

CCT Weekly Practice (Mathematics)

Fascinating Triangular numbers



Numbers such as 1, 3, 6, 10... which can be arranged in a compact triangular pattern are termed as triangular numbers.

Mr. John, applied this concept of triangular numbers to arrange students in the school ground for PT drills. He asked 3 students to stand in a triangular number arrangement keeping one foot distance between two consecutive students. He named this group as G1.



Next, he selected 6 students and gave them direction to arrange themselves in triangular number form keeping same one foot distance between each other. This group was named as G2. Continued in the same manner, the students formed many groups.

Question 1

If perimeters of the groups so formed are denoted by P1, P2, P3 etc. then find values of P3, P5 and P7.

- a) 3ft, 5ft and 7 ft
- b) 10ft, 21ft and 34 ft
- c) 9ft, 25ft and 49ft
- d) 9ft, 15ft and 21ft.

Question 2

If A1, A2, A3... represent areas of G1, G2, G3 respectively then find the value of $A(n) - A(n-1)$.

- a) $\frac{\sqrt{3}}{4} (2n - 1)$
- b) $\frac{\sqrt{3}}{4} (2n + 1)$
- c) $\frac{\sqrt{3}}{4} (2n^2 - 2n + 1)$
- d) $\frac{\sqrt{3}}{4} (n^2 - 2n + 1)$

Question 3

Each of the students of G4 group shook hands with everyone else standing in that group exactly once, how many handshakes were there altogether?

- a) 15
- b) 30
- c) 105
- d) 210

Question 4

Rohan and Naval are best friends. Rohan is standing at one vertex of G 14 group whereas Naval is standing in the middle of the last row. Mr. John asked Rohan to hand over the baton to Naval and come back at his own place in minimal time. What minimum distance (in nearest whole number) should Rohan cover to do this?

- a) 32 ft
- b) 12 ft
- c) 24 ft
- d) 26 ft

Question 5

On the occasion yoga day students performed different asanas for their mental and physical well-being. For this purpose they stood in triangular number arrangement starting with a group of 3, 6, 10...students keeping 2 feet gap between two consecutive candidates.



Areas of new triangles are denoted as N_1, N_2, N_3 etc. Few of these values are being equal to some of A_1, A_2, A_3 ...What is the relation between N and A if there are “ k ” number of new triangular arrangements?

- a) $N(k) = A(2k)$
- b) $N(k) = 2A(k)$
- c) $N(k) = A(k^2)$
- d) $N(k) = A(k^3)$

Prepared by:

Ms. Kakali Bagachi, Hariyana Vidya Mandir, Kolkata.

Answer Key

1. (d) 9ft, 15ft, 21 ft

Arrangement	No of students in each side	Length of each side(ft)	Perimeter (ft)
G1	2	1	$P1 = 3 \times 1 = 3$
G2	3	2	$P2 = 3 \times 2 = 6$
G3	4	3	$P3 = 3 \times 3 = 9$
G4	5	4	$P4 = 3 \times 4 = 12$
G5	6	5	$P5 = 3 \times 5 = 15$
G6	7	6	$P6 = 3 \times 6 = 18$
G7	8	7	$P7 = 3 \times 7 = 21$

2. (a) $\frac{\sqrt{3}}{4} (2n - 1)$

G (n) has “n+1” no of students in each side, each pair having one foot gap in between. So no of line segments joining “n+1” no of collinear points is n. Hence length of each side is n ft. So, A(n) will be $\frac{\sqrt{3}n^2}{4}$. Similarly, A (n-1) will be $\frac{\sqrt{3}(n-1)^2}{4}$.

Therefore, $A(n) - A(n-1) = \frac{\sqrt{3}n^2}{4} - \frac{\sqrt{3}(n-1)^2}{4} = \frac{\sqrt{3}}{4} (2n - 1)$

3. (c) 105 number of handshakes = $\frac{n(n-1)}{2}$ and here n= 15 as in group 4 the total number of students are 15.
4. (c) 24 ft

In G14 there are 15 students in the last row. Rohan will have to handover the baton to Naval who is the 8th student in that row and will have to come back at his own position. That means Rohan will move median wise. He will cover minimum distance. Altitude is the minimum distance from vertex (Rohan) to the opposite side of a triangle. As, G14 is an equilateral triangle, so, median and altitude will coincide. Hence, minimum distance to be covered by Rohan is equal to 2 x length of altitude i.e. $2 \times \frac{\sqrt{3}}{2} \times 14 \text{ ft} = 14\sqrt{3} \text{ ft} = 24\text{ft}$

5. (a) $N(k) = A(2k)$

Here side of each triangular arrangement will be doubled. Length of sides will be 2ft, 4ft, 6ft etc. So, $N1 = \frac{\sqrt{3}(2)^2}{4} = A2$

$$N2 = \frac{\sqrt{3}(4)^2}{4} = A4$$

$$N3 = \frac{\sqrt{3}(6)^2}{4} = A6 \quad \text{Following this pattern we get, } N(k) = A(2k)$$

Prepared By:-

Ms. Kakali Bagachi, Hariyana Vidya Mandir, Kolkata.