text{length} \times \text{breadth} \times \text{height} = 1000 \times 24 \times 400\text{cm}^3$$

Now, each brick is a cuboid with length=24cm, breadth=12cm, height= 8cm

$$\text{Volume of each brick} = l \times b \times h = 24 \times 12 \times 8\text{cm}^3$$

$$\text{So, no. of brick require} = \text{volume of the wall} / \text{Volume of each brick}$$

$$= 1000 \times 24 \times 400 / 24 \times 12 \times 8$$

$$= 4166.6$$

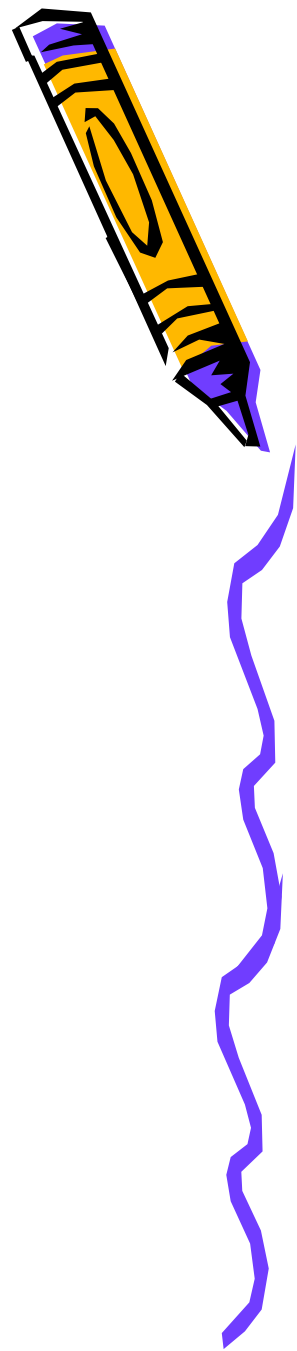
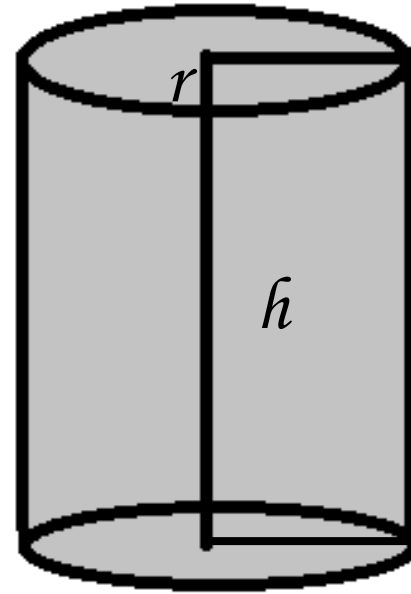
So, the wall requires 4167 bricks.



CYLINDER:-

A right circular cylinder is a solid generated by the revolution of a rectangle about one of its side.

It is a folded rectangle with both circular ends.



CURVED SURFACE AREA OF CYLINDER

Curved surface area of the cylinder :-

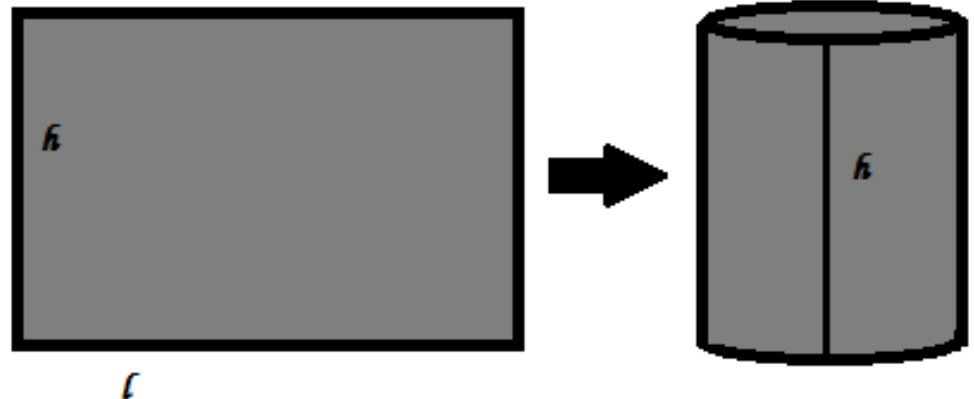
= Area of the rectangular sheet

*= length * breadth*

*= perimeter of the base of the cylinder * h*

*= $2\pi r * h$*

= $2\pi rh$



EXAMPLE :-



1. Shubhi had to make a model of a cylindrical kaleidoscope for her project. She wanted to use chart paper to use chart paper to make the curved surface of it. What would be the area of chart paper required by her, if she wanted to make a kaleidoscope of length-25cm with a 3.5cm radius ?

Sol. – Radius of the base of the cylindrical kaleidoscope (r) = 3.5cm
Height (length) of kaleidoscope (h) = 25cm

$$\begin{aligned}\text{Area of paper required} &= \text{curved surface area of kaleidoscope} \\ &= 2\pi rh \\ &= 2 \times \frac{22}{7} \times 3.5 \times 25 \text{ cm}^2 \\ &= 550 \text{ cm}^2\end{aligned}$$



TOTAL SURFACE AREA OF CYLINDER:-

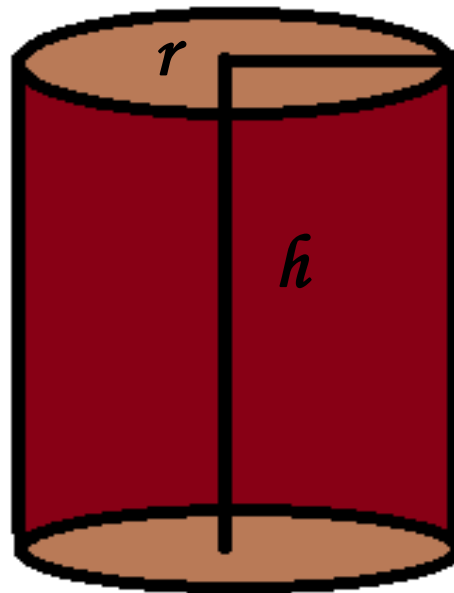
Total surface area of a cylinder :

= area of the rectangular sheet + 2 (area of the circular regions)

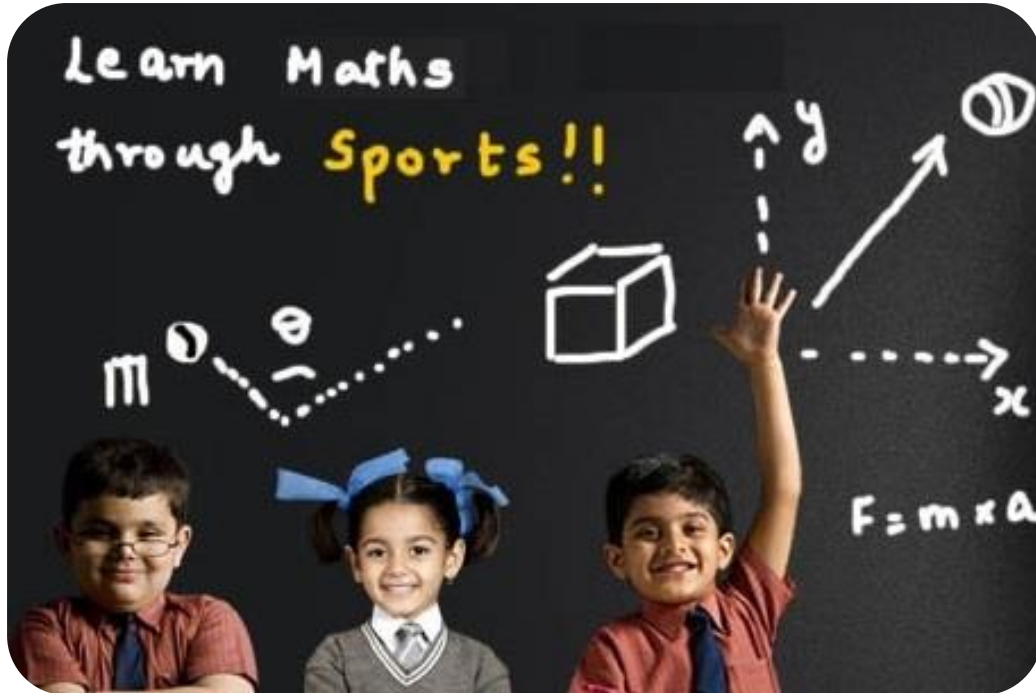
= perimeter of the base of cylinder $\times h$ + 2 (area of circular base)

$$= 2\pi rh + 2\pi r^2$$

$$= 2\pi r (r + h)$$

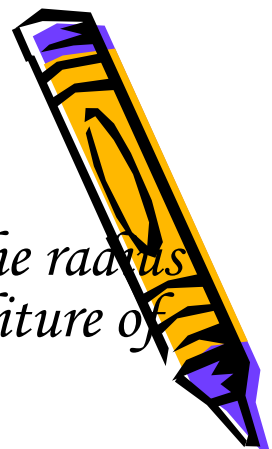


MATHS PROJECT WORK



SURFACE AREA AND VOLUME





EXAMPLE :-

1. A barrel is to be painted from inside and outside. It has no lid. The radius of its base and height is 1.5m and 2m respective. Find the expenditure of painting at the rate of Rs. 8 per square meter.

Sol. – Given, $r = 1.5\text{m}$, $h = 2\text{m}$

$$\text{Base area of barrel} = \pi r^2$$

$$\text{Base area to be painted (inside and outside)} = 2 \pi r^2$$

$$= 2 * 3.14 * (1.5)^2 \text{ cm}^2$$

$$= 2 * 3.14 * 2.25 = 14.13 \text{ cm}^2$$

$$\text{Curved surface area of barrel} = 2 \pi r h$$

$$\text{Area to be painted} = 2 * 2 \pi r h$$

$$= 4 * 3.14 * 1.5 * 2 \text{ cm}^2$$

$$= 12 * 3.14 \text{ cm}^2 = 37.68 \text{ cm}^2$$

$$\text{Total area to be painted} = (37.68 + 14.13) \text{ cm}^2 = 51.81 \text{ cm}^2$$

$$\text{Expenditure on painting} = \text{Rs. } 8 * 51.81$$

$$= \text{Rs. } 414.48$$



VOLUME OF CYLINDER:-

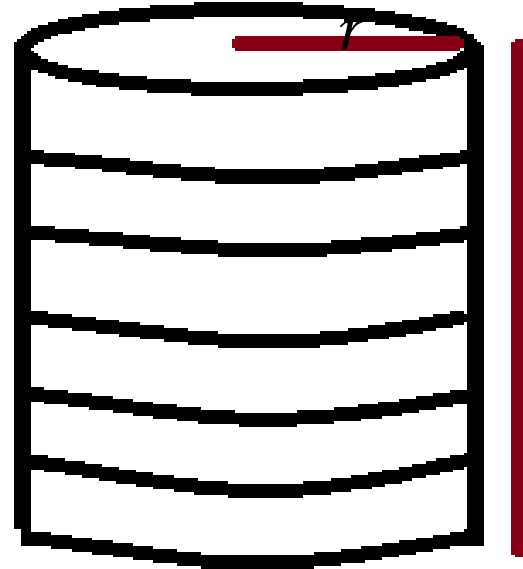
*Volume of a cylinder can be built up using circles
Of same size.*

So, the volume of cylinder can be obtained as :-

*base area * height*

*= area of circular base * height*

= $\pi r^2 h$



EXAMPLE :-



1. A measuring jar of one liter for measuring milk is of right circular cylinder shape. If the radius of the base is 5cm, find the height of the jar.

Sol. – Radius of the cylindrical jar = 5cm

Let 'h' be its height

$$\text{Volume} = \pi r^2 h$$

$$\text{Volume} = 1 \text{ liter} = 1000 \text{ cm}^3$$

$$\pi r^2 h = 1000$$

$$h = 1000 / \pi r^2$$

$$h = 1000 * 7 / 22 * 5 * 5 \text{ cm}$$

$$= 1000 * 7 / 22 * 25 \text{ cm}$$

$$= 140 / 11 \text{ cm} = 12.73 \text{ cm}$$

Height of the jar is 12.73 cm .



EXAMPLE :-



1. Find the weight of a hollow cylindrical lead pipe 26cm long and $1/2$ cm thick. Its external diameter is 5cm. (Weight of 1cm^3 of lead is 11.4 gm)

Sol. – Thickness = $1/2\text{cm}$

External radius of cylinder = $R = (2 + 1/2)\text{cm} = 5/2\text{cm}$

Internal radius of cylinder = $r = (5/2 - 1/2) = 2\text{cm}$

$$\begin{aligned}\text{Volume of lead} &= \pi(R^2 - r^2) * h = \pi[(5/2)^2 - 2^2] * 26 \\ &= 22/7 * [25/4 - 4] * 26 \\ &= 22/7 * (25 - 16/4) * 26 \\ &= 11 * 9 * 13/7 = 1287/7 \text{ cm}^3\end{aligned}$$

Weight of 1cm^3 of lead = 11.4 gm

$$\begin{aligned}\text{Weight of cylinder} &= 11.4 * 1287/7 \text{ gm} = 14671.8/7 \text{ gm} \\ &= 2095.9714 \text{ gm} \\ &= 2095.9714/1000 \text{ kg} = 2.0959714 \text{ kg} \\ &= 2.096 \text{ kg}\end{aligned}$$

Therefore, weight of the cylindrical pipe is 2.096kg



RIGHT CIRCULAR CONE :-

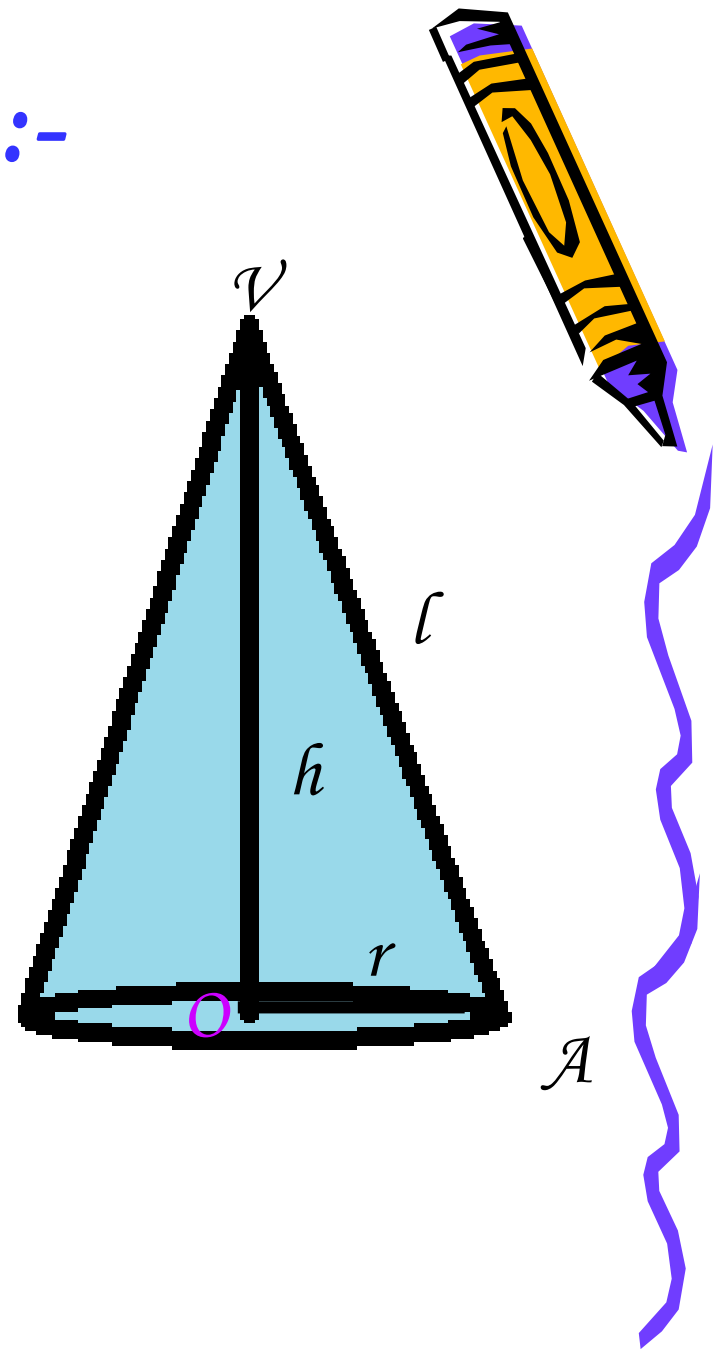
If a right angled triangle is revolved about one of its sides containing a right angle, the solid thus formed is called a right circular cone.

The point V is the vertex of cone.

The length $OV = h$, height of the cone

The base of a cone is a circle with O as center and OA as radius.

The length $VA = l$, is the slant height of the cone.



CURVED SURFACE AREA OF CONE:-

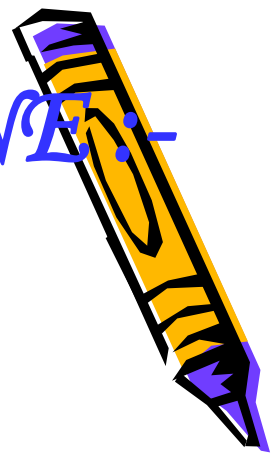
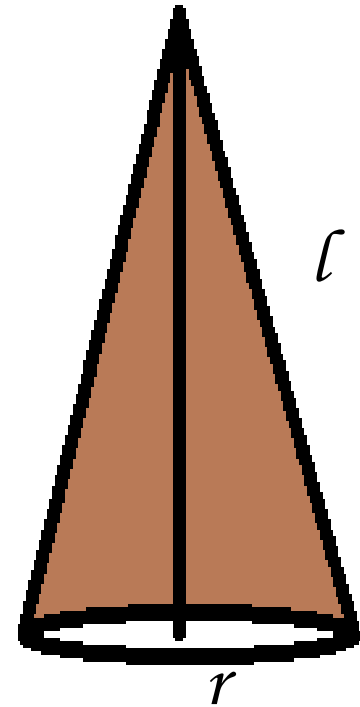
It is the area of the curved part of the cone. (Excluding the circular base)

Formula :-

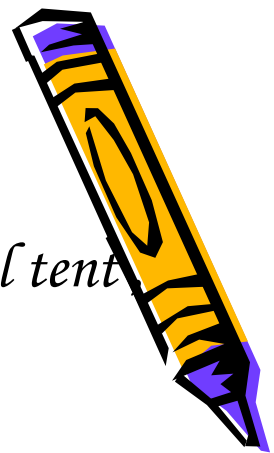
$1/2 \star \text{perimeter of the base} \star \text{slant height}$

$$= \frac{1}{2} \star 2\pi r \star l$$

$$= \pi r l$$



EXAMPLE :-



2. How many meters of cloth 5m wide will be required to make a conical tent the radius of whose base is 7m and whose height is 24m ?

Sol. – Radius of base = 7m

Vertical height, 'h' = 24m

$$\begin{aligned}\text{Slant height 'l'} &= \sqrt{h^2 + r^2} = \sqrt{(24)^2 + (7)^2} \\ &= \sqrt{576 + 49} = \sqrt{625} = 25 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Curved surface area} &= \pi r l \\ &= 22/7 * 7 * 25 \text{ m}^2 = 550 \text{ m}^2\end{aligned}$$

Width of cloth = 5m

$$\begin{aligned}\text{Length required to make conical tent} &= 550/5 \text{ m} \\ &= 110 \text{ m}\end{aligned}$$



TOTAL SURFACE AREA OF CONE :-

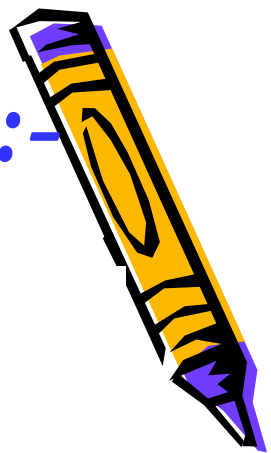
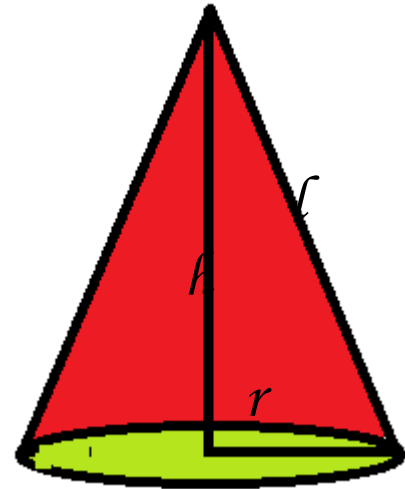
Total surface area of the cone :-

= Curved surface area of cone + circular base

(Red coloured area + green coloured area)

$$= \pi r l + \pi r^2$$

$$= \pi r (l + r)$$



EXAMPLE :-



1. Total surface area of a cone is 770cm^2 . If the slant height of cone is 4 times the radius of its base , then find the diameters of the base.

Sol. – $\text{Total surface area of cone} = 770\text{ cm}^2$

$$\begin{aligned} &= \pi r (r + l) = 770 \\ &= l = 4 * \text{radius} \\ &= 4r \\ &= \pi r (r + 4r) = 770 \\ &= 5\pi r^2 = 770 \\ &= r^2 = 770 * 7 / 5 * 22 = 7 * 7 \\ &= r = 7\text{cm} \end{aligned}$$

Therefore, diameter of the base of the cone is 14cm.



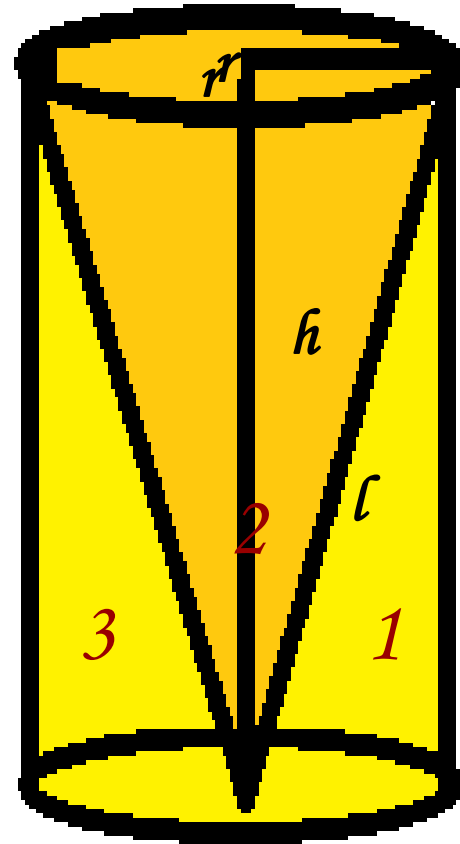
VOLUME OF THE CONE :-

Formula :- $\frac{1}{3} \pi r^2 h$

Derivation :- If a cylinder and cone of same base Radius and height are taken , and if cone is put Under the cylinder then it will occupy only One –third part of it .

Therefore, volume of cone is $\frac{1}{3}$ of the volume of Cylinder.

$$= \frac{1}{3} \pi r^2 h$$



EXAMPLE :-



1. The radius and perpendicular height of a cone are in the ratio 5 : 12. If the volume of the cone is 314cm^3 , find its perpendicular height and slant height.

Sol. —

Let the radius of the cone = $5x$

Perpendicular height of the cone = $12x$

Volume of the cone = 314 m^3

Hence,

$$\frac{1}{3}\pi r^2 h = 314$$

$$= \pi r^2 h = 942$$

$$= 3.14 (5x)^2 (12) = 942$$

$$= 3 \times 314 x^3 = 942$$

$$= x^3 = 1$$

$$= x = 1$$

Therefore, perpendicular height of the cone = 12m

And radius of the cone = 5m

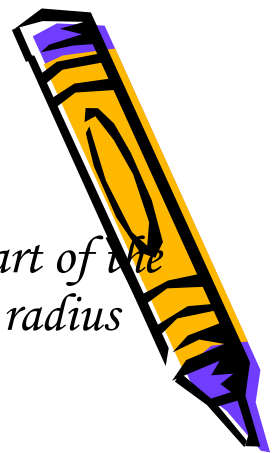
$$\text{Slant height of cone} = \sqrt{r^2 + h^2} = \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144} = \sqrt{169}$$

$$= 13\text{m}$$



EXAMPLE :-



2. A wooden right circular cone has a base of radius 3cm and height 4cm. The upper part of the cone cut is in such a way that the conical piece will have height 1cm and base radius 0.75cm. Find the volume of the remaining portion.

Sol. – For complete cone, $r = 3\text{cm}$

height ' h ' = 4cm

Volume of the complete cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} * \pi * 3 * 3 * 4 = 12 \pi \text{ cm}^3$$

For the upper part of cone, radius = 0.75cm, height = 1cm

$$= \frac{1}{3}\pi R^2 H$$

$$= \frac{1}{3} * \pi * 0.75 * 0.75 * 1$$

$$= 0.1875\pi \text{ cm}^3$$

Volume of the remaining portion of the cone –

= Volume of the complete cone – volume of the cut cone

$$= 12 \pi - 0.1875\pi = 11.8125\pi = 11.8125 * 3.14$$

$$= 37.09 \text{ cm}^3$$

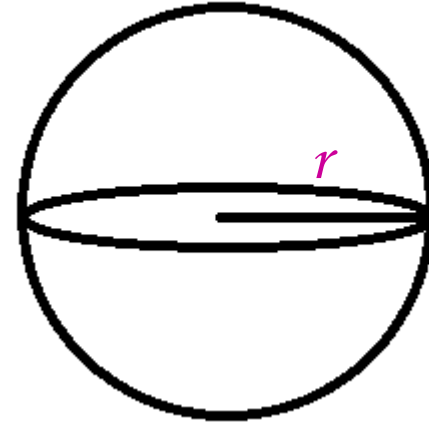


SPHERE : -

The set of all points in space equidistant from a fixed point, is called a sphere .

The fixed point is called the center of the sphere.

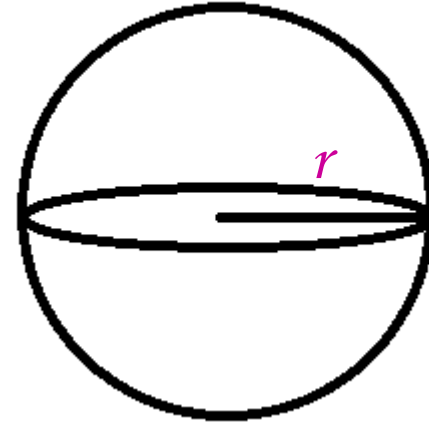
A line segment passing through the center of the sphere with its end points on the sphere is called a diameter of the sphere.



SURFACE AREA OF SPHERE :-

Surface area of the sphere :-

$$=4\pi r^2$$



EXAMPLE :-



1. If the diameter of a sphere is 'd' and curved surface area 'S', then show that $S = \pi d^2$. Hence, find the surface area of a sphere whose diameter is 4.2 cm.

Sol. — $d = 2r$

Curved surface area of sphere = S

$$= 4\pi r^2 = \pi \star 4r^2 = \pi(2r)^2$$

$$= \pi d^2$$

Here, $d = 4.2\text{cm}$

$$\text{Surface area of the sphere} = \pi d^2 = 22/7 \star (4.2)^2$$

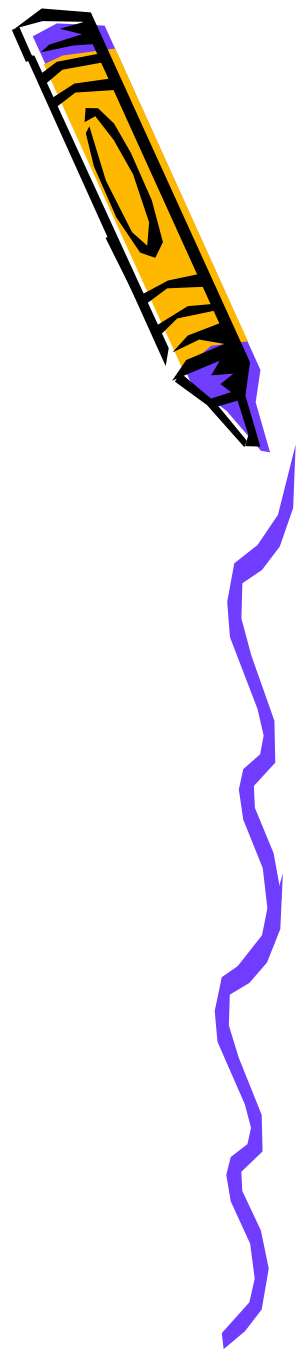
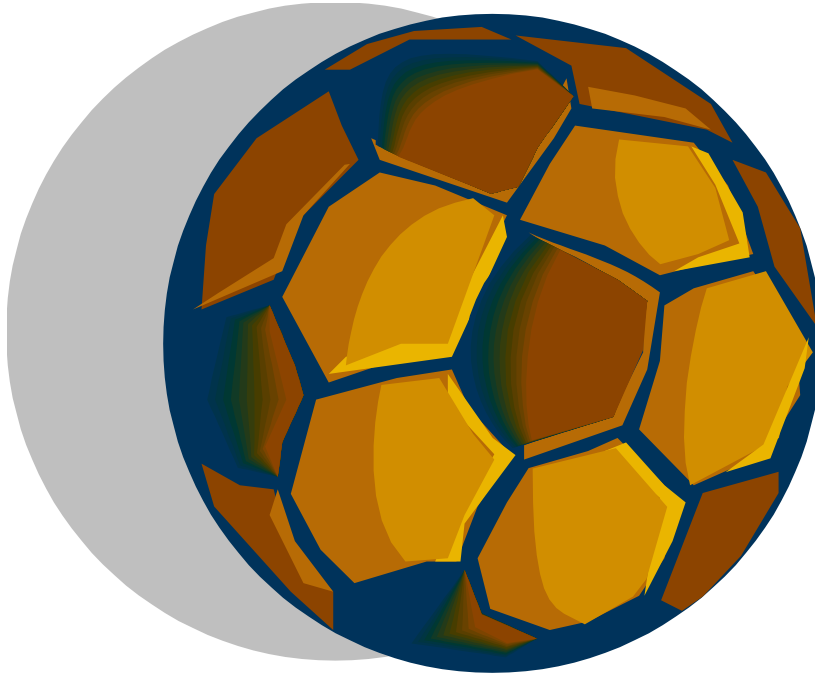
$$= 55.44\text{cm}^2$$



VOLUME OF THE SPHERE :-

Volume of the sphere :-

$$= \frac{4}{3}\pi R^3$$



EXAMPLE :-



1. How many spherical bullets can be made out of lead whose edge measures 44cm, each bullet being 4cm in diameter.

Sol. – Let the total no. of bullets be x

Radius of spherical bullet = $4/2 \text{ cm} = 2\text{cm}$

Volume of a spherical bullet = $\frac{4}{3} \pi \times (2)^3 \text{ cm}^3$
 $= (\frac{4}{3} \times \frac{22}{7} \times 8) \text{ cm}^3$

Volume of solid cube = $(44)^3 \text{ cm}^3$

$$\begin{aligned} \text{Number of spherical bullets recast} &= \frac{\text{volume of cube}}{\text{volume of one bullet}} = \frac{44 \times 44 \times 44 \times 3 \times 7}{4 \times 22 \times 8} \\ &= 33 \times 77 \\ &= 2541 \end{aligned}$$



EXAMPLE :-

2. If the radius of a sphere is doubled, what is the ratio of the volume of the first sphere to that of the second?

Sol. – For the first sphere,

$$\text{Radius} = r$$

$$\text{Volume} = V_1$$

For the second sphere,

$$\text{Radius} = 2r$$

$$\text{Volume} = V_2$$

Then,

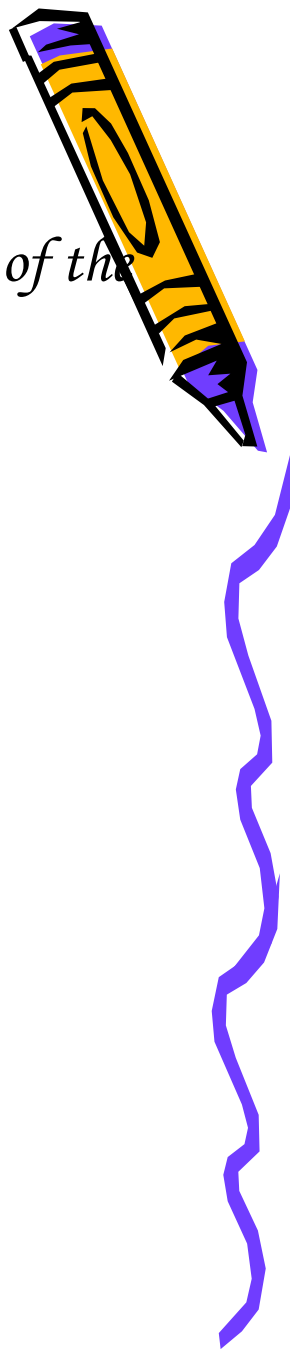
$$V_1 = \frac{4}{3}\pi r^3$$

$$V_2 = \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi(8r^3)$$

$$\text{Therefore, } \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi 8r^3} = \frac{1}{8}$$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi 8r^3} = \frac{1}{8}$$

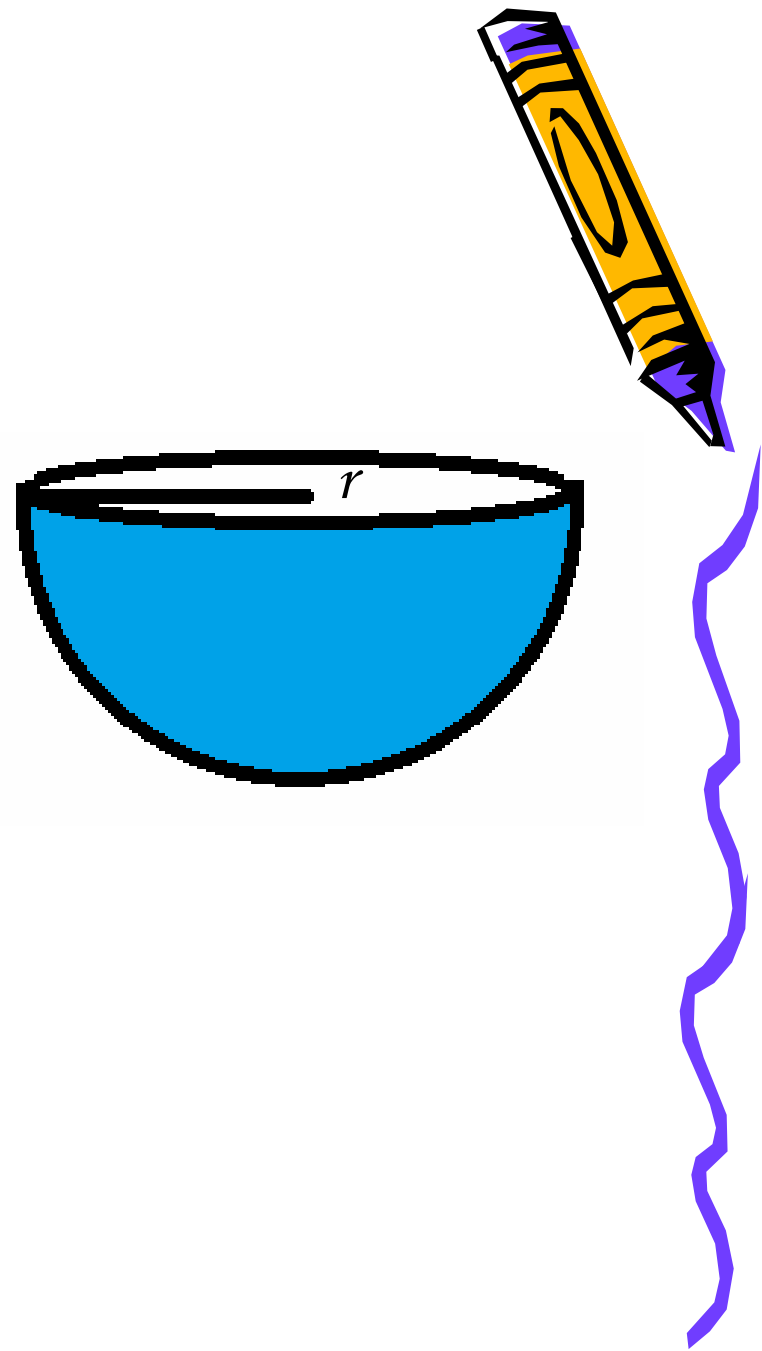
$$\text{Ratio} = 1:8$$



HEMISPHERE :-

A plane passing through the centre of a sphere divides the sphere into two equal parts .

Each part is known as hemi- sphere.



CURVED SURFACE AREA OF HEMISPHERE :-

Formula : - $2\pi r^2$

Derivation :-

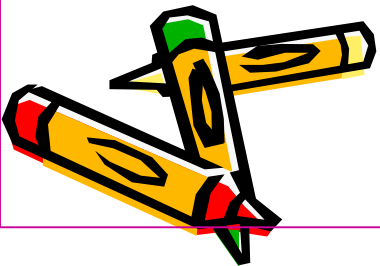
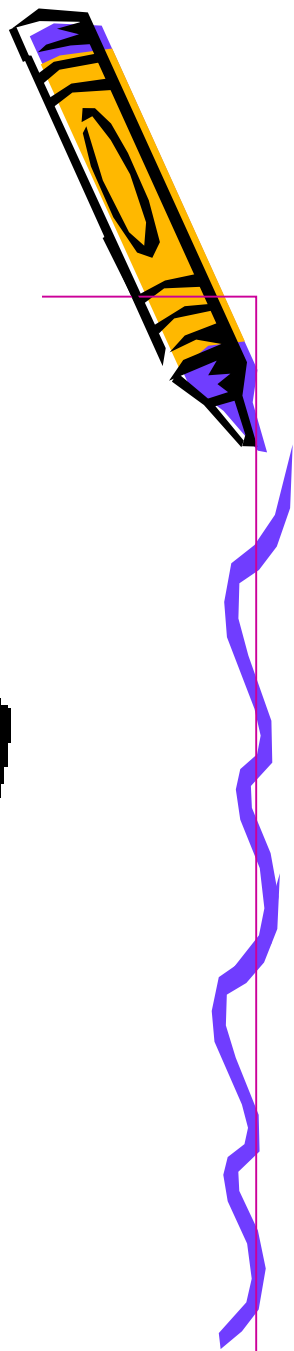
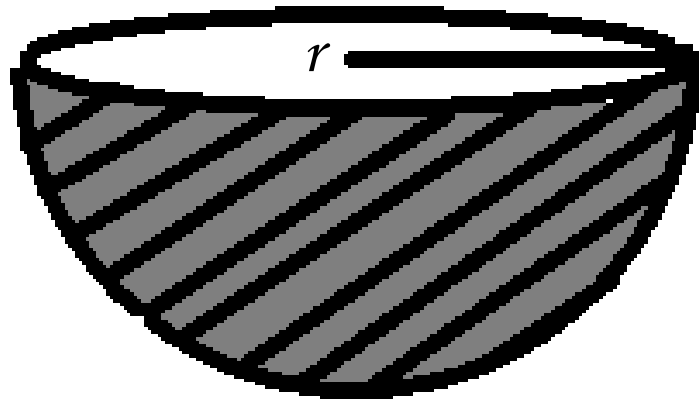
Since,

hemisphere is half of sphere-

Therefore,

Surface area of sphere = $4\pi r^2$

Half of it = $2\pi r^2$



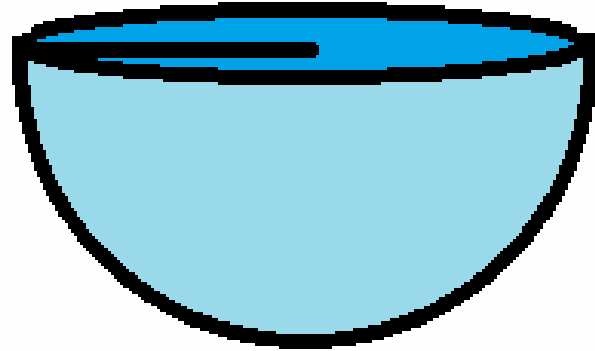
TOTAL SURFACE AREA OF HEMISPHERE

Total surface area of hemisphere:

= Curved surface area + circular base

$$= 2\pi r^2 + \pi r^2$$

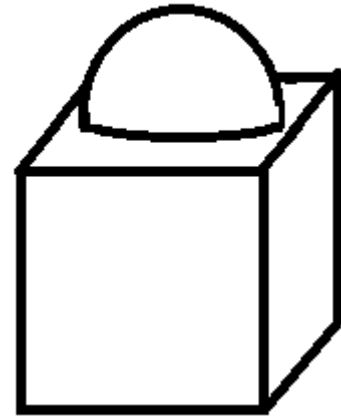
$$= 3\pi r^2$$



SURFACE AREAS AND VOLUMES OF COMBINATION OF SOLIDS :-

SOME EXAMPLES :-

1. The decorative block is made up of two solids – a cube with edge 5cm and a hemisphere fixed on the top has a diameter of 4.2cm. Find the total surface area of the solid.



*Sol. – Total surface area of cube = $6 \times 5 \times 5$
 $= 150 \text{ cm}^2$*

The surface area of block = Total surface area of block + curved surface area of the hemisphere – area of the base of the hemisphere

$$= 150 - \pi r^2 + 2\pi r^2 = (150 + \pi r^2) \text{ cm}^2$$

$$= 150 \text{ cm}^2 + [22/7 \times 4.2/2 \times 4.2/2] \text{ cm}^2$$

$$= (150 + 13.86) \text{ cm}^2 = 163.86 \text{ cm}^2$$



EXAMPLE :-

1. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 28 cm respectively. The cost to paint 1 cm² of the surface is Rs 0.05. Find the cost to paint the vessel all over.

Sol. –

External area which is to be painted

$$\begin{aligned} &= 2\pi R^2 = 2 \times \frac{22}{7} \times \frac{28}{2} \times \frac{28}{2} \text{ cm}^2 \\ &= 6160 \text{ cm}^2 \end{aligned}$$

Internal area which is to be painted

$$\begin{aligned} &= 2\pi r^2 = 2 \times \frac{22}{7} \times \frac{24}{2} \times \frac{24}{2} \text{ cm}^2 \\ &= 5541 \frac{3}{7} \text{ cm}^2 \end{aligned}$$

$$\text{Area of the ring at top} = \frac{22}{7} \{ (28)^2 - (24)^2 \}$$

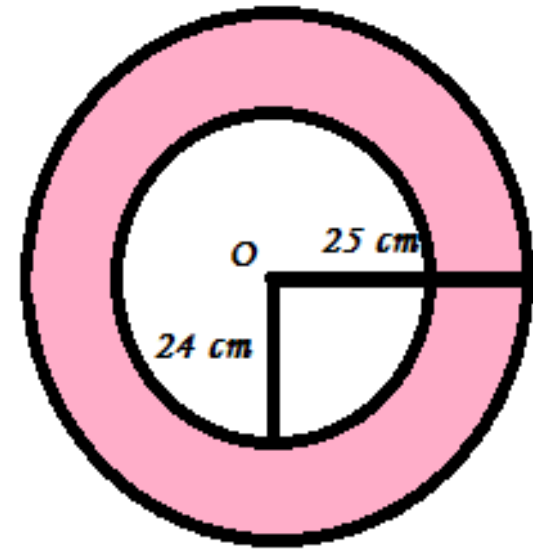
$$\begin{aligned} &= \frac{22}{7} [(14)^2 - (12)^2] = \frac{22}{7} (14 + 12)(14 - 12) = \frac{22}{7} \times 26 \times 2 \\ &= 1831 \frac{2}{7} \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Total area to be painted} &= \frac{6160}{1} + \frac{5541 \frac{3}{7}}{1} + \frac{1831 \frac{2}{7}}{1} = \frac{13532 \frac{5}{7}}{1} \text{ cm}^2 \\ &= 13532 \frac{5}{7} \text{ cm}^2 \end{aligned}$$

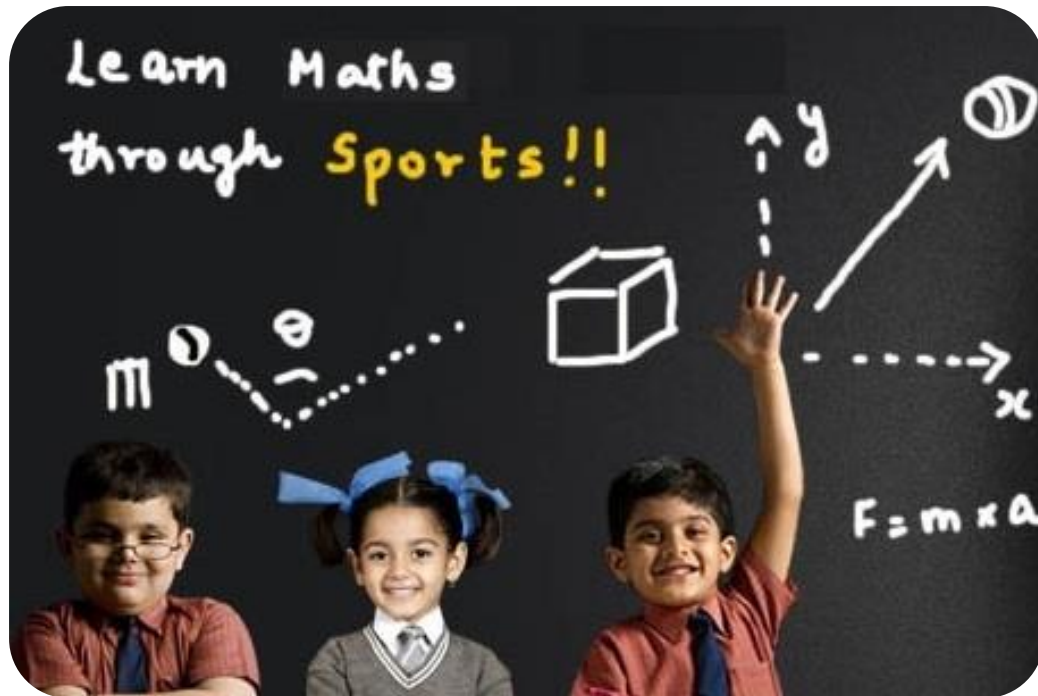
Cost of painting @ Rs. 0.05/cm²

$$= \text{Rs. } 13532 \frac{5}{7} \times 0.05 = \text{Rs. } 676 \frac{25}{7}$$

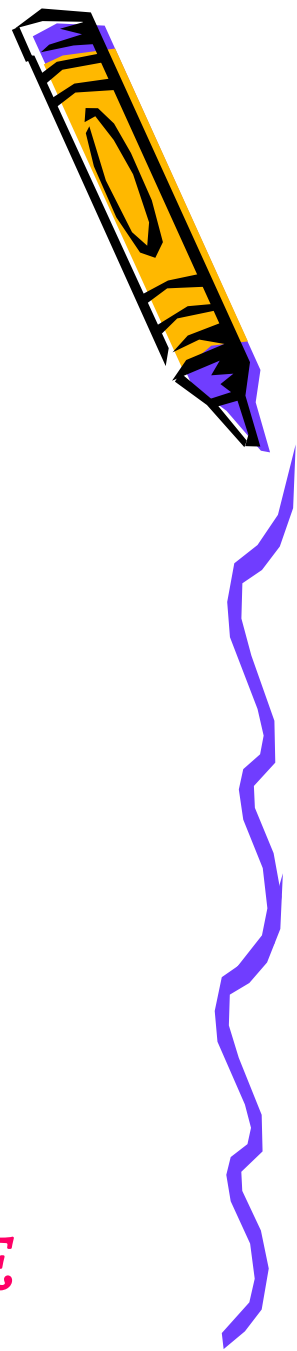
$$= \text{Rs. } 968 \frac{5}{7}$$



MATHS

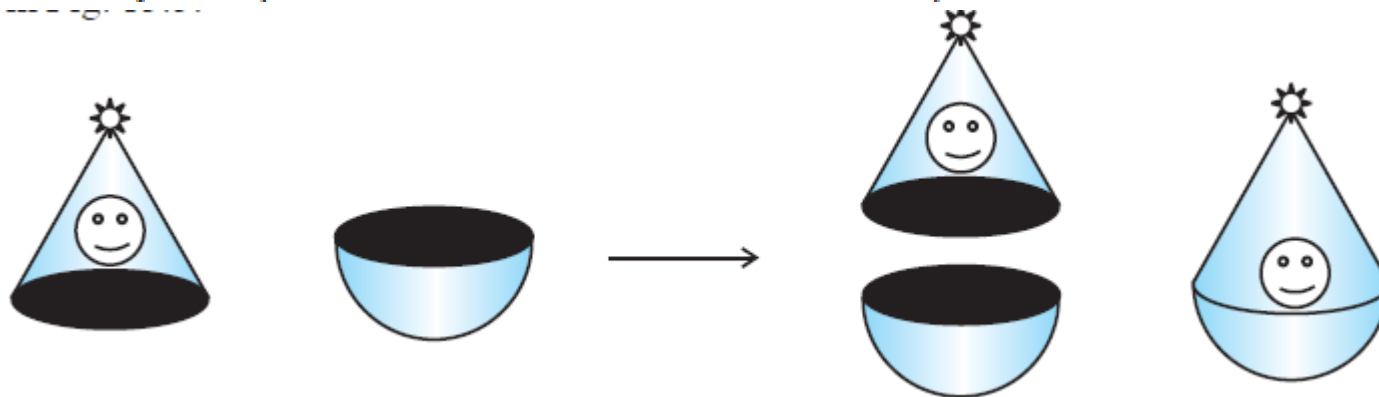


SURFACE AREA AND VOLUME



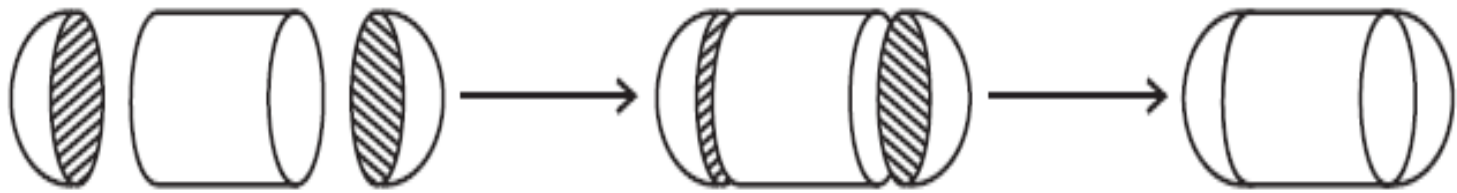
COMBINATION OF SOLIDS

In our daily life we come across many combined solids like,
a cylinder over a hemisphere,
a capsule which is combination of a cylinder attached with two
hemispheres on both the ends,
a toy shaped with a cone over a hemispherical bottom etc.



SURFACE AREAS OF COMBINATION OF SOLIDS

While calculating the surface area of the combined figures, we should only calculate the areas that are visible to our eyes. For example, if a cone is surmounted by a hemisphere and if we have to find out the total surface area of this solid, we need to just find out the curved surface area of the hemisphere and curved surface area of the cone separately and add them together. Note that we are leaving the base area of both the cone and the hemisphere in this case since both the bases are attached together and are not visible to our eyes.



EXAMPLE :-

2. Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45m and its radius is 30cm. Find its total surface area.

Sol.- Let 'h' be the height of the cylinder

'r' be the common radius of the cylinder, and hemisphere

Total surface area of the bird bath = CSA of cylinder + CSA of hemisphere

$$= 2\pi rh + 2\pi r^2 = 2\pi r (r + h)$$

$$= 2 \times \frac{22}{7} \times 30(145 + 30)\text{cm}^2$$

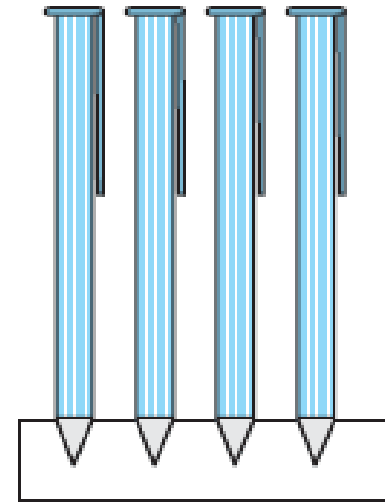
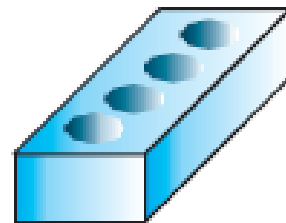
$$= 33000\text{cm}^2$$

$$= 3.3\text{m}^2$$



VOLUMES OF COMBINATION OF SOLIDS

Unlike the surface area of the combination of solids, while calculating the volume of the combined solids we need to find the volume of each solid and add them together.



EXAMPLE :-

3. A solid consisting of a right circular cone of height 120cm and radius 60cm standing on a hemisphere of radius 60cm is placed in a right circular cylinder full of water such that it touches the bottom. Find the volume of water if the radius of the cylinder is 60cm and its height is 180 cm.

Sol. – We are given :-

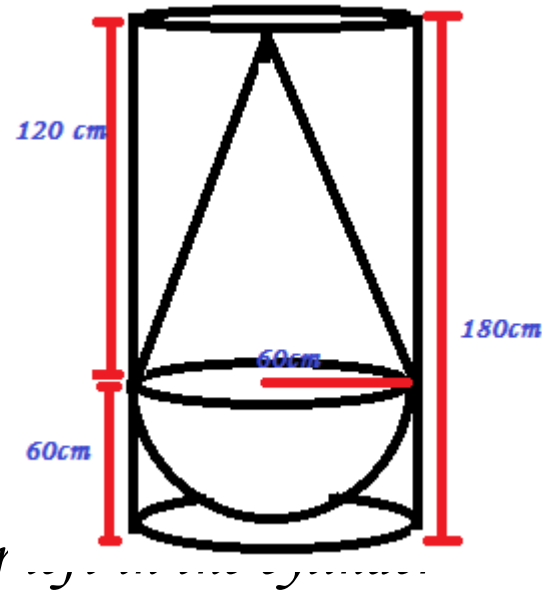
Height of the cone ' h ' = 120cm

Radius of the cone ' r ' = 60cm

Radius of the hemisphere ' R ' = 60cm

Radius of the cylinder ' R_2 ' = 60cm

Height of the cylinder ' h ' = 180cm



EXAMPLE :-

Volume of water left = Volume of cylinder - (volume of cone + volume of hemisphere)

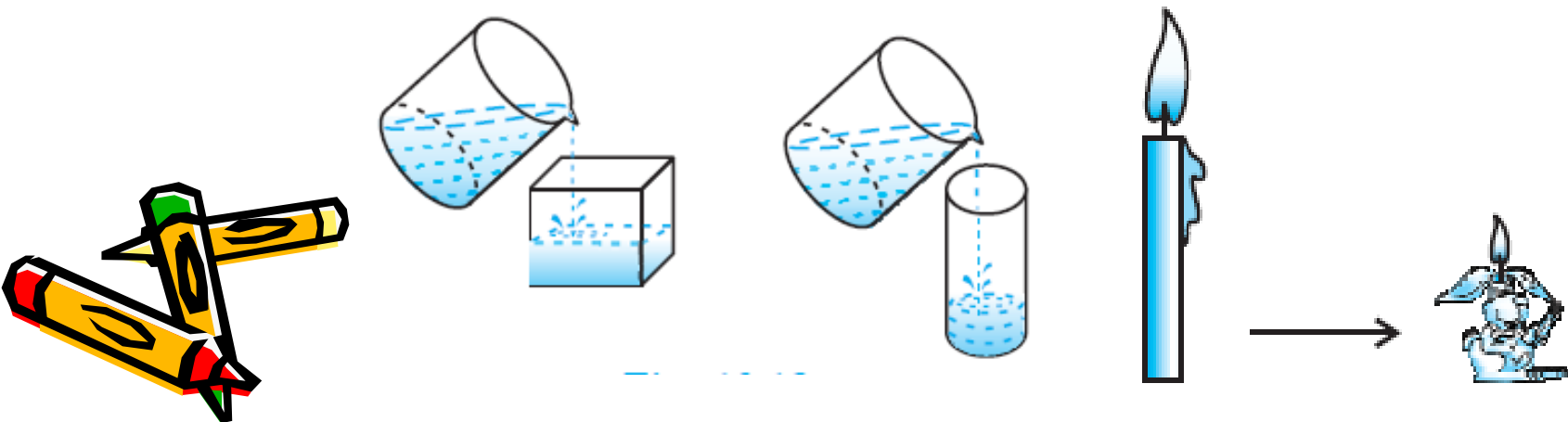
$$\begin{aligned} &= \pi R^2 h - \left(\frac{1}{3} r^2 h + \frac{2}{3} \pi R^3 \right) \\ &= \pi R^2 h - \left[\frac{1}{3} \pi (r^2 h + 2 R^3) \right] \\ &= \pi R^2 h - \left[\frac{1}{3} \pi (60 * 60 * 120 + 2 * 60 * 60 * 60) \right] \\ &= \pi R^2 h - \left[\frac{1}{3} \pi (432000 + 432000) \right] \\ &= \pi R^2 h - \left[\frac{1}{3} \pi 864000 \right] = \pi R^2 - (\pi * 288000) \\ &= \pi (R^2 h - 288000) = \pi (3600 * 180 - 288000) \\ &= \pi (648000 - 288000) = \pi * 360000 \\ &= 3.14 * 360000 = 1130400 \text{ cm}^3 \\ &= 1130400 / 1000000 \\ &= 1.13 \text{ m}^3 \end{aligned}$$



CONVERSION OF SOLIDS FROM ONE FIGURE INTO OTHER



You must have seen in our daily life that some solids are melted and recast into another solid for various purposes. For example a logwood is cut to make small pencils, which are in the shape of a cylinder surmounted by a cone. Also, for your school activities you must have converted a cylindrical shaped candle to form a spherical ball or conical lamp etc. by melting them. Now, in this topic we are going to learn as to how to find the volume and surface area of the converted solid from the original solid.



EXAMPLE :-

2. How many silver coins, 1.75cm in diameter and thickness 2mm, must be melted to form a cuboid of dimension 5.5cm*10cm*3.5cm?

Sol.- Radius of the coin ' r ' = 1.75/2 cm

Height of the cone ' h ' = 2mm = 0.2cm

$$\begin{aligned}\text{Volume of coin} &= \pi r^2 h = \frac{22}{7} * \frac{1.75}{2} * \frac{1.75}{2} * 0.2 \\ &= 0.481\text{cm}\end{aligned}$$

Length of cuboid ' l ' = 5.5cm

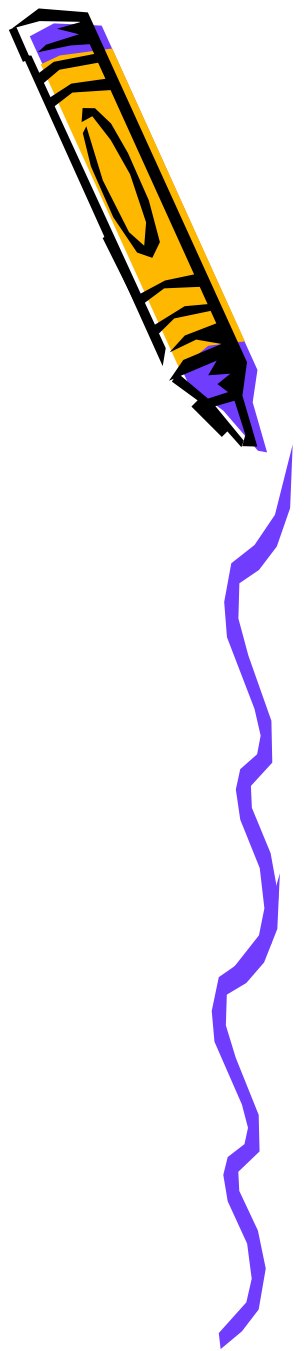
Breadth of cuboid ' b ' = 10 cm

Height of cuboid ' h ' = 3.5cm



$$\begin{aligned}\text{Volume of the cuboid} &= l * b * h \\ &= 5.5 * 10 * 3.5 \\ &= 192.5 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{Number of coins needed} &= \frac{\text{volume of cuboid}}{\text{volume of coin}} \\ &= \frac{192.5 * 100}{0.481 * 100} \\ &= 400 \text{ coins}\end{aligned}$$



FRUSTUM: -

If a cone is cut by a plane parallel to the base, the part between the base and the plane is called frustum of the cone

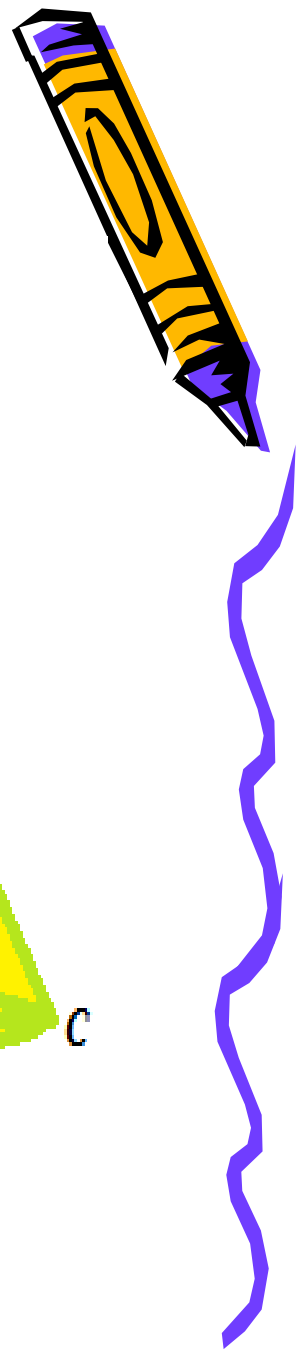
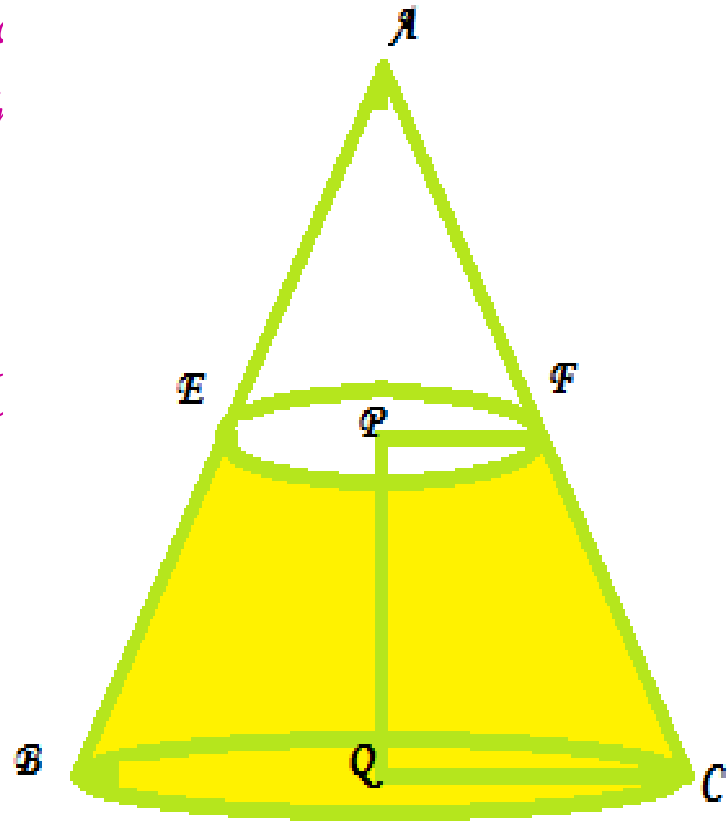
Here, EBSF is frustum from the cone ABC

$PQ = R = \text{radius}$

$QC = r = \text{radius}$

$PQ = h = \text{height}$

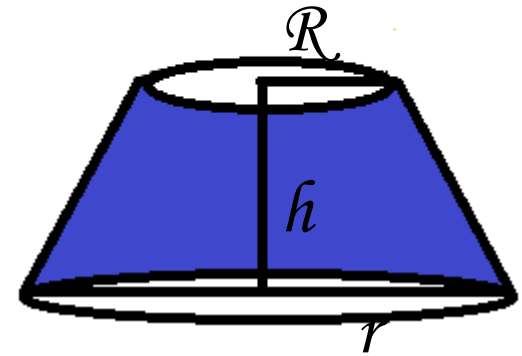
$FC = l = \text{slant height}$



SURFACE AREA OF THE FRUSTUM:-

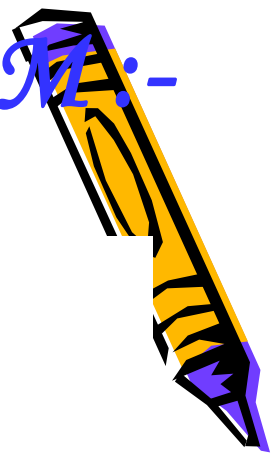
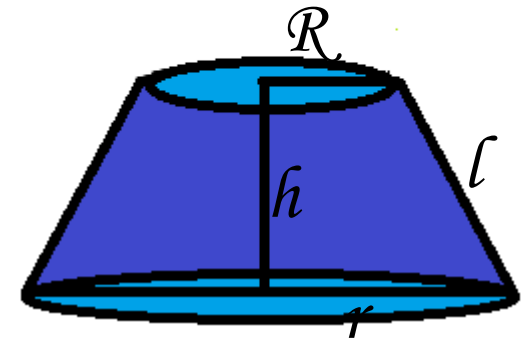
#CURVED SURFACE AREA OF FRUSTUM

$$= \pi l (\mathcal{R} + r) + \pi \mathcal{R}^2 + r\pi^2$$
$$l = \sqrt{h^2 + (\mathcal{R} - r)^2}$$



#TOTAL SURFACE AREA OF FRUSTUM:-

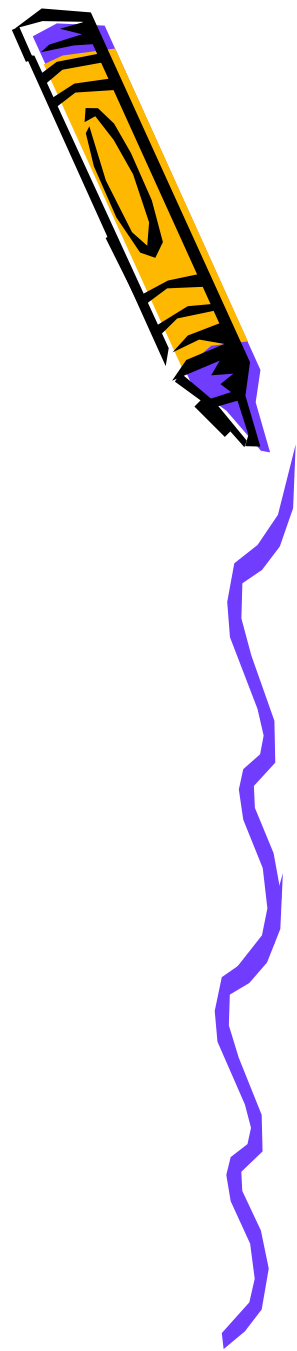
$$= \pi (\mathcal{R} + r) l$$
$$l = \sqrt{h^2 + (\mathcal{R} - r)^2}$$



VOLUME OF THE FRUSTUM :-

Volume of frustum of cone :

$$= \frac{1}{3} \pi h (R^2 + r^2 + Rr)$$



EXAMPLE :-

1. A friction clutch is in the form of frustum of a cone the diameter being 16 cm and 10 cm and length 8 cm. Find its curved surface.

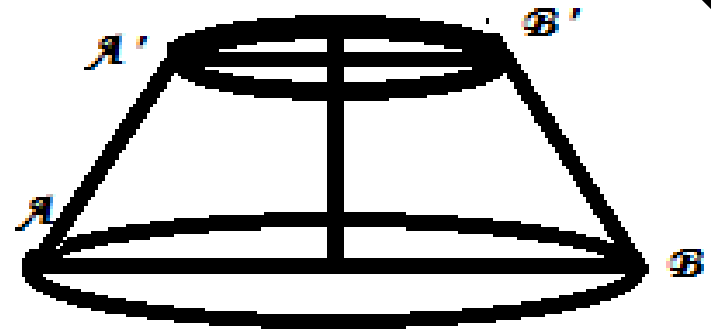
Sol. – Let $ABB'A'$ be the friction clutch

Let ' l ' be its slant height

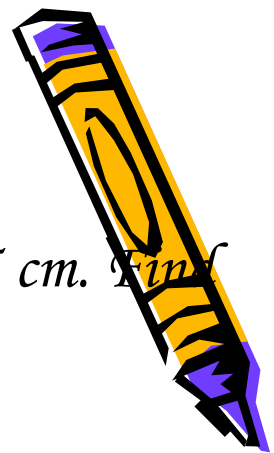
$$\begin{aligned} l &= \sqrt{8^2 + (8-5)^2} \\ &= \sqrt{64 + 9} = \sqrt{73} \text{ cm} \end{aligned}$$

Bearing surface = Lateral surface area of $ABB'A'$

$$\begin{aligned} &= \pi l (R + r) = \frac{22}{7} * \sqrt{73} (8 + 5) \text{ cm}^2 \\ &= 349 \text{ cm sq.} \end{aligned}$$



EXAMPLE :-



1. The radii of the ends of a bucket of height 24 cm are 15 cm and 5 cm. Find its capacity.

Sol. – Capacity of the bucket = Volume of the frustum

$$= \frac{\pi h}{3} [r^2 + R^2 + Rr]$$

$$= \frac{22}{7} \times \frac{24}{3} [(15)^2 + 5^2 + 15 \times 5] \text{ cm}^3$$

$$= \frac{22 \times 8}{7} [225 + 25 + 75] \text{ cm}^3$$

$$= \frac{176}{7} \times 325 \text{ cm}^3$$

$$= 8171.43 \text{ cm}^3$$





THANK YOU

EXAMPLE :-

4. A solid iron pole consists of a cylinder of height 220cm and base diameter 24cm, which is surmounted by another cylinder of height 60cm and radius 8cm. Find the mass of the pole, given that 1cm^3 of iron has approximately 8gms mass.

Sol.- *Dimension of smaller cylinder-*

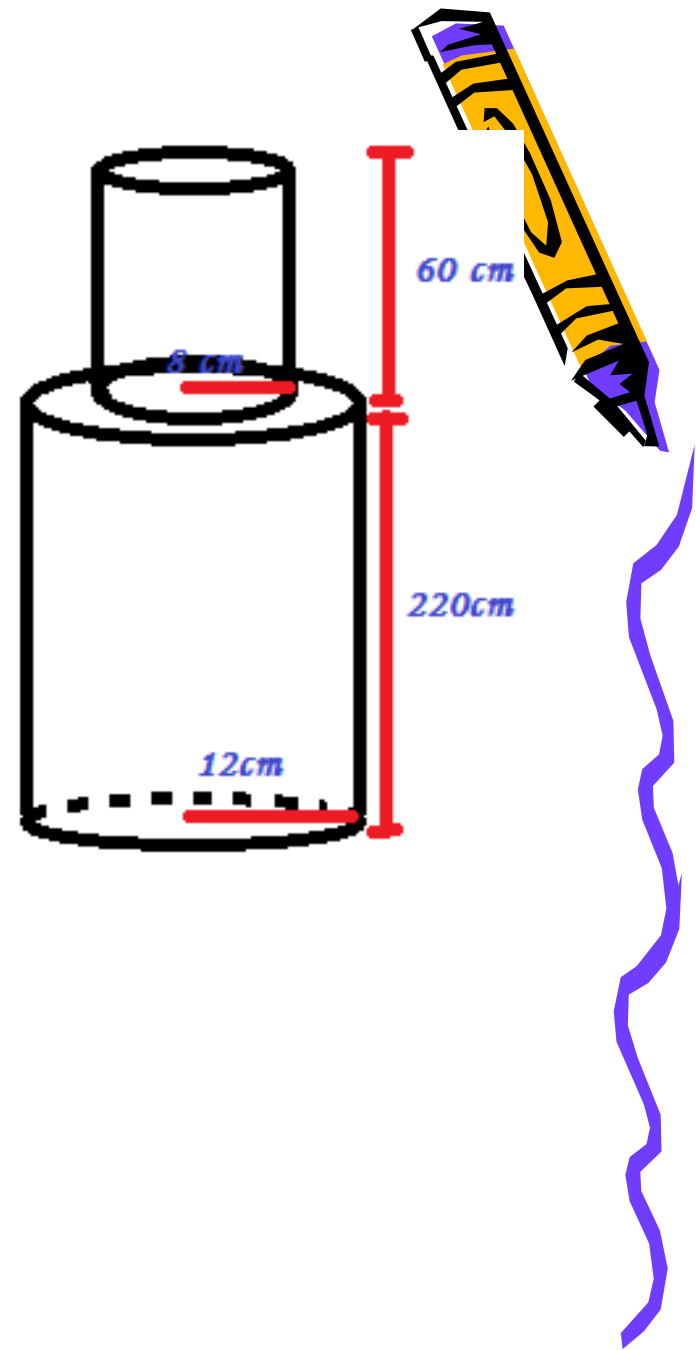
$$\text{Radius 'r'} = 8\text{cm}$$

$$\text{Height 'h'} = 60\text{ cm}$$

Dimension of large cylinder-

$$\text{Radius 'R'} = 12\text{cm}$$

$$\text{Height ;H'} = 220\text{cm}$$



Volume of the statue = Volume of large cylinder + Volume of small cylinder

$$= \pi R^2 H + \pi r^2 h$$

$$= 3.14 * 8 * 8 * 60 + 3.14 * 12 * 12 * 220$$

$$= 3.14 * 64 * 60 + 3.14 * 144 * 220$$

$$= 2009.6 * 60 + 690.8 * 144$$

$$= 12057.6 + 99476.2 = 111533.8 \text{ cm}^3$$

Mass of the statue-

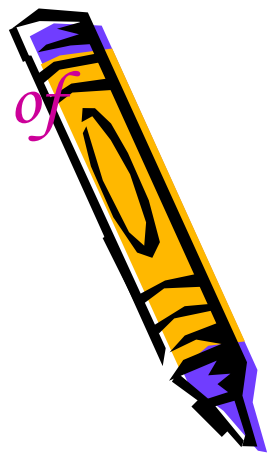
$$\text{Mass of } 1 \text{ cm}^3 = 8 \text{ gms}$$

$$\text{Mass of } 111533.8 \text{ cm}^3 = 111533.8 * 8$$

$$= 892260.4 \text{ gms}$$

$$\text{In kg- } 892260.4 / 1000$$

$$= 892.26 \text{ kgs}$$



CONVERSION OF SOLIDS FROM ONE FIGURE INTO OTHER:-

1. A metallic right circular cone 20cm high whose Vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $1/16$ cm, find the length of wire.

Sol.- $\triangle ABC$ $H=20\text{cm}$

$$\beta = 60^\circ$$

$$r = \tan 30 = r/20$$

$$= 1/\sqrt{3} = r/20$$

$$= r = 20/\sqrt{3} \text{ cm}$$

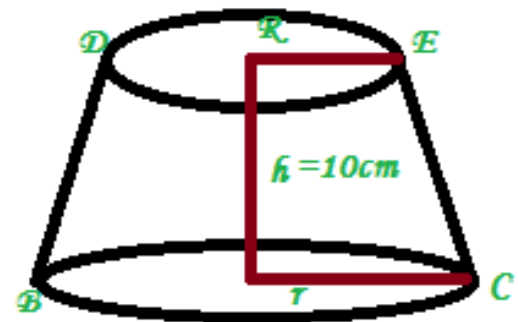
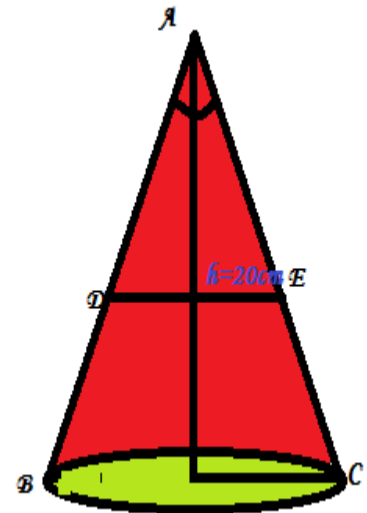
$$\triangle ADE \quad \alpha = 30^\circ$$

$$R = \tan 30 = R/10$$

$$= 1/\sqrt{3} = R/10$$

$$= R = 10/\sqrt{3} \text{ cm}$$

$$\text{Frustum } BCED \quad h = 10\text{cm}$$



Volume of the frustum BCED –

$$= 1/3\pi h (\mathcal{R}^2 + r^2 + \mathcal{R}r)$$

$$= 1/3\pi \star 10 [(20/\sqrt{3})^2 + (10/\sqrt{3})^2 + (20/\sqrt{3})(10/\sqrt{3})]$$

$$= 10/3 \pi (200/3 + 100/3 + 200/3) = 10\pi/3 (700/3)$$

$$= \frac{10 \star 22}{3 \star 7} \star \frac{700}{3}$$

$$= 22000/7 \text{ cm}^3$$

Area of the wire :-

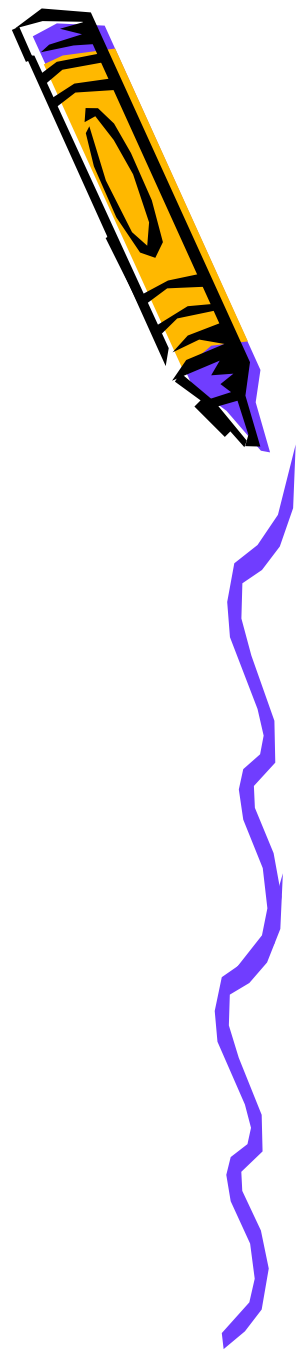
$$d = 1/16 \text{ cm}$$

$$r = 1/32 \text{ cm}$$

$$l = ?$$

$$\pi r^2 = 22/7 \star 1/32 \star 1/32$$

$$= 22/7168 \text{ cm}^2$$



$$\text{Length of the wire} = \frac{\text{Volume of frustum}}{\text{Area of the wire}}$$

$$= \frac{22000/9 \text{ cm}^3}{22/7168 \text{ cm}^2}$$

$$= \frac{22000}{9} \star \frac{7168}{22}$$

$$= 7168000/9 \text{ cm}$$

$$= 71680 /9 \text{ m}$$

$$= 7964.4 \text{ m}$$

