

Activity 28

OBJECTIVE

To find analytically $\lim_{x \rightarrow c} f(x) = \frac{x^2 - c^2}{x - c}$

MATERIAL REQUIRED

Pencil, white paper, calculator.

METHOD OF CONSTRUCTION

1. Consider the function f given by $f(x) = \frac{x^2 - 9}{x - 3}$
2. In this case $c = 3$ and the function is not defined at $x = 3$.

DEMONSTRATION

1. Take some values of c less than $c = 3$ and some other values of c more than $c = 3$.
2. In both cases, the values to be taken have to be very close to $c = 3$.
3. Calculate the corresponding values of f at each of the values of c taken close to $c = 3$.

DEMONSTRATION : TABLE 1

1. Write the values of $f(x)$ in the following tables:

Table 1

x	2.9	2.99	2.999	2.9999	2.99999	2.999999
$f(x)$	5.9	5.99	5.999	5.9999	5.99999	5.999999

Table 2

x	3.1	3.01	3.001	3.0001	3.00001	3.000001
$f(x)$	6.1	6.01	6.001	6.0001	6.00001	6.000001

OBSERVATION

1. Values of $f(x)$ as $x \rightarrow 3$ from the left, as in Table 1 are coming closer and closer to _____.
2. Values of $f(x)$ as $x \rightarrow 3$ from the right, as in Table 2 are coming closer and closer to _____ from tables (2) and (3), $\lim_{x \rightarrow 3} f(x) = \frac{x^2 - 9}{x - 3} = \underline{\hspace{2cm}}$.

APPLICATION

This activity can be used to demonstrate the concept of a limit $\lim_{x \rightarrow c} f(x)$ when $f(x)$ is not defined at $x = c$.

Activity 29

OBJECTIVE

Verification of the geometrical significance of derivative.

MATERIAL REQUIRED

Graph sheets, adhesive, hardboard, trigonometric tables, geometry box, wires.

METHOD OF CONSTRUCTION

1. Paste three graph sheets on a hardboard and draw two mutually perpendicular lines representing x -axis and y -axis on each of them.
2. Sketch the graph of the curve (circle) $x^2 + y^2 = 25$ on one sheet.
3. On the other two sheets sketch the graphs of $(x-3)^2 + y^2 = 25$ and the curve $xy = 4$ (rectangular hyperbola).

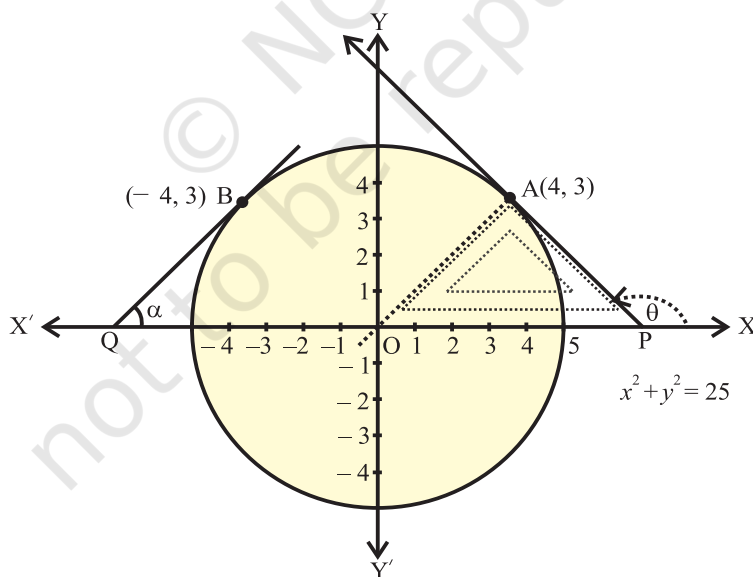


Fig. 29.1

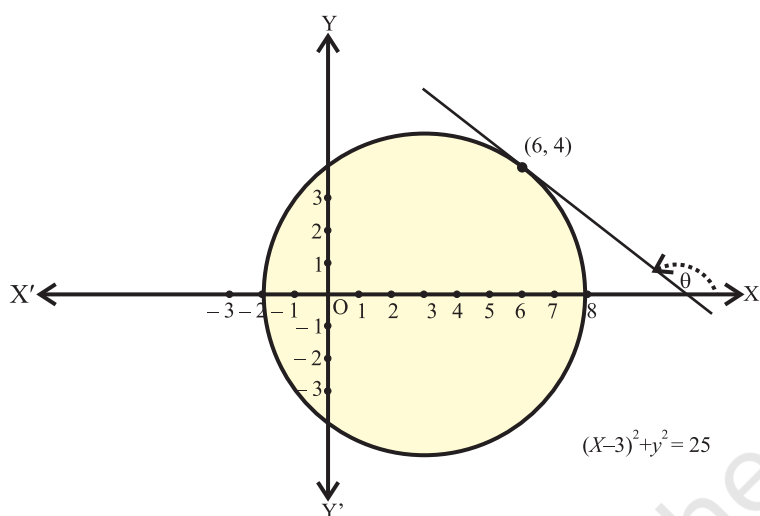


Fig 29.2

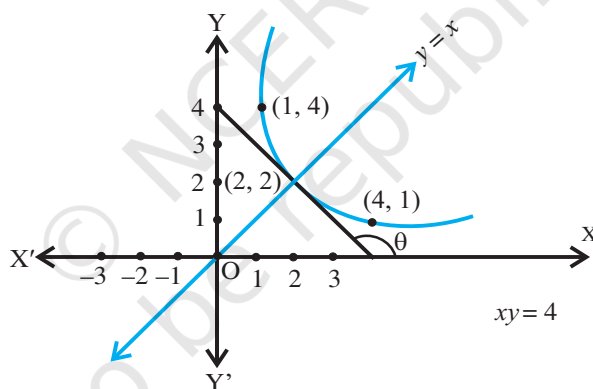


Fig. 29.3

DEMONSTRATION

1. Take first sheet on which, the graph of the circle $x^2 + y^2 = 25$ has been drawn (see Fig.29.1).
2. Take a point A (4, 3) on the circle.
3. With the help of a set square, place a wire in the direction OA and other perpendicular to OA at the point A to meet x-axis at a point (say P).

4. Measure the angle between the wire and the positive direction of x -axis at P (say θ).
5. Then find $\tan \theta$ (with the help of trigonometric tables)

$$\text{Now, } x^2 + y^2 = 25 \Rightarrow y = \sqrt{25 - x^2} \Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{25 - x^2}}.$$

Find $\frac{dy}{dx}$ at the point (4, 3) and verify that $\left(\frac{dy}{dx}\right)$ at (4, 3) = $\tan \theta$.

6. Similarly, take another point (−4, 3) on the circle. Verify that $\frac{dy}{dx}$ at (−4, 3) = $\tan \alpha$ where α is the angle made by the tangent to the circle at the point (−4, 3) with the positive direction of x -axis. (see Fig. 29.1).
7. Take other sheet with the graph of $(x - 3)^2 + y^2 = 25$ and take the point (6, 4) on it and repeat the above process using set square and wires as shown in Fig. 29.2, i.e. verify that $\frac{dy}{dx}$ at (6, 4) = $\tan \theta$.
8. Now take the third sheet, showing the graph of the curve $xy = 4$. Take the point (2, 2) on it. Place one perpendicular side of set square along the line $y = x$ and a wire along the other side touching the curve at the point (2, 2) and find the angle made by the wire with the positive direction of x -axis as shown in Fig. 29.3. Let it be θ . Verify that $\frac{dy}{dx}$ at (2, 2) = $\tan \theta$.

OBSERVATION

1. For the curve $x^2 + y^2 = 25$, $\frac{dy}{dx}$ at the point (3, 4) = _____. Value of θ = _____
 $\tan \theta$ = _____ $\frac{dy}{dx}$ at (3, 4) = _____.

2. For the curve $x^2 + y^2 = 25$, $\frac{dy}{dx}$ at $(-4, 3) = \underline{\hspace{2cm}}$, $\tan \alpha = \underline{\hspace{2cm}}$

, $\frac{dy}{dx}$ at $(-4, 3) = \underline{\hspace{2cm}}$.

3. For the curve $(x - 3)^2 + y^2 = 25$, $\frac{dy}{dx}$ at $(6, 4) = \underline{\hspace{2cm}}$, value of $\theta = \underline{\hspace{2cm}}$,

$\tan \theta = \underline{\hspace{2cm}}$, $\frac{dy}{dx}$ at $(6, 4) = \underline{\hspace{2cm}}$.

4. For the curve $xy = 4$, $\left(\frac{dy}{dx}\right)$ at $(2, 2) = \underline{\hspace{2cm}}$,

$\theta = \underline{\hspace{2cm}}$, $\tan \theta = \underline{\hspace{2cm}}$.

NOTE

The activity may be repeated by taking point $(4, 3)$ on first sheet, $(0, 4)$ on second sheet and $(1, 4)$ on the third sheet.

APPLICATION

Same activity can be used to verify the result that the slope of the tangent at a point is equal to the value of the derivative at that point for other curves.