

Activity 18

OBJECTIVE

To demonstrate that the Arithmetic mean of two different positive numbers is always greater than the Geometric mean.

MATERIAL REQUIRED

Coloured chart paper, ruler, scale, sketch pens, cutter.

METHOD OF CONSTRUCTION

1. From chart paper, cut off four rectangular pieces of dimension $a \times b$ ($a > b$).
2. Arrange the four rectangular pieces as shown in figure. 18.

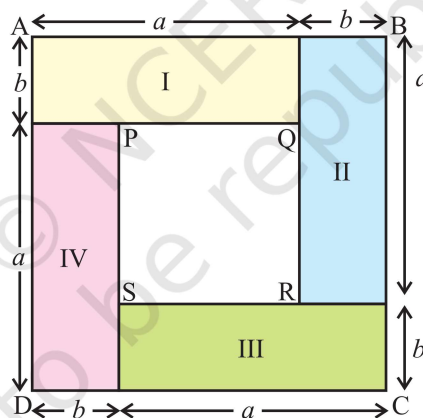


Fig. 18

DEMONSTRATION

1. ABCD is a square of side $(a + b)$ units.
2. Area ABCD = $(a + b)^2$ sq. units.
3. Area of four rectangular pieces = $4(ab) = 4ab$ sq. units.

4. PQRS is a square of side $(a - b)$ units.

5. Area ABCD = Sum of the areas of four rectangular pieces + area of square PQRS.

$\therefore$ Area ABCD > sum of the areas of four rectangular pieces

i.e., $(a + b)^2 > 4ab$

$$\text{or } \left(\frac{a+b}{2} \right)^2 > ab$$

$$\therefore \frac{a+b}{2} > \sqrt{ab}, \text{ i.e., A.M.} > \text{G.M.}$$

OBSERVATION

Take $a = 5\text{cm}$, $b = 3\text{cm}$

$\therefore AB = a + b = \underline{\hspace{2cm}}$ units.

Area of ABCD = $(a + b)^2 = \underline{\hspace{2cm}}$ sq. units.

Area of each rectangle = $ab = \underline{\hspace{2cm}}$ sq. units.

Area of square PQRS = $(a - b)^2 = \underline{\hspace{2cm}}$ sq. units.

Area ABCD = 4 (area of rectangular piece) + Area of square PQRS

$$\underline{\hspace{2cm}} = 4 (\underline{\hspace{2cm}}) + (\underline{\hspace{2cm}})$$

$$\therefore \underline{\hspace{2cm}} > 4 (\underline{\hspace{2cm}})$$

$$\text{i.e. } (a + b)^2 > 4ab \quad \text{or } \left(\frac{a+b}{2} \right)^2 > ab$$

$$\text{or } \frac{a+b}{2} > \sqrt{ab} \quad \therefore \text{AM} > \text{GM}$$

Activity 19

OBJECTIVE

To establish the formula for the sum of the cubes of the first n natural numbers.

MATERIAL REQUIRED

Thermocol sheet, thermocol balls, pins, pencil, ruler, adhesive, chart paper, cutter.

METHOD OF CONSTRUCTION

1. Take (or cut) a square sheet of thermocol of a convenient size and paste a chart paper on it.
2. Draw horizontal and vertical lines on the pasted chart paper to form 225 small squares as shown in Fig. 19.

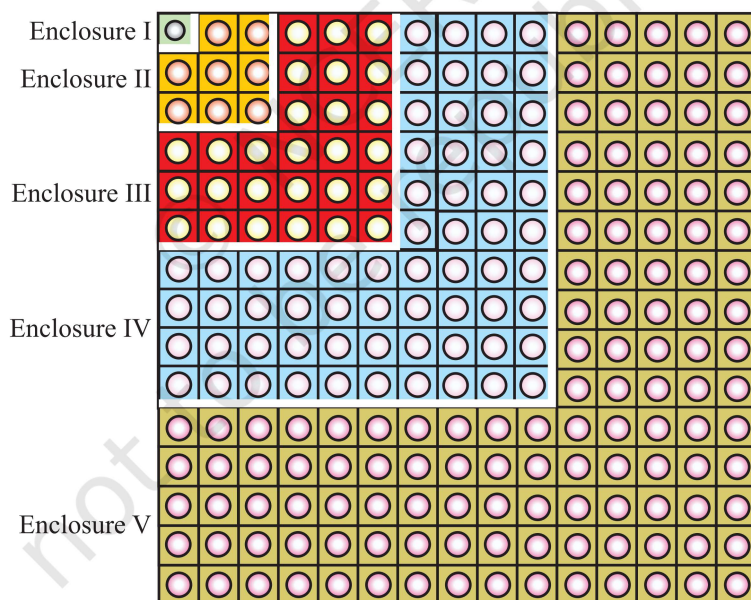


Fig. 19

3. Fix a thermocol ball with the help of a pin at the square on the upper left most corner.

4. Fix 2^3 , i.e., 8, thermocol balls with the help of 8 pins on the same square sheet in 8 squares adjacent to the previous square as shown in the figure.
5. Fix 3^3 , i.e., 27 thermocol balls with the help of 27 pins on the same square sheet in 27 squares adjacent to the previous 8 squares.
6. Continue fixing the thermocol balls in this way till all the squares are filled (see. Fig. 19).

DEMONSTRATION

1. Number of balls in Enclosure I $= 1^3 = 1 = \left(\frac{1 \times 2}{2}\right)^2$.
2. Number of balls in Enclosure II $= 1^3 + 2^3 = 9 = \left(\frac{2 \times 3}{2}\right)^2$.
3. Number of balls in Enclosure III $= 1^3 + 2^3 + 3^3 = 36 = \left(\frac{3 \times 4}{2}\right)^2$.
4. Number of balls in Enclosure IV $= 1^3 + 2^3 + 3^3 + 4^3 = 100 = \left(\frac{4 \times 5}{2}\right)^2$.
5. Total number of balls in Enclosure V $= 1^3 + 2^3 + 3^3 + 4^3 + 5^3$
 $= 225 = \left(\frac{5 \times 6}{2}\right)^2$.

OBSERVATION

By actual counting of balls

1. Number of balls in Enclosure I $= 1^3 = \underline{\hspace{2cm}} = \left(\frac{1 \times 2}{2}\right)^2$.

2. Number of balls in Enclosure II = $1^3 + 2^3 = \underline{\hspace{2cm}} = \left(\frac{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}{\underline{\hspace{1cm}}} \right)^2$.

3. Number of balls in Enclosure III

$$= 1^3 + 2^3 + \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \left(\frac{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}{2} \right)^2.$$

4. Number of balls in Enclosure IV

$$= 1^3 + 2^3 + (\underline{\hspace{1cm}})^3 + (\underline{\hspace{1cm}})^3 = \underline{\hspace{2cm}} = \left(\frac{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}{2} \right)^2.$$

5. Number of balls in Enclosure V

$$= (\underline{\hspace{1cm}})^3 + (\underline{\hspace{1cm}})^3 + (\underline{\hspace{1cm}})^3 + (\underline{\hspace{1cm}})^3 + (\underline{\hspace{1cm}})^3 = \underline{\hspace{2cm}} = \left(\frac{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}{2} \right)^2.$$

APPLICATION

This result can be used in finding the sum of cubes of first n natural numbers, i.e.,

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2.$$