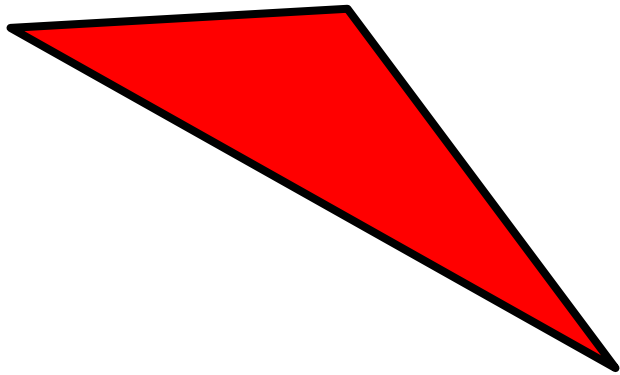
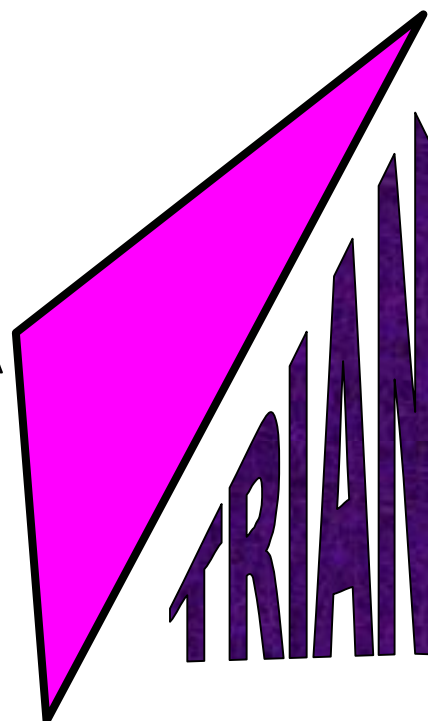
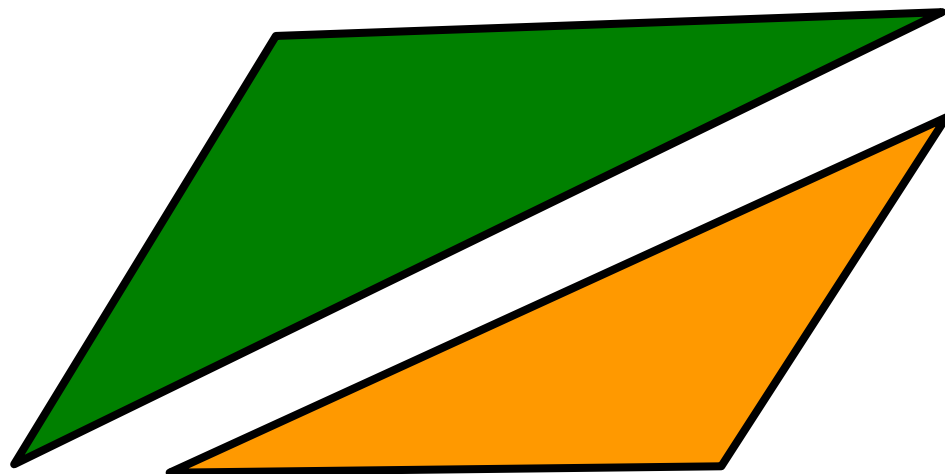


SIMILAR

The word "SIMILAR" is written in a bold, purple, stylized font. It is surrounded by four triangles: a purple triangle at the top left, a red triangle at the bottom left, a green triangle at the top right, and an orange triangle at the bottom right.

TRIANGLES

The word "TRIANGLES" is written in a bold, purple, stylized font. It is surrounded by two triangles: a pink triangle at the top left and a yellow triangle at the bottom right.

In geometry, two polygons are similar when one is a replica (scale model) of the other.

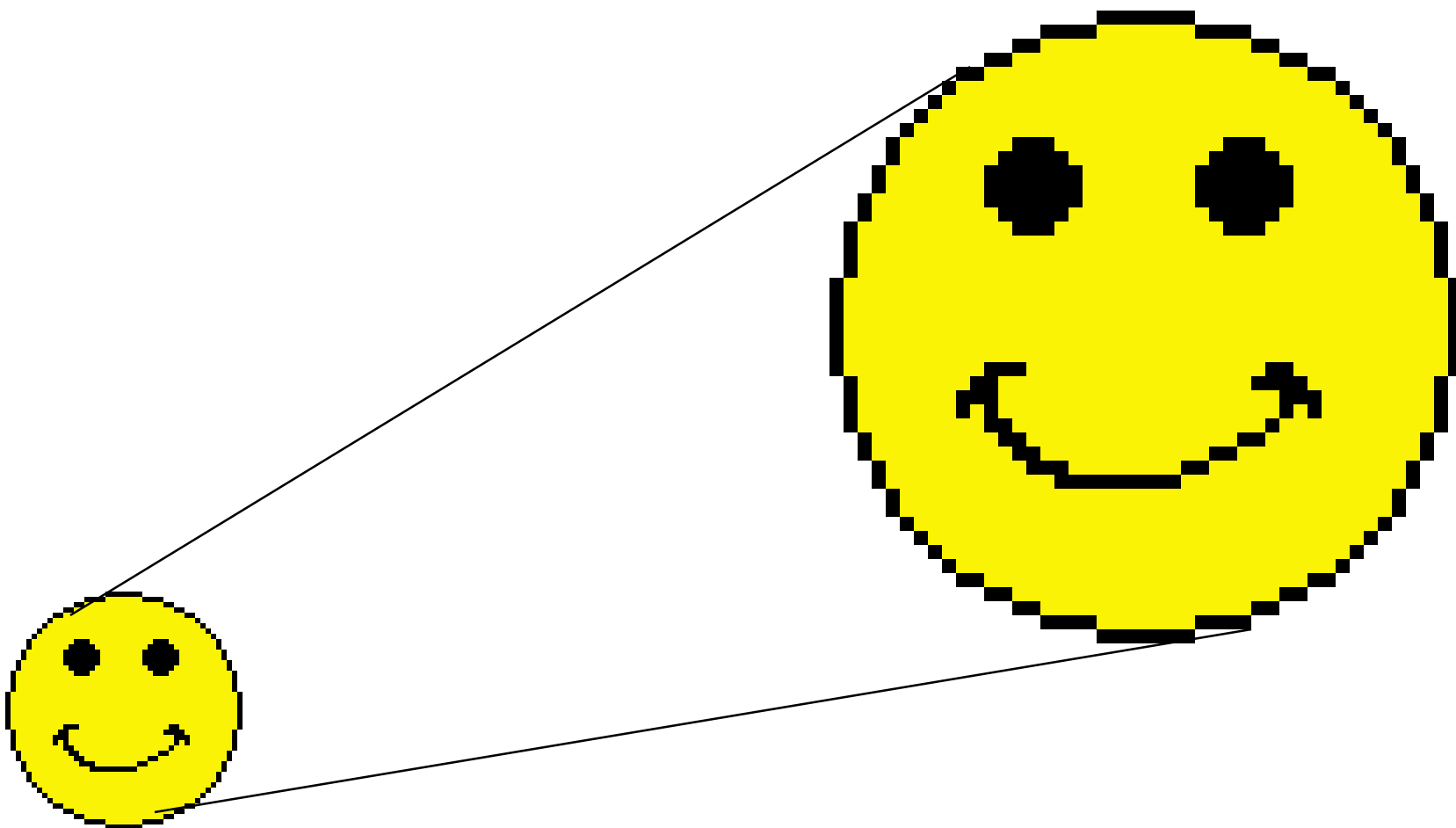
Consider Dr. Evil and Mini Me from Mike Meyers' hit movie Austin Powers. Mini Me is supposed to be an exact replica of Dr. Evil.



BACK

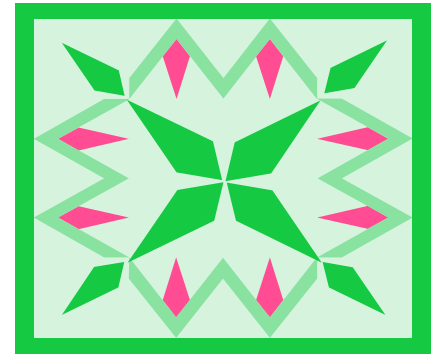
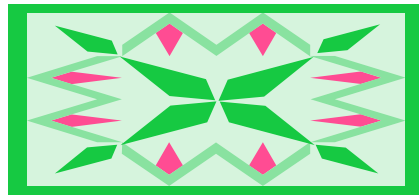
NEXT

EXIT

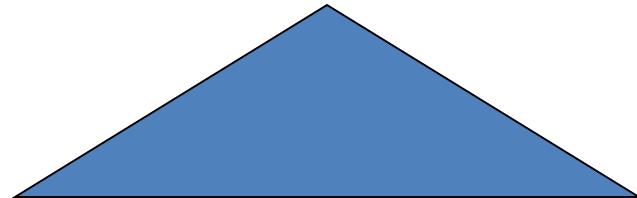
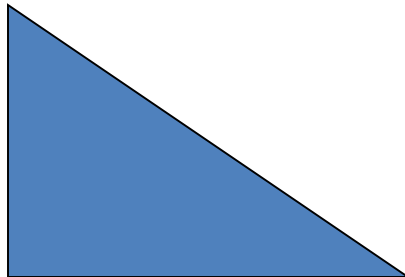


The following are non-similar figures.

I



II

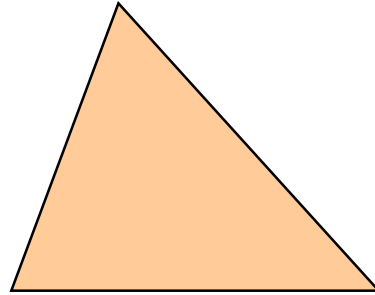


BACK

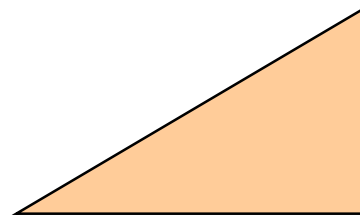
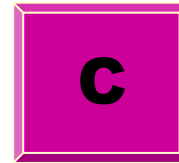
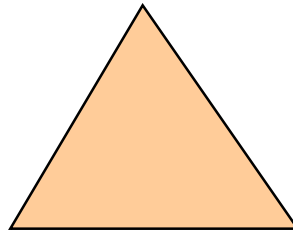
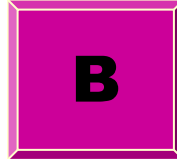
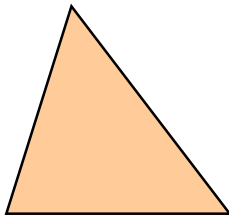
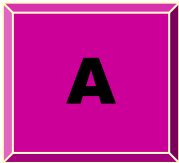
NEXT

EXIT

1.



Which of the following is similar to the above triangle?



BASIC PROPORTIONALITY THEOREM

Theorem 6.1 :-

If a line is drawn parallel to one side of a triangle to intersect the other two side in distinct points, the other two sides are divided in the same ratio.

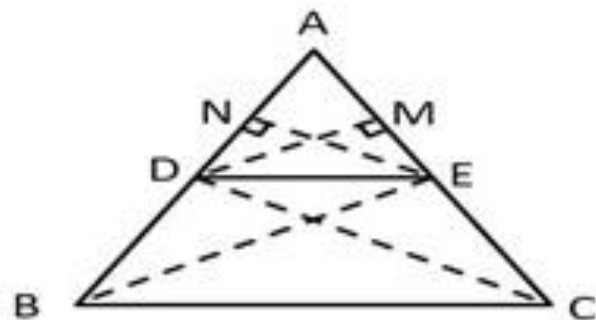
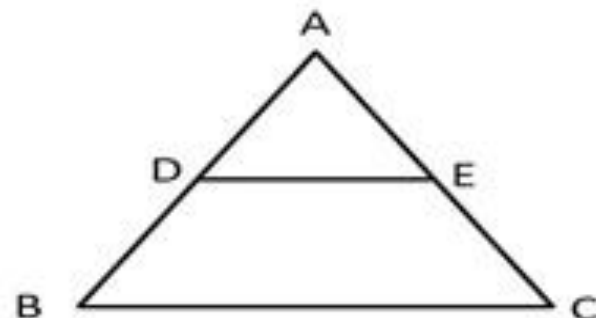
Given :- ΔABC where $DE \parallel BC$

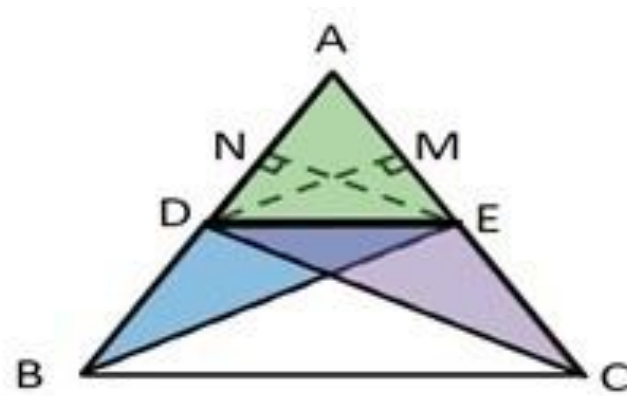
To Prove :- $\frac{AD}{DB} = \frac{AE}{EC}$

Construction :- Join BE and CD

Draw $DM \perp AC$ and $EN \perp AB$.

Proof:





Now,

$$\begin{aligned}\text{ar (ADE)} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times AD \times EN \quad \dots(1)\end{aligned}$$

$$\begin{aligned}\text{ar (BDE)} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times DB \times EN \quad \dots(2)\end{aligned}$$

Divide (1) and (2)

$$\frac{\text{ar (ADE)}}{\text{ar (BDE)}} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN}$$

$$\frac{\text{ar (ADE)}}{\text{ar (BDE)}} = \frac{AD}{DB} \quad \dots(A)$$

$$\begin{aligned}\text{ar (ADE)} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times AE \times DM \quad \dots(3)\end{aligned}$$

$$\begin{aligned}\text{ar (DEC)} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times EC \times DM \quad \dots(4)\end{aligned}$$

Divide (3) and (4)

$$\frac{\text{ar (ADE)}}{\text{ar (DEC)}} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM}$$

$$\frac{\text{ar (ADE)}}{\text{ar (DEC)}} = \frac{AE}{EC} \quad \dots(B)$$

Now,

$\triangle BDE$ and $\triangle DEC$ are on the same base DE and between the same parallel BC and DE .

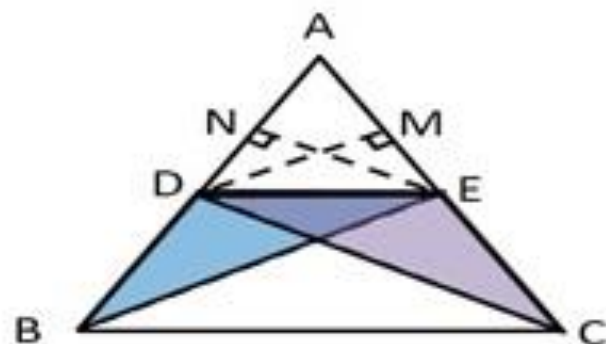
$$\therefore \text{ar}(\triangle BDE) = \text{ar}(\triangle DEC)$$

Hence,

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DEC)}$$

$$\frac{AD}{DB} = \frac{AE}{EC} \quad (\text{From (A) and (B)})$$

Hence Proved.



CONVERSE OF BPT

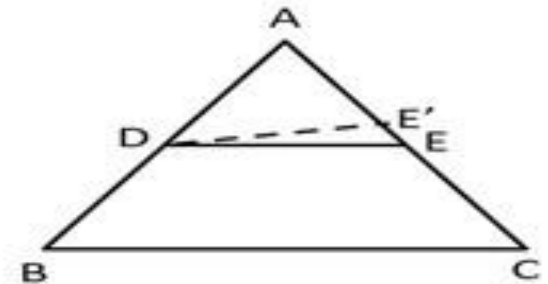
Theorem 6.2 :-

If line a divides any two side of a triangle in the same ratio, then the line is parallel to third side.

Given :- ΔABC and a line DE intersecting AB at D and AC at E , such that $\frac{AD}{DB} = \frac{AE}{EC}$

To Prove :- $DE \parallel BC$

Construction :- Draw DE' parallel to BC .



Proof :-

Since $DE' \parallel BC$,

By Theorem 6.1 : If a line is drawn parallel to one side of a triangle to intersecting other two sides not distinct points, the other two sides are divided in the same ratio.

$$\therefore \frac{AD}{DB} = \frac{AE'}{E'C} \quad \dots(1)$$

And given that,

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \dots(2)$$

From (1) and (2)

$$\frac{AE}{EC} = \frac{AE'}{E'C}$$

Adding 1 on both sides

$$\frac{AE}{EC} + 1 = \frac{AE'}{E'C} + 1$$

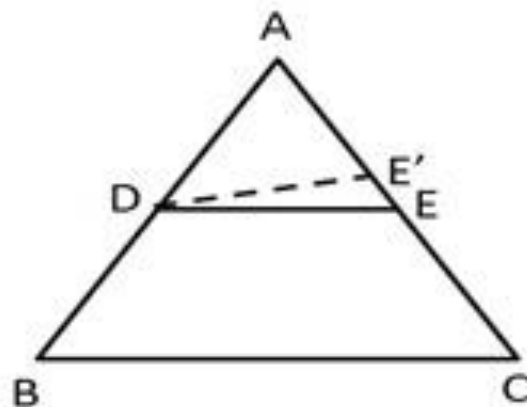
$$\frac{AE + EC}{EC} = \frac{AE' + E'C}{E'C}$$

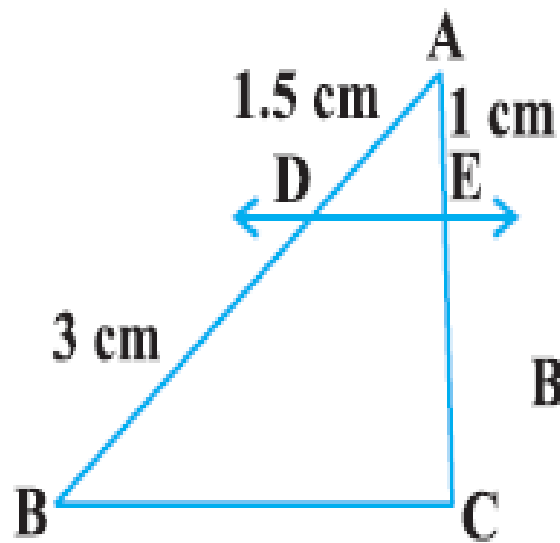
$$\frac{AC}{EC} = \frac{AC}{E'C}$$

$$EC = E'C$$

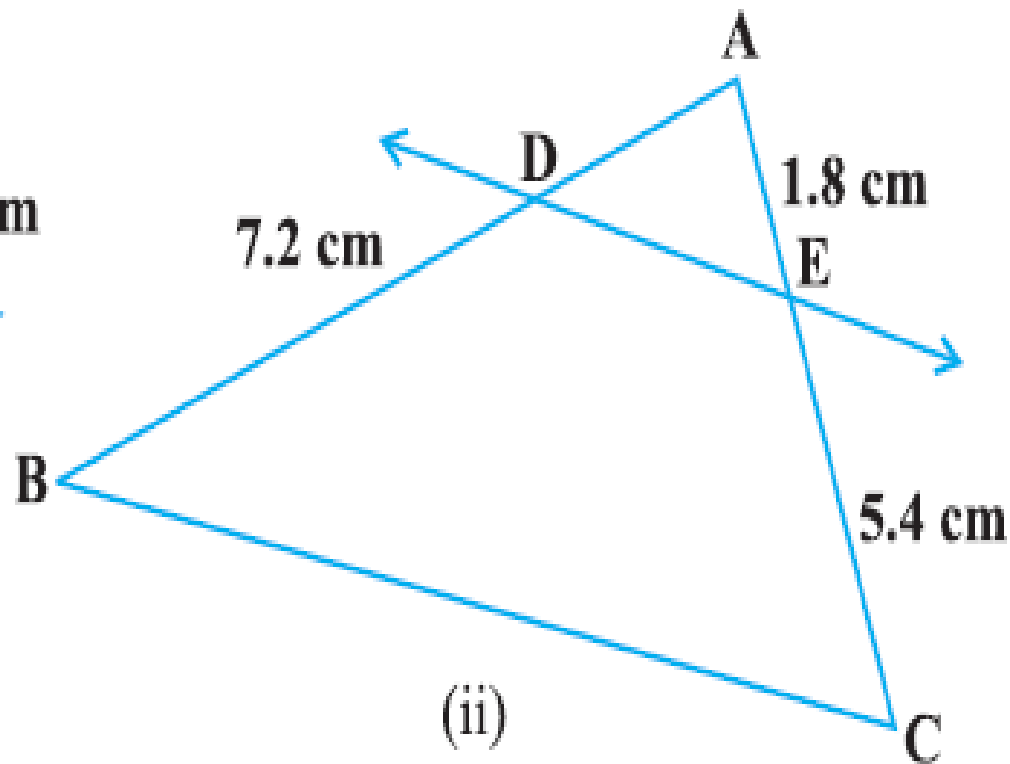
Thus, E and E' coincides.

Hence $DE \parallel BC$.



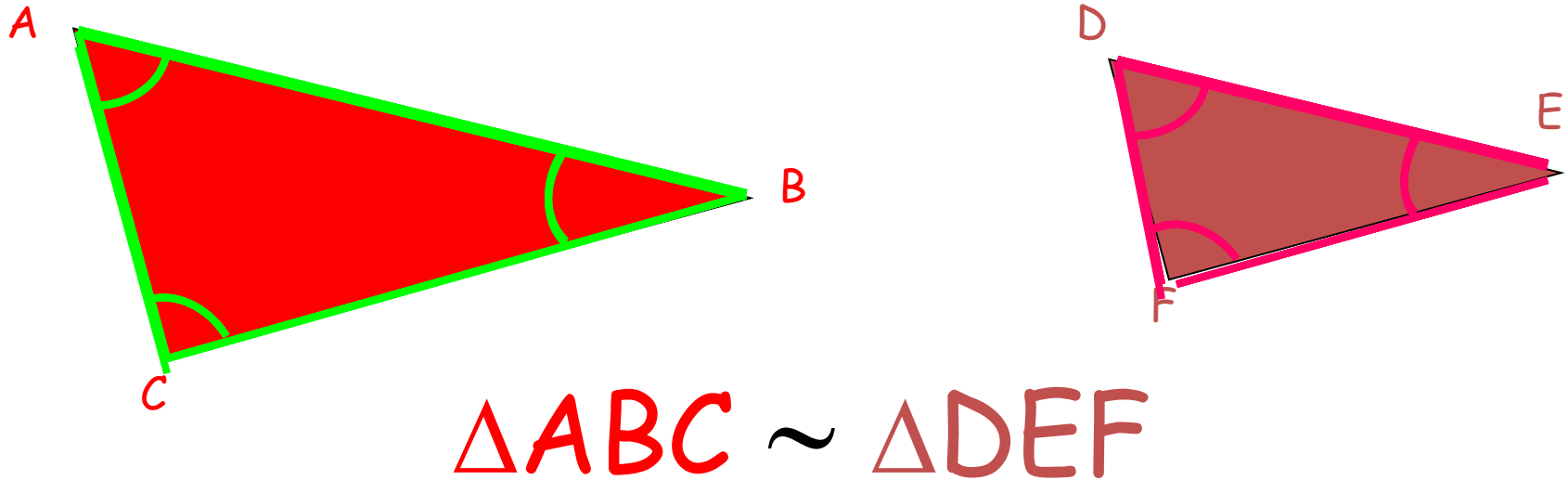


(i)



(ii)

Similar triangles are triangles that have the same shape but not necessarily the same size.



When we say that triangles are similar there are several repercussions that come from it.

$$\angle A \cong \angle D$$

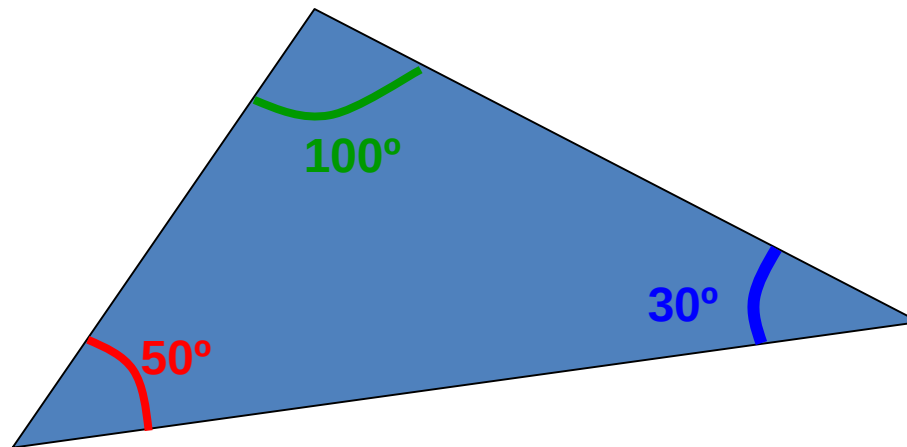
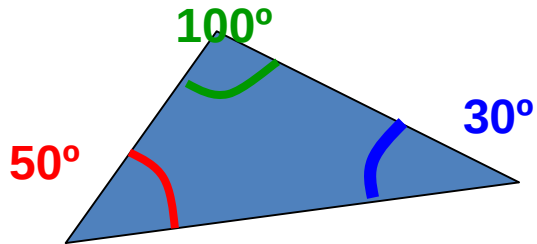
$$\angle B \cong \angle E$$

$$\angle C \cong \angle F$$

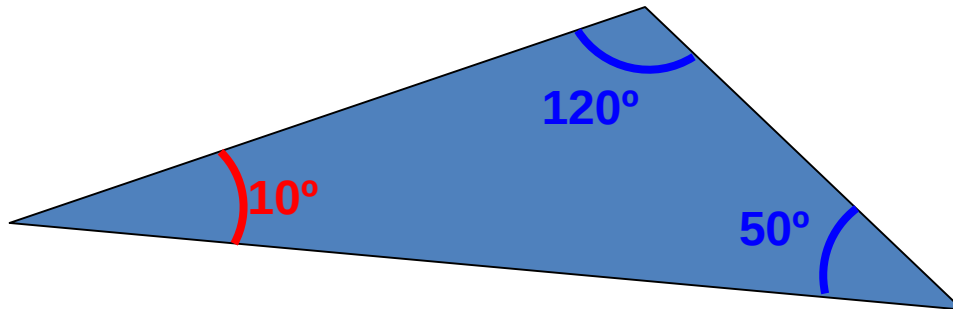
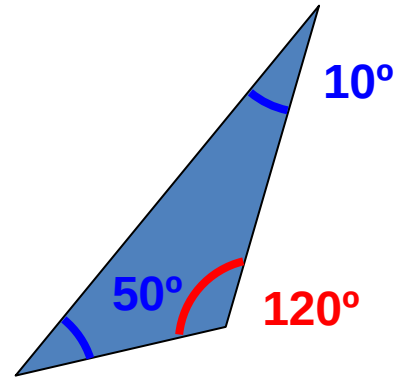
$$\frac{\overline{AB}}{\overline{DE}} = \frac{\overline{BC}}{\overline{EF}} = \frac{\overline{AC}}{\overline{DF}}$$

Similar Triangles

Triangles are similar if matching angles remain the same.



Show that these triangles are similar



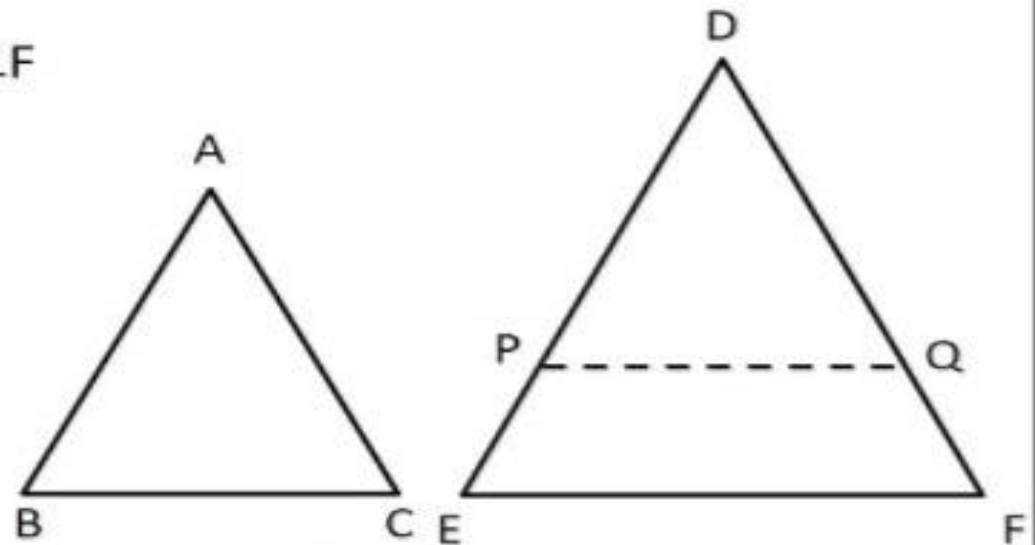
Theorem 6.3 (AAA Criteria)

If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.

Given :- Two triangles $\triangle ABC$ and $\triangle DEF$ such that

$$\angle A = \angle D, \angle B = \angle E \text{ \& \& } \angle C = \angle F$$

To Prove :- $\triangle ABC \sim \triangle DEF$



Construction :- Draw P and Q on DE & DF

such that $DP = AB$ and $DQ = AC$ respectively

and join PQ.

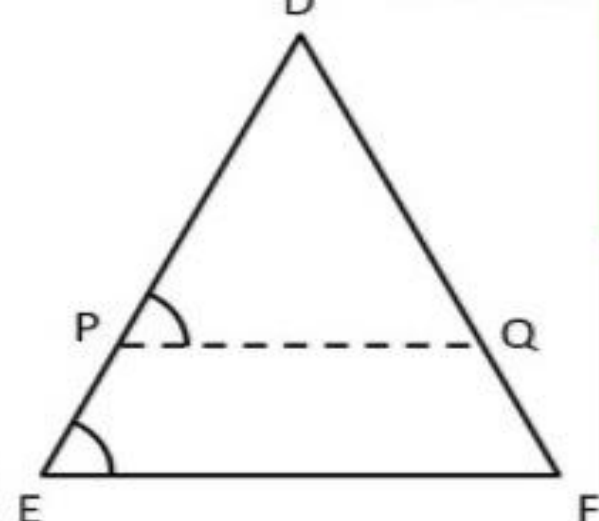
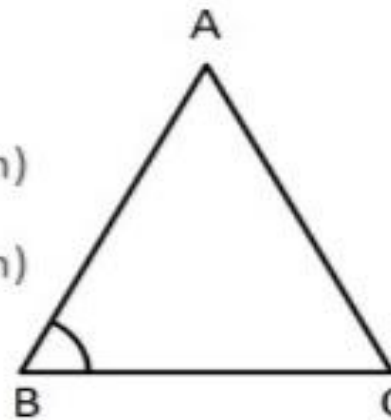
Proof :-

In $\triangle ABC$ and $\triangle DPQ$

$$AB = DP \quad (\text{Construction})$$

$$AC = DQ \quad (\text{Construction})$$

$$\angle A = \angle D \quad (\text{Given})$$



$$\Rightarrow \triangle ABC \cong \triangle DPQ \quad (\text{By SAS Criterion})$$

$$\Rightarrow \angle B = \angle P \quad (\text{By CPCT})$$

$$\text{But } \angle B = \angle E \quad (\text{Given})$$

$$\text{Thus, } \angle P = \angle E$$

But they are corresponding angles.

For lines AB & CD with transversal PS, corresponding angles are equal

Hence, $PQ \parallel EF$.

Since, $PQ \parallel EF$.

By **Theorem 6.1**, : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

$$\therefore \frac{DP}{PE} = \frac{DQ}{QF}$$

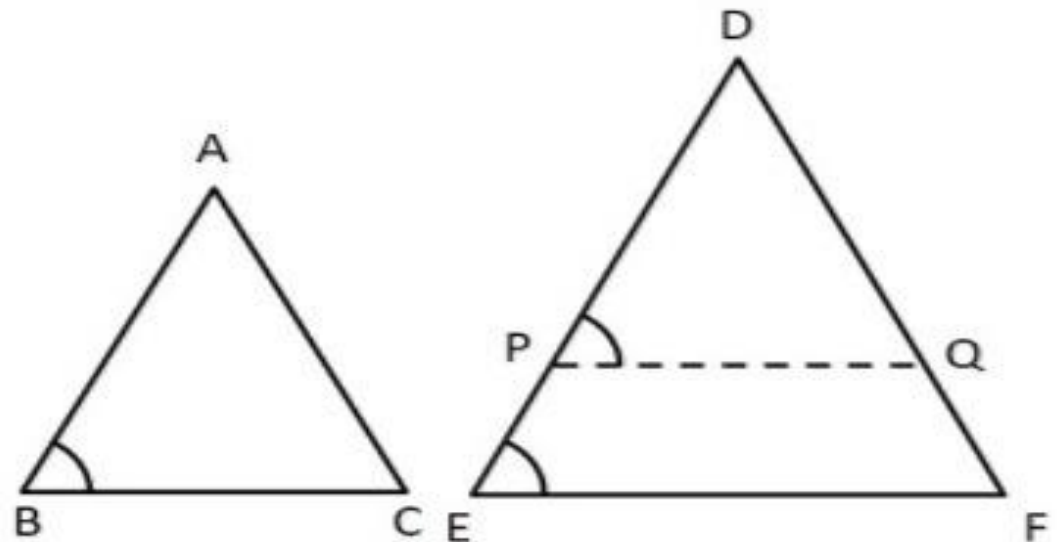
$$\Rightarrow \frac{PE}{DP} = \frac{QF}{DQ}$$

Adding 1 on both sides.

$$\frac{PE}{DP} + 1 = \frac{QF}{DQ} + 1$$

$$\frac{PE + DP}{DP} = \frac{QF + DQ}{DQ}$$

$$\frac{DE}{DP} = \frac{DF}{DQ}$$



$$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$$

And by construction $DP = AB$ and $DQ = AC$

$$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$$

Similarly, we can prove that

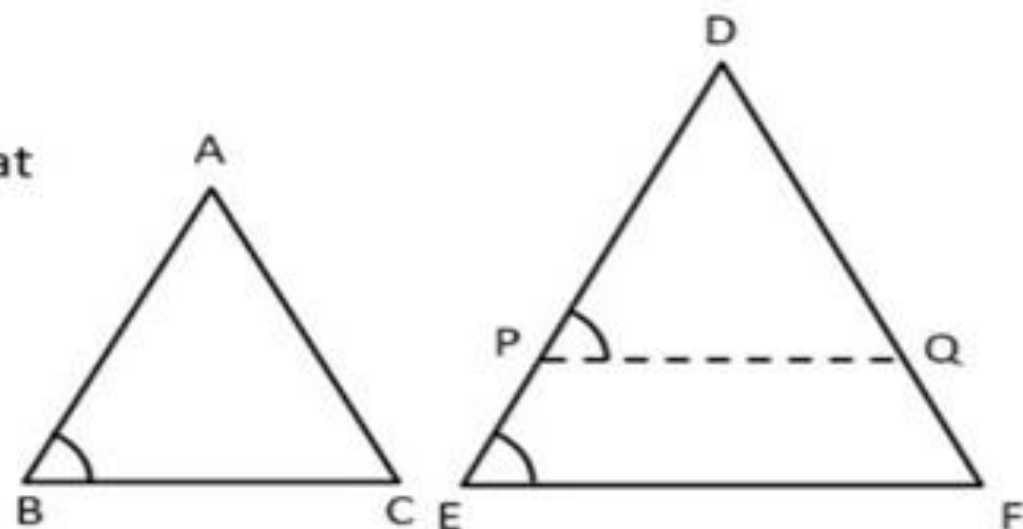
$$\frac{AB}{DE} = \frac{BC}{EF}$$

Therefore,

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

& $\triangle ABC \sim \triangle DEF$

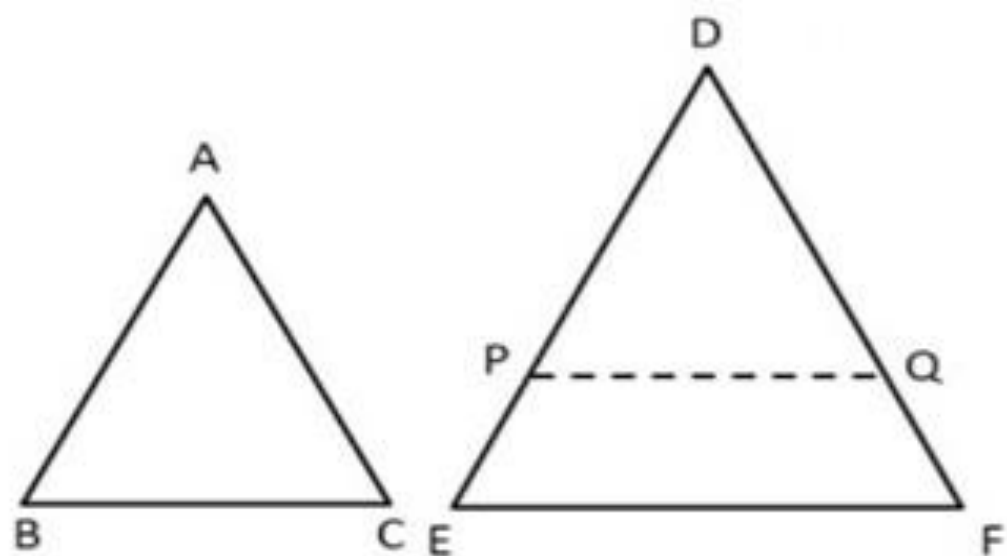
Hence Proved.



Proof :-

Given

$$\frac{AB}{DE} = \frac{CA}{FD}$$



And $DP = AB$, $DQ = AC$ (By Construction)

$$\therefore \frac{DP}{DE} = \frac{DQ}{DF}$$

$$\frac{DE}{DP} = \frac{DF}{DQ}$$

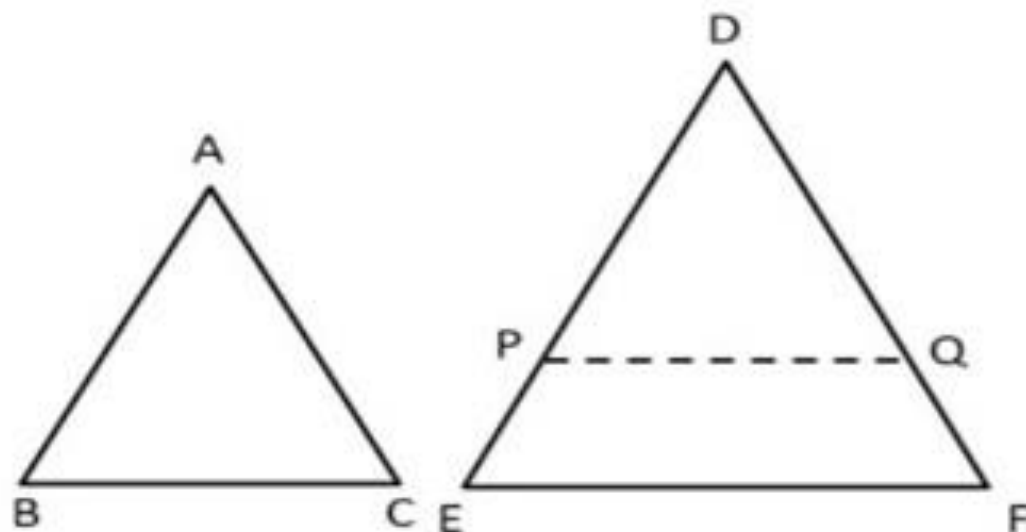
Subtracting 1 on both sides

$$\frac{DE}{DP} - 1 = \frac{DF}{DQ} - 1$$

$$\frac{DE - DP}{DP} = \frac{DF - DQ}{DQ}$$

$$\frac{PE}{DP} = \frac{QF}{DQ}$$

$$\frac{DP}{PE} = \frac{DQ}{QF}$$



Using **theorem 6.2** : If a line divides any two sides of a triangle in the same ratio, then the line is parallel to third side.

$\therefore PQ \parallel EF$.

Thus, $\angle P = \angle E$ and $\angle Q = \angle F$ (Corresponding Angles) ...(1)

Now, In $\triangle DPQ$ and $\triangle DEF$

$$\angle P = \angle E \quad (\text{From (1)})$$

$$\angle Q = \angle F \quad (\text{From (1)})$$

$$\Rightarrow \triangle DPQ \sim \triangle DEF \quad (\text{By AA Similarity Criterion})$$

$$\therefore \frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{PF} \quad (\text{Corresponding sides of similar triangles are proportional}) \quad \dots(2)$$

$$\text{Also, } \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} \quad (\text{Given})$$

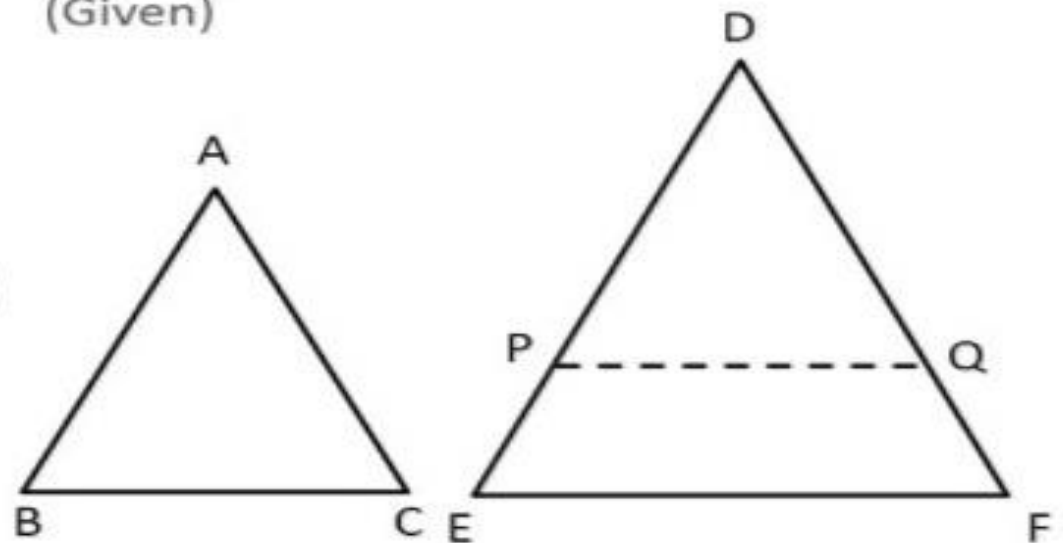
$$\text{and } AB = DP \text{ \& } CA = QD$$

$$\Rightarrow \frac{DP}{DE} = \frac{BC}{EF} = \frac{QD}{FD} \quad \dots(3)$$

From (2) and (3)

$$\frac{BC}{EF} = \frac{PQ}{EF}$$

$$\Rightarrow BC = PQ \quad \dots(4)$$



In $\triangle ABC$ and $\triangle DPQ$

$$AB = DP \quad (\text{Construction})$$

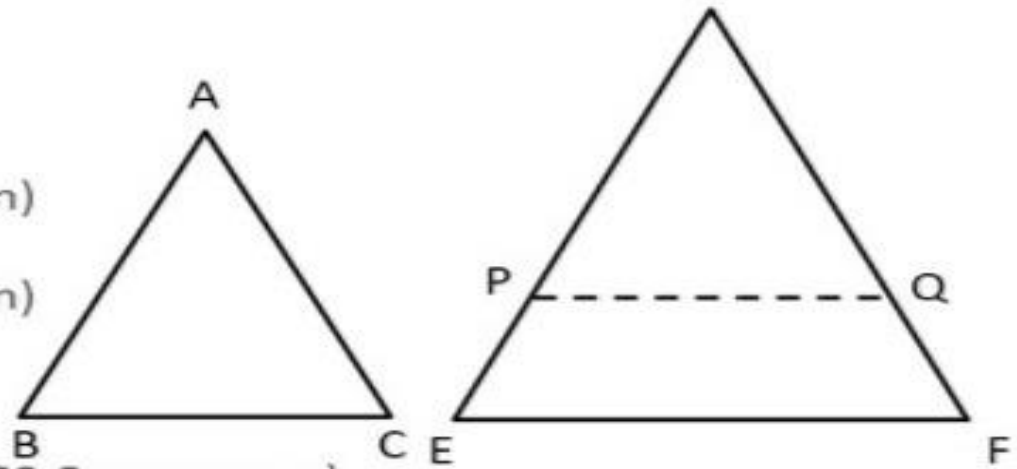
$$AC = DQ \quad (\text{Construction})$$

$$BC = PQ \quad (\text{From (4)})$$

$$\Rightarrow \triangle ABC \cong \triangle DPQ \quad (\text{By SSS Congruency})$$

$$\therefore \angle B = \angle P \quad (\text{By CPCT})$$

$$\angle C = \angle Q \quad (\text{By CPCT})$$



But From (1)

$$\angle P = \angle E \text{ and } \angle Q = \angle F$$

Therefore,

$$\angle B = \angle P = \angle E \quad \dots(4)$$

$$\text{and } \angle C = \angle Q = \angle F \quad \dots(5)$$

Therefore,

In ΔABC & ΔDEF

$$\angle B = \angle E \quad (\text{From (4)})$$

$$\angle C = \angle F \quad (\text{From (5)})$$

$$\therefore \Delta ABC \sim \Delta DEF \quad (\text{By AA Similarity Criteria})$$

Hence Proved

Theorem 6.5 (SAS Criteria)

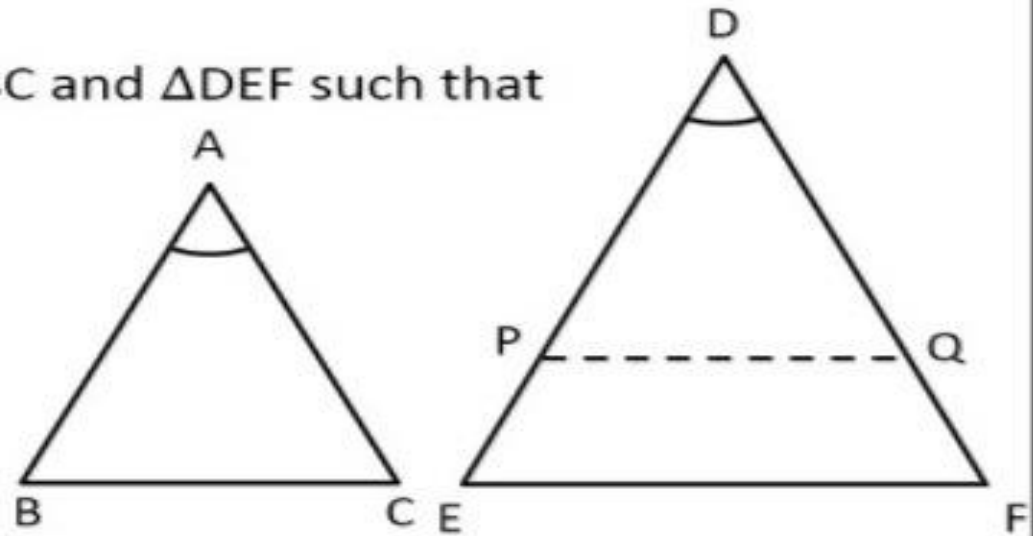
If one angle of a triangle is equal to one angle of the other triangle and sides including these angles are proportional then the triangles are similar.

Given :- Two triangles $\triangle ABC$ and $\triangle DEF$ such that

$$\angle A = \angle D$$

$$\frac{AB}{DE} = \frac{AC}{DF}$$

To Prove : $\triangle ABC \sim \triangle DEF$



Construction :- Draw P and Q on DE & DF

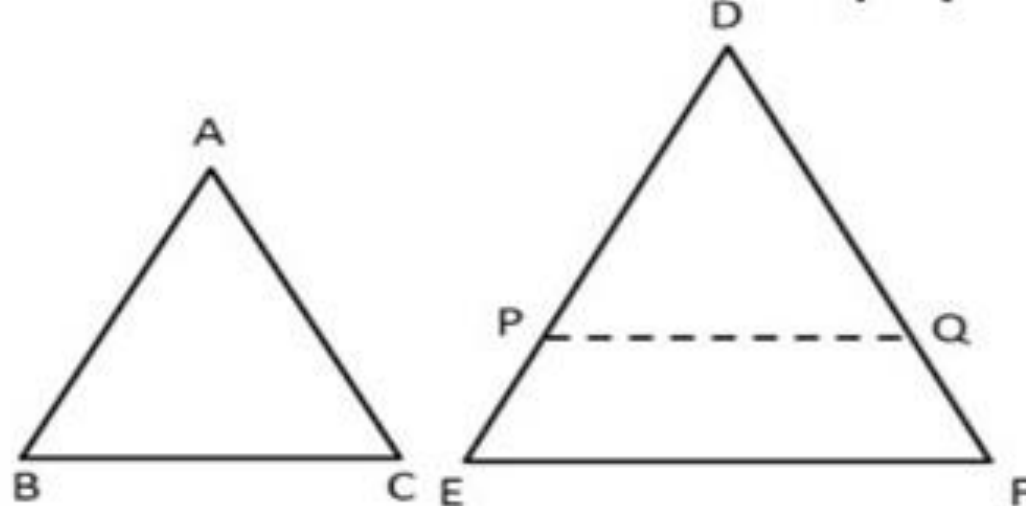
such that $DP = AB$ and $DQ = AC$ respectively

and join PQ.

Proof :-

Given

$$\frac{AB}{DE} = \frac{CA}{FD}$$



And $DP = AB$, $DQ = AC$

(By Construction)

$$\therefore \frac{DP}{DE} = \frac{DQ}{DF}$$

$$\frac{DE}{DP} = \frac{DF}{DQ}$$

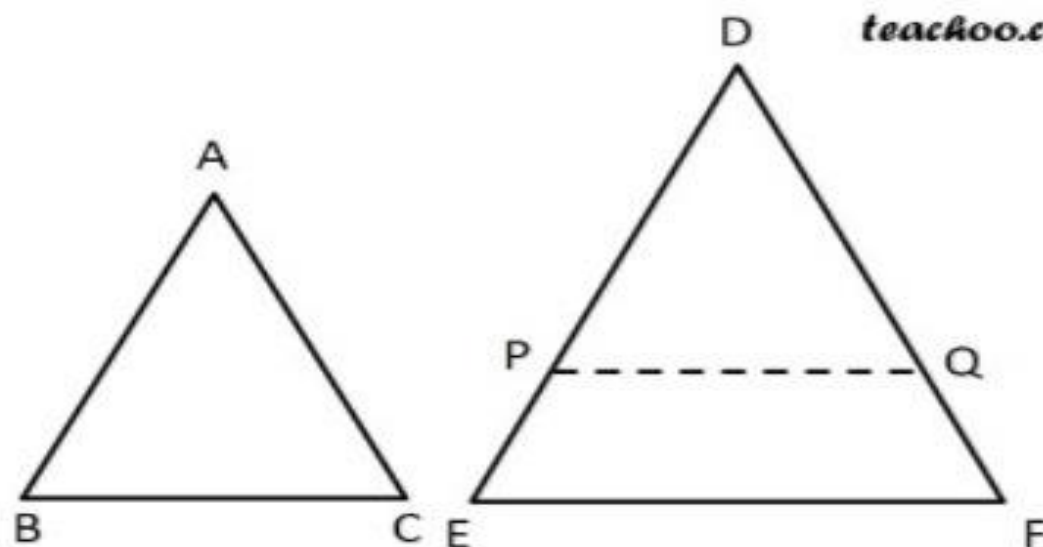
Subtracting 1 on both sides

$$\frac{DE}{DP} - 1 = \frac{DF}{DQ} - 1$$

$$\frac{DF - DP}{DP} = \frac{DF - DQ}{DQ}$$

$$\frac{PE}{DP} = \frac{QF}{DQ}$$

$$\frac{DP}{PE} = \frac{DQ}{QF}$$



Using **theorem 6.2** : If a line divides any two sides of a triangle in the same ratio, then the line is parallel to third side.

$\therefore PQ \parallel EF$.

Thus, $\angle P = \angle E$ and $\angle Q = \angle F$ (Corresponding Angles) ... (1)

In $\triangle ABC$ and $\triangle DPQ$

$$AB = DP \quad (\text{Construction})$$

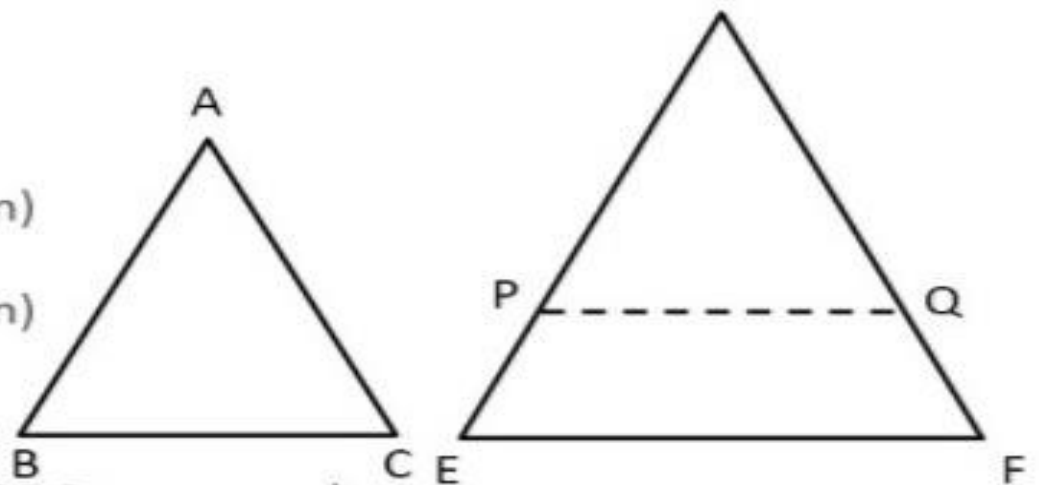
$$AC = DQ \quad (\text{Construction})$$

$$BC = PQ \quad (\text{From (4)})$$

$$\Rightarrow \triangle ABC \cong \triangle DPQ \quad (\text{By SSS Congruency})$$

$$\therefore \angle B = \angle P \quad (\text{By CPCT})$$

$$\angle C = \angle Q \quad (\text{By CPCT})$$



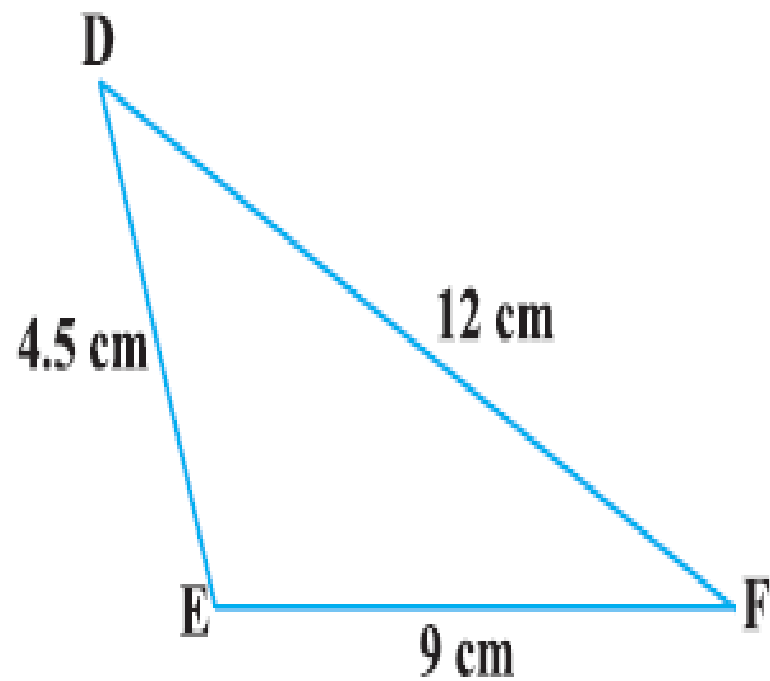
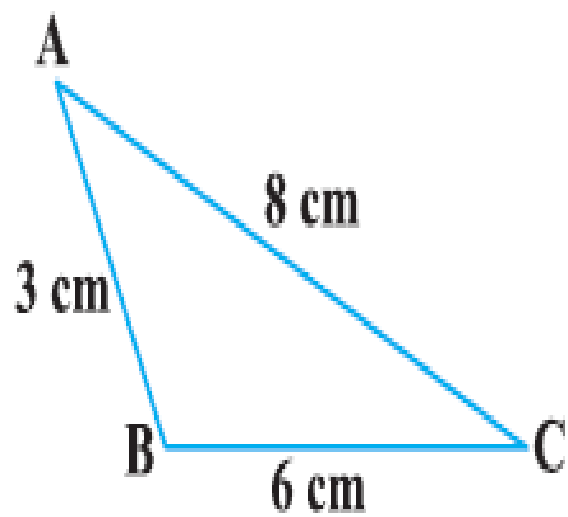
But From (1)

$$\angle P = \angle E \text{ and } \angle Q = \angle F$$

Therefore,

$$\angle B = \angle P = \angle E \quad \dots(4)$$

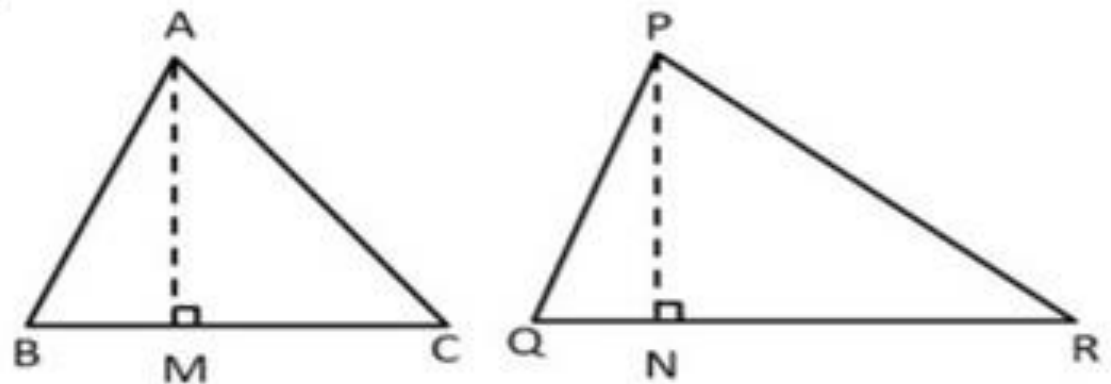
$$\text{and } \angle C = \angle Q = \angle F \quad \dots(5)$$



Theorem 6.6 :-

The ratio of the areas of two similar triangles is equal to the square of ratio of their corresponding sides.

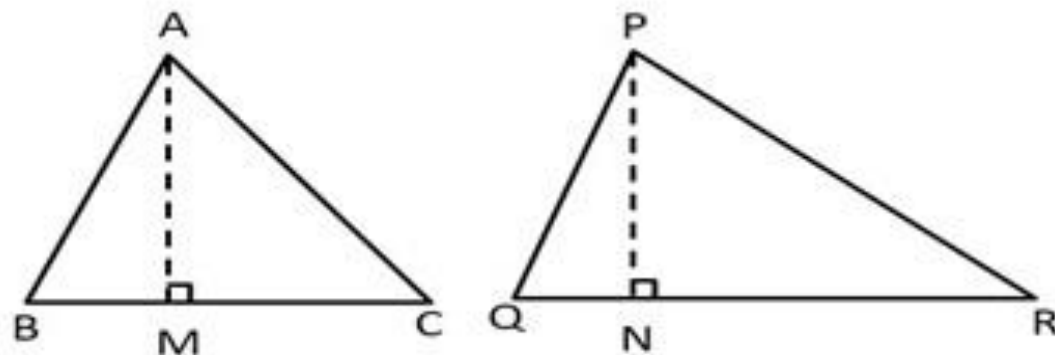
Given :- $\Delta ABC \sim \Delta PQR$



To Prove :- $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Construction :- Draw $AM \perp BC$ and $PN \perp QR$.

Proof:



$$\begin{aligned}\text{ar (ABC)} &= \frac{1}{2} \times \text{Base} \times \text{Height} \\ &= \frac{1}{2} \times BC \times AM \quad \dots(1)\end{aligned}$$

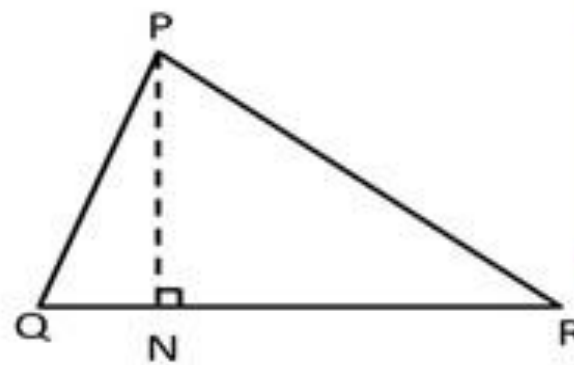
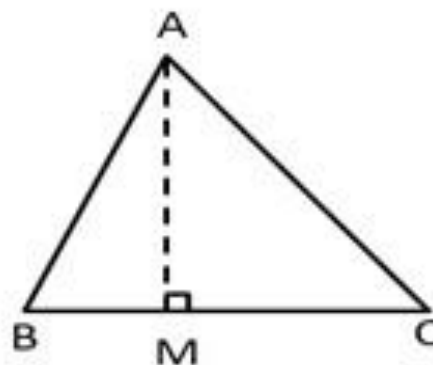
$$\begin{aligned}\text{ar (PQR)} &= \frac{1}{2} \times \text{Base} \times \text{Height} \\ &= \frac{1}{2} \times QR \times PN \quad \dots(2)\end{aligned}$$

Divide (1) & (2)

$$\frac{\text{ar (ABC)}}{\text{ar (PQR)}} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN}$$

$$\frac{\text{ar (ABC)}}{\text{ar (PQR)}} = \frac{BC \times AM}{QR \times PN} \quad \dots(A)$$

Now, $\triangle ABM$ and $\triangle PQN$



$$\angle B = \angle Q \quad \begin{array}{l} \text{(As } \triangle ABC \sim \triangle PQR \\ \text{\& Angles of similar triangles are equal)} \end{array}$$

$$\angle M = \angle N \quad \text{(Both } 90^\circ \text{)}$$

$$\triangle ABM \sim \triangle PQN \quad \text{(By AA Similarity)}$$

$$\therefore \frac{AB}{PQ} = \frac{AM}{PN} \quad \begin{array}{l} \text{(Corresponding sides of} \\ \text{similar triangles are proportional)} \end{array} \quad \dots(B)$$

From (A)

$$\frac{\text{ar (ABC)}}{\text{ar (PQR)}} = \frac{BC \times AM}{QR \times PN}$$

$$\frac{\text{ar (ABC)}}{\text{ar (PQR)}} = \frac{BC}{QR} \times \frac{AB}{PQ} \quad \begin{array}{l} \text{(From B)} \end{array} \quad \dots(C)$$

Now,

Given $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

Putting in (C)

$$\Rightarrow \frac{\text{ar}(ABC)}{\text{ar}(PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ} = \left(\frac{AB}{PQ}\right)^2$$

Now, again using $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

$$\Rightarrow \frac{\text{ar}(ABC)}{\text{ar}(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

Hence Proved.

EXAMPLE

- Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2 .
If $EF = 15.4 \text{ cm}$, find BC .

Theorem 6.7 :-

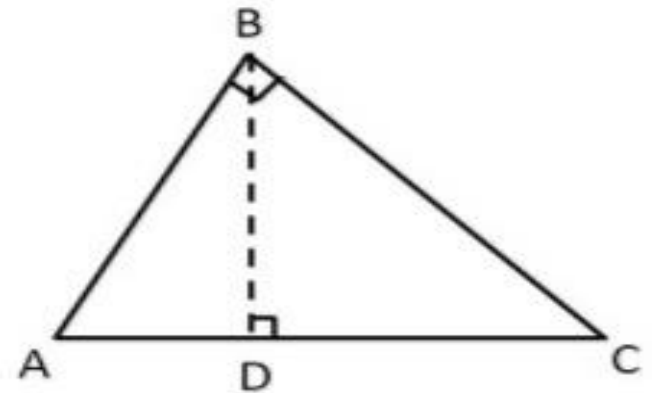
If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then right triangle on both sides of the perpendicular are similar to the whole triangle and to each other

Theorem 6.8 (Pythagoras Theorem) :

If a right triangle, the square of the hypotenuse is equal to the sum of the squares of other two sides.

Given :- $\triangle ABC$ right angle at B

To Prove :- $AC^2 = AB^2 + BC^2$



Construction :- Draw $BD \perp AC$

Proof :- Since $BD \perp AC$

Using **Theorem 6.7**: If a perpendicular is drawn from the vertex of the right angle of the a right triangle to the hypotenuse then triangle on both side of the perpendicular are similar to whole triangle and to each other.

$\Delta ADB \sim \Delta ABC$

Since, sides of similar triangles are in the same ratio,

$$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$$

$$\Rightarrow AD \cdot AC = AB^2 \quad \dots(1)$$

$\Delta BDC \sim \Delta ABC$

Since, sides of similar triangles are in the same ratio

$$\Rightarrow \frac{CD}{BC} = \frac{BC}{AC}$$

$$\Rightarrow CD \cdot AC = BC^2 \quad \dots(2)$$

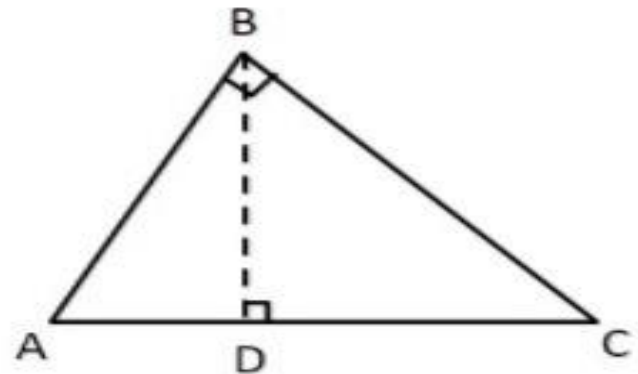
Adding (1) and (2)

$$AD \cdot AC + CD \cdot AC = AB^2 + BC^2$$

$$AC (AD + CD) = AB^2 + BC^2$$

$$AC \times AC = AB^2 + BC^2$$

$$AC^2 = AB^2 + BC^2$$



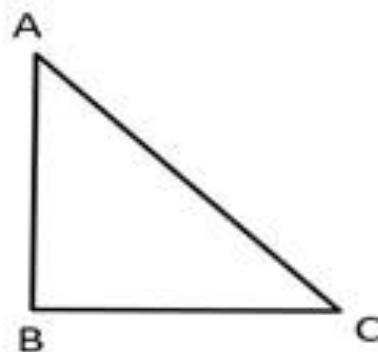
Hence Proved

Theorem 6.9 :-

In a right triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

Given :- A triangle ABC in which

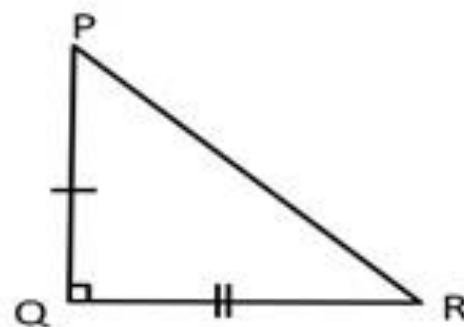
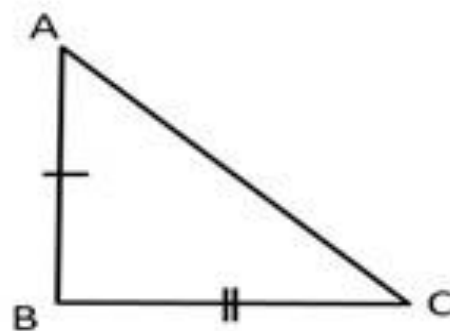
$$AC^2 = AB^2 + BC^2$$



To Prove :- $\angle B = 90^\circ$

Construction :- Draw ΔPQR right angled at Q, such that

$PQ = AB$ and $QR = BC$.



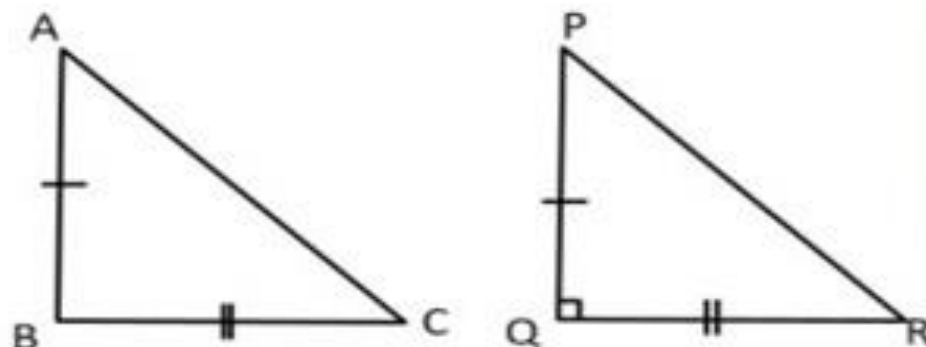
Proof :-

In ΔPQR

$$\angle Q = 90^\circ$$

By Pythagoras theorem,

$$PR^2 = PQ^2 + QR^2$$



Since $PQ = AB$ and $QR = BC$ (By Construction)

$$\Rightarrow PR^2 = AB^2 + BC^2 \quad \dots(1)$$

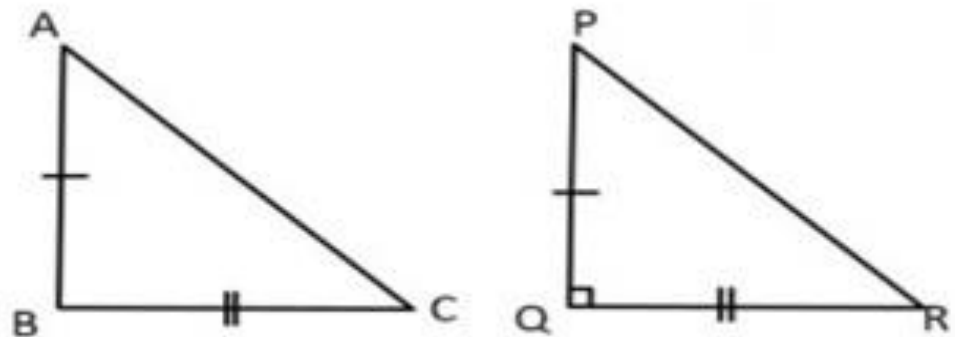
Also, given that

$$AC^2 = AB^2 + BC^2 \quad \dots(2)$$

From (1) & (2)

$$PR^2 = AC^2$$

$$PR = AC \quad \dots(3)$$



Now, In $\triangle ABC$ & $\triangle PQR$

$$AC = PR \quad (\text{From (3)})$$

$$AB = PQ \quad (\text{Construction})$$

$$BC = QR \quad (\text{Construction})$$

$$\therefore \triangle ABC \cong \triangle PQR \quad (\text{By SSS Congruence})$$

$$\Rightarrow \angle B = \angle Q \quad (\text{By CPCT})$$

$$\text{Since } \angle Q = 90^\circ \quad (\text{By Construction})$$

$$\angle B = 90^\circ$$

Hence Proved.

EXAMPLE

- A ladder is placed against a wall such that its foot is at a distance of 2.5 m from the wall and its top reaches a window 6 m above the ground. Find the length of the ladder.

