

Activity 7

OBJECTIVE

To verify the relation between the degree measure and the radian measure of an angle.

MATERIAL REQUIRED

Bangle, geometry box, protractor, thread, marker, cardboard, white paper.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Draw a circle using a bangle on the white paper.
3. Take a set square and place it in two different positions to find diameters PQ and RS of the circle as shown in the Fig.7.1 and 7.2

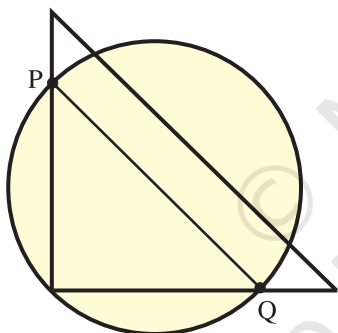


Fig. 7.1

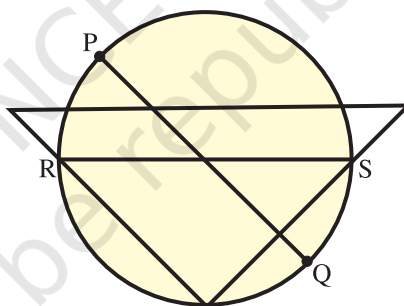


Fig. 7.2

4. Let PQ and RS intersect at C. The point C will be the centre of the circle (Fig. 7.3).
5. Clearly $CP = CR = CS = CQ = \text{radius}$.

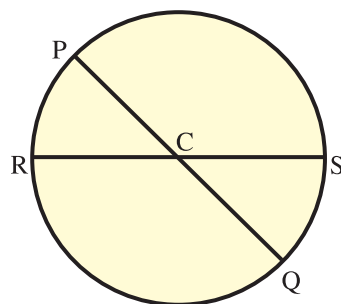


Fig. 7.3

DEMONSTRATION

- Let the radius of the circle be r and l be an arc subtending an angle θ at the centre C , as shown

in Fig. 7.4. $\theta = \frac{l}{r}$ radians.

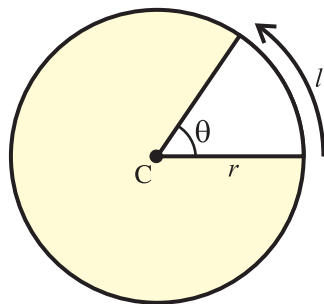


Fig. 7.4

- If Degree measure of $\theta = \frac{l}{2\pi r} \times 360$ degrees

$$\text{Then } \frac{l}{r} \text{ radians} = \frac{l}{2\pi r} \times 360 \text{ degrees}$$

$$\text{or } 1 \text{ radian} = \frac{180}{\pi} \text{ degrees} = 57.27 \text{ degrees.}$$

OBSERVATION

Using thread, measure arc lengths RP, PS, RQ, QS and record them in the table given below :

S.No	Arc	length of arc (l)	radius of circle (r)	Radian measure
1.	$\widehat{RP}$	-----	-----	$\angle RCP = \frac{\widehat{RP}}{r} = \underline{\hspace{1cm}}$
2.	$\widehat{PS}$	-----	-----	$\angle PCS = \frac{\widehat{PS}}{r} = \underline{\hspace{1cm}}$
3.	$\widehat{SQ}$	-----	-----	$\angle SCQ = \frac{\widehat{SQ}}{r} = \underline{\hspace{1cm}}$
4.	$\widehat{QR}$	-----	-----	$\angle QCR = \frac{\widehat{QR}}{r} = \underline{\hspace{1cm}}$

2. Using protractor, measure the angle in degrees and complete the table.

Angle	Degree measure	Radian Measure	Ratio = $\frac{\text{Degree measure}}{\text{Radian measure}}$
$\angle RCP$	-----	-----	-----
$\angle PCS$	-----	-----	-----
$\angle QCS$	-----	-----	-----
$\angle QCR$	-----	-----	-----

3. The value of one radian is equal to _____ degrees.

APPLICATION

This result is useful in the study of trigonometric functions.

Activity 8

OBJECTIVE

To find the values of sine and cosine functions in second, third and fourth quadrants using their given values in first quadrant.

MATERIAL REQUIRED

Cardboard, white chart paper, ruler, coloured pens, adhesive, steel wires and needle.

METHOD OF CONSTRUCTION

1. Take a cardboard of convenient size and paste a white chart paper on it.
2. Draw a unit circle with centre O on chart paper.
3. Through the centre of the circle, draw two perpendicular lines $X'OX$ and YOY' representing x -axis and y -axis, respectively, as shown in Fig.8.1.

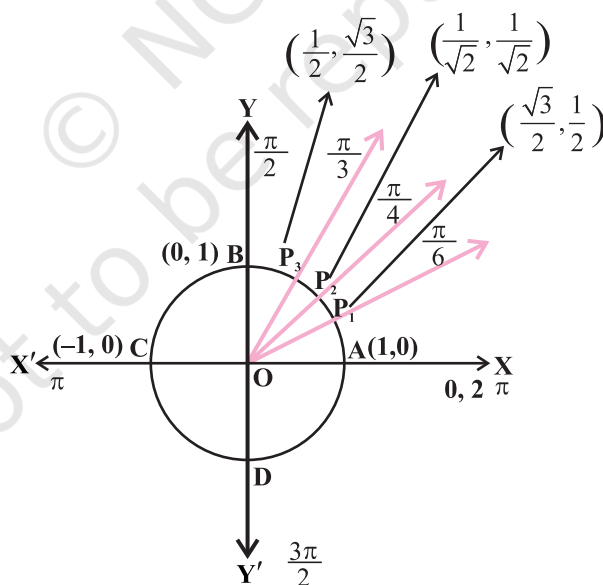


Fig. 8.1

- Mark the points as A, B, C and D, where the circle cuts the x -axis and y -axis, respectively, as shown in Fig. 8.1.
- Through O, draw angles P_1OX , P_2OX , and P_3OX of measures $\frac{\pi}{6}$, $\frac{\pi}{4}$ and $\frac{\pi}{3}$, respectively.
- Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle.

DEMONSTRATION

- The coordinates of the point P_1 are $\left(\sqrt{\frac{3}{2}}, \frac{1}{2}\right)$ because its x -coordinate is $\cos \frac{\pi}{6}$ and y -coordinate is $\sin \frac{\pi}{6}$. The coordinates of the points P_2 and P_3 are $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, respectively.
- To find the value of sine or cosine of some angle in the second quadrant (say) $\frac{2\pi}{3}$, rotate the needle in anti clockwise direction making an angle P_4OX of measure $\frac{2\pi}{3} = 120^\circ$ with the positive direction of x -axis.
- Look at the position OP_4 of the needle in

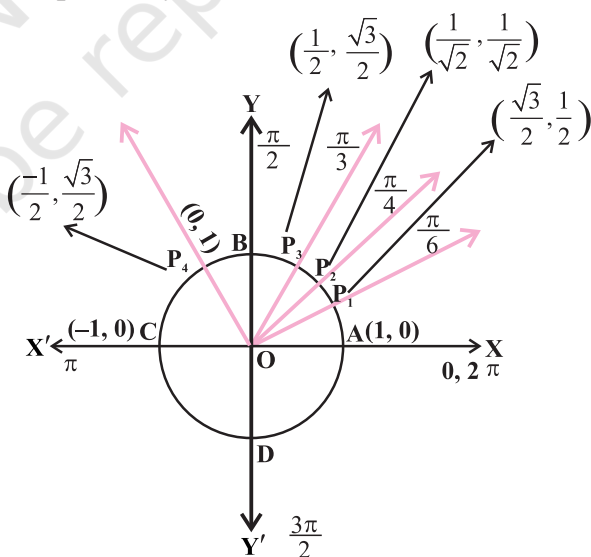


Fig. 8.2

Fig.8.2. Since $\frac{2\pi}{3} = \pi - \frac{\pi}{3}$, OP_4 is the mirror image of OP_3 with respect to

y-axis. Therefore, the coordinate of P_4 are $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Thus

$$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2} \text{ and } \cos \frac{2\pi}{3} = -\frac{1}{2}.$$

4. To find the value of sine or cosine of some angle say, $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$, i.e., $-\frac{2\pi}{3}$ (say) in the third quadrant, rotate the needle in anti clockwise direction making as an angle of $\frac{4\pi}{3}$ with the positive direction of x-axis.

5. Look at the new position OP_5 of the needle, which is shown in Fig. 8.3.

Point P_5 is the mirror image of the point P_4 (since $\angle P_4OX' = \angle P_5OX'$) with respect to x-axis. Therefore, coordinates of P_5 are

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ and hence

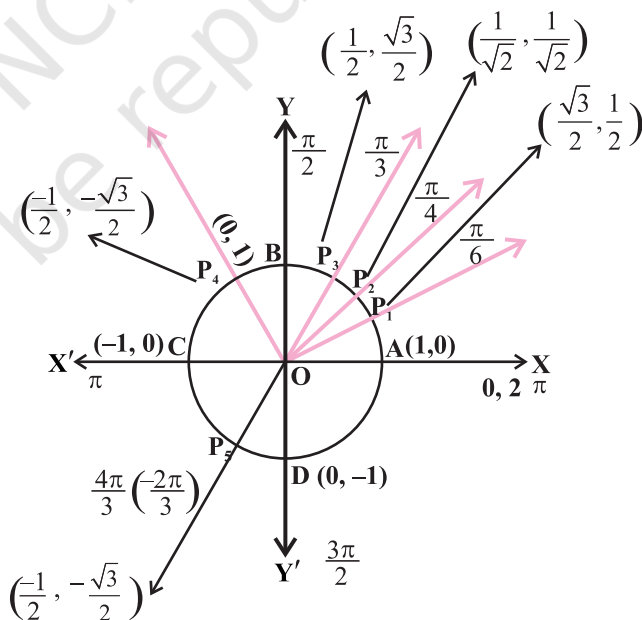


Fig. 8.3

$$\sin\left(-\frac{2\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \text{ and } \cos\left(-\frac{2\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}.$$

6. To find the value of sine or cosine of some angle in the fourth quadrant, say

$\frac{7\pi}{4}$, rotate the needle in anti clockwise direction making an angle of $\frac{7\pi}{4}$ with the positive direction of x -axis represented by OP_6 , as shown in

Fig. 8.4. Angle $\frac{7\pi}{4}$ in anti clockwise direction = Angle $-\frac{\pi}{4}$ in the clockwise direction.

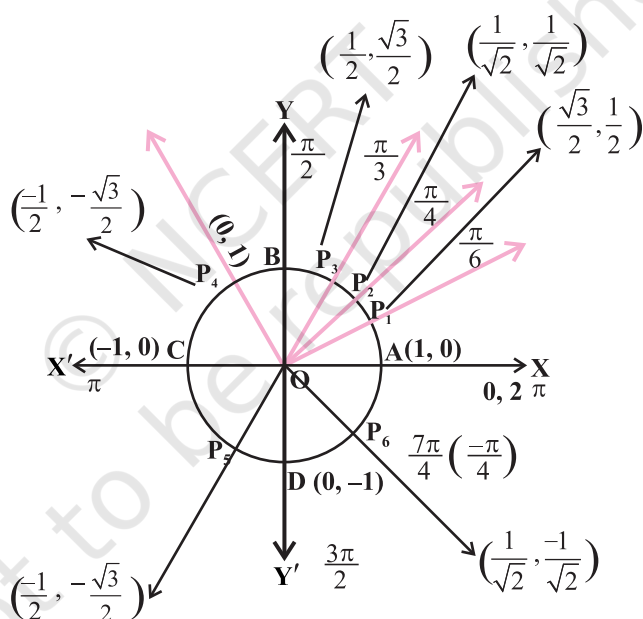


Fig. 8.4

From Fig. 8.4, P_6 is the mirror image of P_2 with respect to x -axis. Therefore,

coordinates of P_6 are $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

Thus $\sin\left(\frac{7\pi}{4}\right) = \sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

and $\cos\left(\frac{7\pi}{4}\right) = \cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

8. To find the value of sine or cosine of some angle, which is greater than one revolution, say $\frac{13\pi}{6}$, rotate the needle in anti clockwise direction since

$\frac{13\pi}{6} = 2\pi + \frac{\pi}{6}$, the needle will reach at the position OP_1 . Therefore,

$$\sin\left(\frac{13\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \text{ and } \cos\left(\frac{13\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}.$$

OBSERVATION

1. Angle made by the needle in one complete revolution is _____.

2. $\cos \frac{\pi}{6} = \text{_____} = \cos\left(-\frac{\pi}{6}\right)$

$\sin \frac{\pi}{6} = \text{_____} = \sin(2\pi + \text{_____}).$

3. sine function is non-negative in _____ and _____ quadrants.
4. cosine function is non-negative in _____ and _____ quadrants.

APPLICATION

1. The activity can be used to get the values for tan, cot, sec, and cosec functions also.
2. From this activity students may learn that
 $\sin(-\theta) = -\sin \theta$ and $\cos(-\theta) = \cos \theta$

This activity can be applied to other trigonometric functions also.

Activity 9

OBJECTIVE

To prepare a model to illustrate the values of sine function and cosine function for different angles which are multiples of $\frac{\pi}{2}$ and π .

MATERIAL REQUIRED

A stand fitted with 0° - 360° protractor and a circular plastic sheet fixed with handle which can be rotated at the centre of the protractor.

METHOD OF CONSTRUCTION

1. Take a stand fitted with 0° - 360° protractor.
2. Consider the radius of protractor as 1 unit.

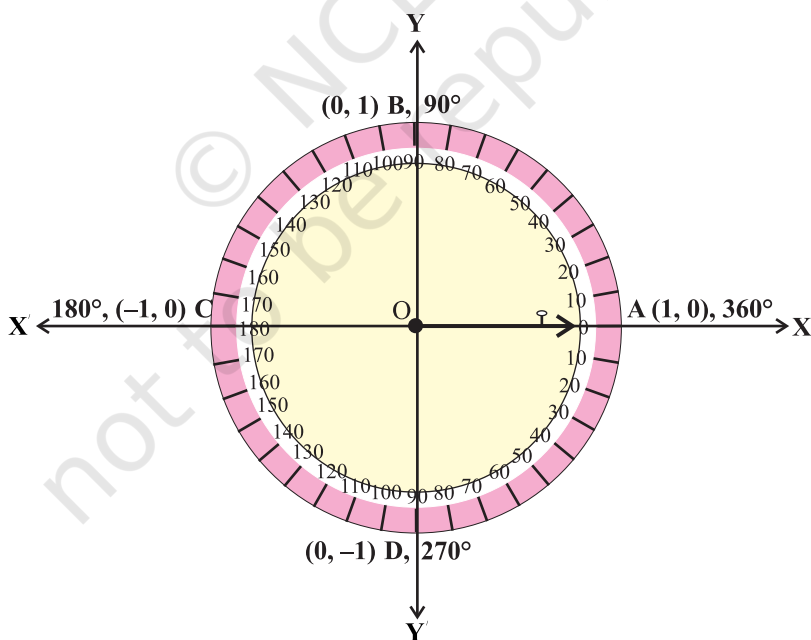


Fig. 9

3. Draw two lines, one joining 0° - 180° line and another 90° - 270° line, obviously perpendicular to each other.
4. Mark the ends of 0° - 180° line as (1,0) at 0° , (-1, 0) at 180° and that of 90° - 270° line as (0,1) at 90° and (0, -1) at 270°
5. Take a plastic circular plate and mark a line to indicate its radius and fix a handle at the outer end of the radius.
6. Fix the plastic circular plate at the centre of the protractor.

DEMONSTRATION

1. Move the circular plate in anticlock wise direction to make different angles like 0 , $\frac{\pi}{2}$, π , $\frac{3\pi}{2}$, 2π etc.
2. Read the values of sine and cosine function for these angles and their multiples from the perpendicular lines.

OBSERVATION

1. When radius line of circular plate is at 0° indicating the point A (1,0),
 $\cos 0 = \underline{\hspace{2cm}}$ and $\sin 0 = \underline{\hspace{2cm}}$.
2. When radius line of circular plate is at 90° indicating the point B (0, 1),
 $\cos \frac{\pi}{2} = \underline{\hspace{2cm}}$ and $\sin \frac{\pi}{2} = \underline{\hspace{2cm}}$.
3. When radius line of circular plate is at 180° indicating the point C (-1,0),
 $\cos \pi = \underline{\hspace{2cm}}$ and $\sin \pi = \underline{\hspace{2cm}}$.
4. When radius line of circular plate is at 270° indicating the point D (0, - 1)
 which means $\cos \frac{3\pi}{2} = \underline{\hspace{2cm}}$ and $\sin \frac{3\pi}{2} = \underline{\hspace{2cm}}$
5. When radius line of circular plate is at 360° indicating the point again at A (1,0), $\cos 2\pi = \underline{\hspace{2cm}}$ and $\sin 2\pi = \underline{\hspace{2cm}}$.

Now fill in the table :

Trigonometric function	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$	4π
$\sin \theta$	—	—	—	—	—	—	—	—	—
$\cos \theta$	—	—	—	—	—	—	—	—	—

APPLICATION

This activity can be used to determine the values of other trigonometric functions for angles being multiple of $\frac{\pi}{2}$ and π .

Activity 10

OBJECTIVE

To plot the graphs of $\sin x$, $\sin 2x$, $2\sin x$ and $\sin \frac{x}{2}$, using same coordinate axes.

MATERIAL REQUIRED

Plyboard, squared paper, adhesive, ruler, coloured pens, eraser.

METHOD OF CONSTRUCTION

1. Take a plywood of size 30 cm $\times$ 30 cm.
2. On the plywood, paste a thick graph paper of size 25 cm $\times$ 25 cm.
3. Draw two mutually perpendicular lines on the squared paper, and take them as coordinate axes.
4. Graduate the two axes as shown in the Fig. 10.
5. Prepare the table of ordered pairs for $\sin x$, $\sin 2x$, $2\sin x$ and $\sin \frac{x}{2}$ for different values of x shown in the table below:

T. ratios	0°	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{9\pi}{12}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$\sin x$	0	0.26	0.50	0.71	0.86	0.97	1.00	0.97	0.86	0.71	0.50	0.26	0
$\sin 2x$	0	0.50	0.86	1.00	0.86	0.50	0	-0.5	-0.86	-1.0	-0.86	-0.50	0
$2 \sin x$	0	0.52	1.00	1.42	1.72	1.94	2.00	1.94	1.72	1.42	1.00	0.52	0
$\sin \frac{x}{2}$	0	0.13	0.26	0.38	0.50	0.61	0.71	0.79	0.86	0.92	0.97	0.99	1.00

DEMONSTRATION

- Plot the ordered pair $(x, \sin x)$, $(x, \sin 2x)$, $(x, \sin \frac{x}{2})$ and $(x, 2\sin x)$ on the same axes of coordinates, and join the plotted ordered pairs by free hand curves in different colours as shown in the Fig.10.

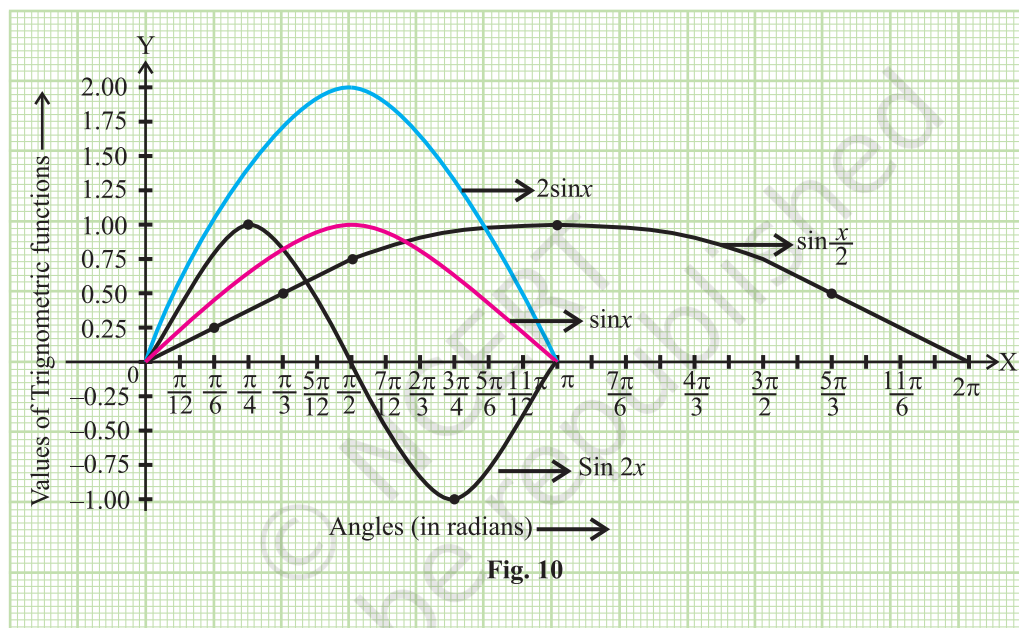


Fig. 10

OBSERVATION

- Graphs of $\sin x$ and $2 \sin x$ are of same shape but the maximum height of the graph of $\sin x$ is _____ the maximum height of the graph of _____.
- The maximum height of the graph of $\sin 2x$ is _____. It is at $x =$ _____.
- The maximum height of the graph of $2 \sin x$ is _____. It is at $x =$ _____.

4. The maximum height of the graph of $\sin \frac{x}{2}$ is _____. It is at

$$\frac{x}{2} = \text{_____}.$$

5. At $x = \text{_____}$, $\sin x = 0$, at $x = \text{_____}$, $\sin 2x = 0$ and at $x = \text{_____}$, $\sin \frac{x}{2} = 0$.

6. In the interval $[0, \pi]$, graphs of $\sin x$, $2 \sin x$ and $\sin \frac{x}{2}$ are _____ x - axes and some portion of the graph of $\sin 2x$ lies _____ x -axes.

7. Graphs of $\sin x$ and $\sin 2x$ intersect at $x = \text{_____}$ in the interval $(0, \pi)$

8. Graphs of $\sin x$ and $\sin \frac{x}{2}$ intersect at $x = \text{_____}$ in the interval $(0, \pi)$.

APPLICATION

This activity may be used in comparing graphs of a trigonometric function of multiples and submultiples of angles.