

# Chapter 3

## Whole Numbers: Operations and Properties

# 3.1 Addition and Subtraction

**Definition:** Let  $a$  and  $b$  be any two whole numbers. If  $A$  and  $B$  are disjoint sets with  $a = n(A)$  and  $b = n(B)$ , then  $a + b = n(A \cup B)$

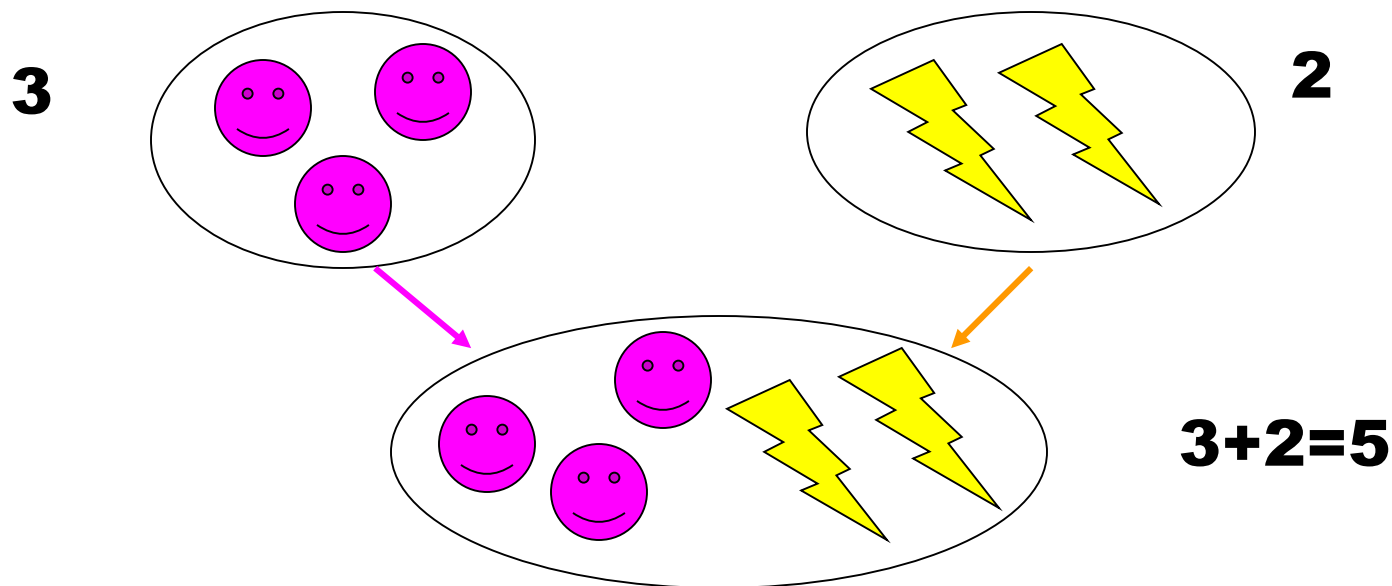
Addition is called a **binary operation** because two numbers are combined to produce a unique number.

# 3.1 Addition and Subtraction

## Set Model

To find  $a + b$ , we need to find two disjoint sets, one with  $a$  objects and one with  $b$ , form their union and count their total.

**Example:** Find  $3+2$  using the set model.



# Measurement Model

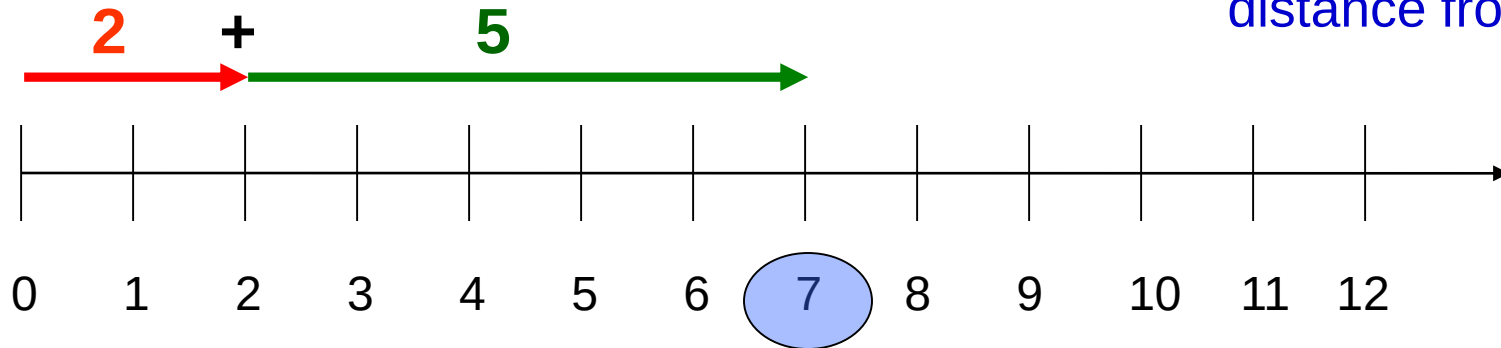
In the **measurement model**, addition of whole numbers is represented by directed arrows of whole number lengths along with the whole number line.

## Example: $2 + 5$

Place an arrow length 2 starting at 0.

Then place an arrow length 5 starting at the end of the first arrow.

The sum is the total distance from 0.



# Properties

1. Closure Property for Whole-Number Addition  
*The sum of any two whole numbers is a whole number.*
2. Commutative Property for Whole-Number Addition  
*Let  $a$  and  $b$  be any whole numbers. Then  $a+b=b+a$ .*
3. Associative Property for Whole-Number Addition  
*Let  $a$ ,  $b$  and  $c$  be any whole numbers.  
Then  $(a+b) + c = a + (b+c)$ .*
4. Identity Property for Whole-Number Addition  
*There is a unique whole number,  $0$ , such that for all whole numbers  $a$ ,  $a + 0 = 0 + a = a$ .*

# Thinking Strategies

1. Commutativity.
2. Adding Zero.
3. Counting on by 1 and 2.
4. Combinations to 10.
5. Doubles.
6. Adding 10.
7. Associativity.
8. Doubles +/-1 and +/-2.

# Subtraction

## **Take-Away Approach:**

Let  $a$  and  $b$  be any whole numbers and  $A$  and  $B$  be sets such that  $a = n(A)$ ,  $b = n(B)$  and  $B \subseteq A$ .

Then

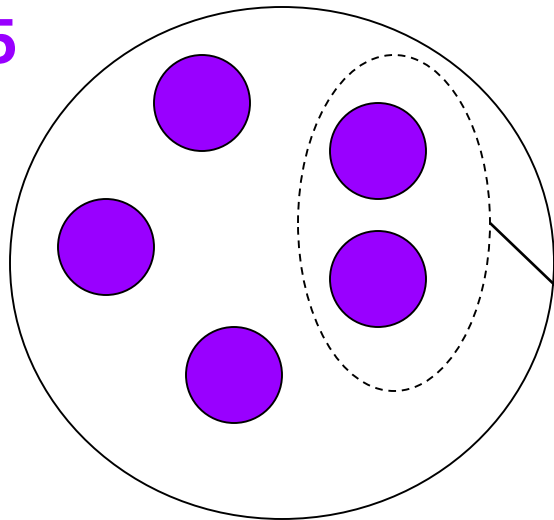
$$a - b = n(A - B).$$

# Subtraction

Take Away Approach:  $5 - 2 = ?$

**Set Model**

5



**Start with 5 objects.**

**Circle two objects.**

2

**Take them “away”.**

3

**Leaves the difference.**

$$5 - 2 = 3$$

# Subtraction

**The number “ $a - b$ ” is called the difference.**

**The expression is read “a minus b”.**

$$a - b$$

**$a$  is called the “minuend”.**

**$b$  is called the “subtrahend”.**

# Subtraction: Alternative Definition

## **Missing-Addend Approach**

Let  $a$  and  $b$  be any whole numbers.

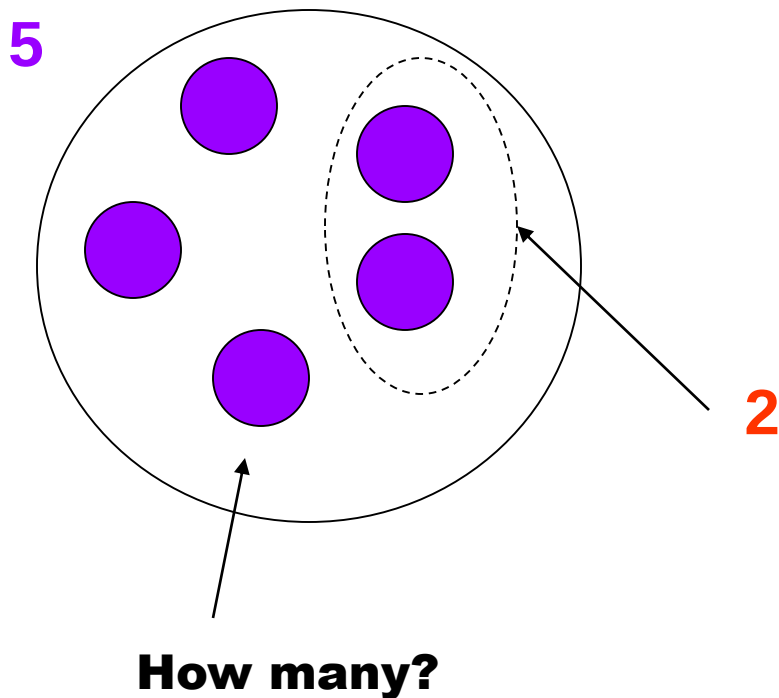
Then  $a - b = c$

if and only if  $a = b + c$  for some whole number  $c$ .

# Subtraction

## Missing-Addend Approach:

**This approach involves changing the subtraction problem to an addition problem.**



$$5 - 2 = ?$$

if and only if

$$2 + ? = 5$$

## 3.2 Multiplication and Division

### **Multiplication**

#### **Repeated-Addition Approach**

Let  $a$  and  $b$  be any whole numbers where  $a \neq 0$ .

Then

$$ab = b + b + \cdots + b$$



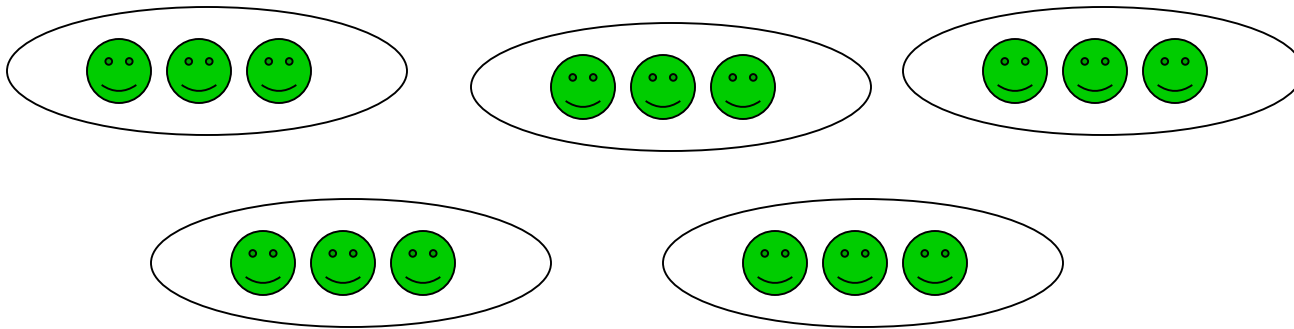
$a$  addends

If  $a = 1$ , then  $ab = 1 \cdot b = b$ ; also  $0 \cdot b = 0$  for all  $b$ .

# Multiplication

## Repeated-Addition Approach

### Set Model



$$5 \times 3$$

This shows that  $3 + 3 + 3 + 3 + 3 = 15$ , or that  $5 \times 3 = 15$ .

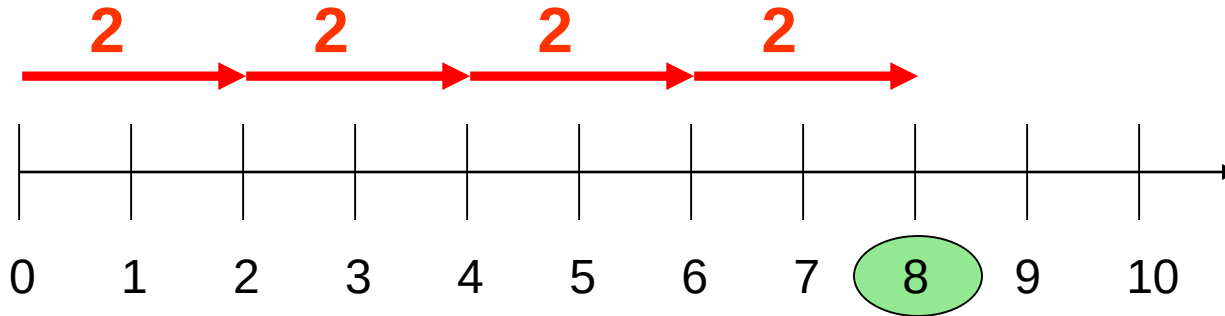
*Five groups of three objects illustrates  $5 \times 3 = 15$ , not  $3 \times 5 = 15$ .*

# Multiplication

## Repeated-Addition Approach

### Measurement Model

$$4 \times 2$$

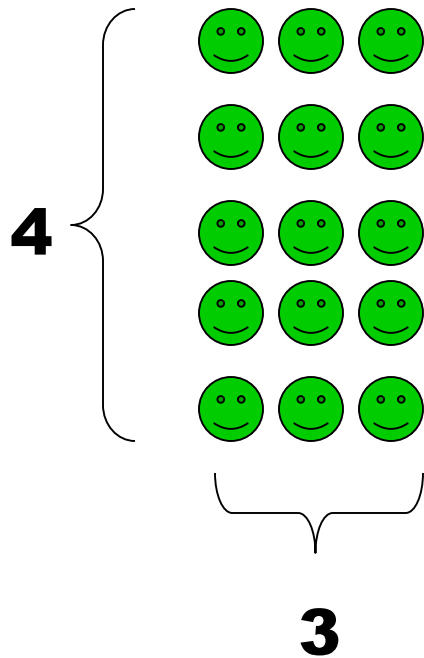


This shows that  $2 + 2 + 2 + 2 = 8$ , or that  $4 \times 2 = 8$ .

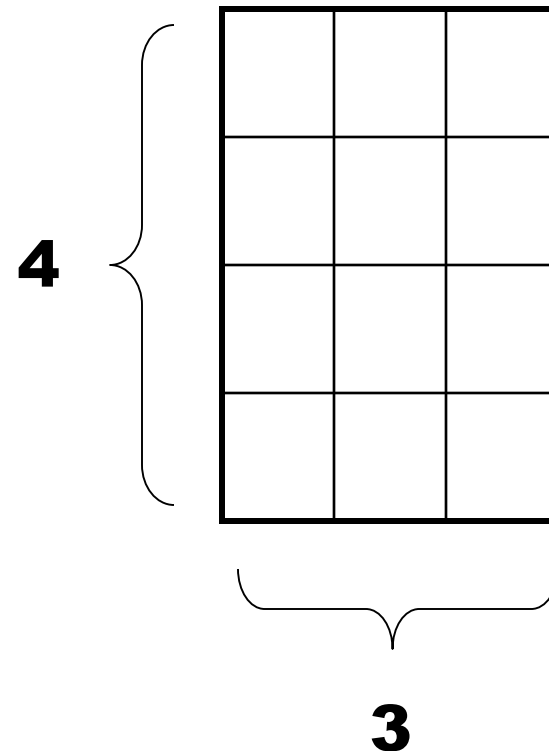
# Multiplication

## Rectangular Array Approach

**Set Model**



**Measurement Model**



# Properties of Whole Number Multiplication

## 1. Closure Property

*The product of any two whole numbers is a whole number.*

## 2. Commutative Property

*Let  $a$  and  $b$  be any whole numbers. Then  $a \times b = b \times a$ .*

## 3. Associative Property

*Let  $a$ ,  $b$  and  $c$  be any whole numbers.*

*Then  $(a \times b) \times c = a \times (b \times c)$ .*

## 4. Identity Property

*There is a unique whole number, 1, such that for all whole numbers  $a$ ,  $a \times 1 = 1 \times a = a$ .*

# New Property

## 5. Distributive Property of Multiplication over Addition

*Let  $a$ ,  $b$  and  $c$  be any whole numbers. Then*

$$a(b + c) = ab + ac$$

## 6. Distributive Property of Multiplication over Subtraction

*Let  $a$ ,  $b$  and  $c$  be any whole numbers. Then*

$$a(b - c) = ab - ac, \text{ where } b \geq c.$$

# Properties

| Property                                     | Addition   | Multiplication |
|----------------------------------------------|------------|----------------|
| Closure                                      | Yes        | Yes            |
| Commutativity                                | Yes        | Yes            |
| Associativity                                | Yes        | Yes            |
| Identity                                     | Yes (0)    | Yes (1)        |
| Distributive of multiplication over addition | <b>Yes</b> |                |

# One Last Property

## 7. Multiplication Property of Zero

*For every whole number,  $a$ ,*

$$a \cdot 0 = 0 \cdot a = 0.$$

# Division

Two very subtle kinds of division, **partitive division** and **measurement division**.

A class of 20 children is to be divided into four teams with the same number of children on each team. How many children are on each team?

*Since the number of divisions, or parts, is known, this is an example of partitive division.*

*A class of 20 children is to be divided into teams of four children each. How many teams are there?*

*Since the size, or measure, of each partition is known, this is an example of measurement division.*

# Division

## Division of Whole Numbers:

### Missing-Factor Approach

If  $a$  and  $b$  are any whole numbers with  $b \neq 0$ ,  
then  $a \div b = c$  if and only if  $a = bc$  for some whole  
number  $c$ .

The diagram shows the equation  $a \div b = c$  with three colored arrows pointing to its components: a blue arrow from the word "Dividend" to  $a$ , a green arrow from the word "Divisor" to  $b$ , and a red arrow from the word "Quotient" to  $c$ .

$$a \div b = c$$

**Dividend**                      **Divisor**                      **Quotient**

# Division and Zero

## **Division Property of Zero:**

If  $a \neq 0$ , then  $0 \div a = 0$ .

***Division by zero is undefined.***

# The Division Algorithm

If  $a$  and  $b$  are any whole numbers with  $d \neq 0$ , then there exist unique whole numbers  $q$  and  $r$  such that

$$a = dq + r, \text{ where } 0 \leq r < d.$$

# Division

## Division of Whole Numbers:

### **Repeated Subtraction Approach**

*Multiplication can be viewed as repeated multiplication.  
Similarly, division can be viewed as repeated subtraction.*