



CHAPTER – 6

Motion

In our daily life we see many objects moving and many at rest. Birds fly, fish swims, car moves, a boy rides a bicycle, dogs and cows move, blood flows through the veins and arteries. Planets, stars, galaxies, atoms and molecules are all in motion. We often see books on table, toys in a box, trees at rest. For an object to be in motion, we notice it, when its position changes with time. A crawling ant is continually changing its position like a man who is walking. Look at the book placed on your study table. The distance of the book from the floor, from walls or from the ceiling of the room remains unchanged since the book is at rest with respect to the room.

However there are situations where the motion is inferred through indirect evidences — for example we use to infer the motion of air by observing the movement of dust and of leaves and branches of trees. Besides, we perceive the phenomena of sunrise, sunset, changing of seasons etc. — what causes them? Is it due to the motion of the earth? If it is true, why don't we perceive the motion of the earth directly?

An object may appear to be moving for one person and stationary for some person. Example; when you are on a moving school bus, the roadside trees appear to be moving backwards. A person standing on the roadside perceives the bus along with the passengers as moving while the roadside trees are at rest. However, you perceive your friends in the moving bus and driver of the bus to be at rest. What do these observations indicate? (Motion is relative)

Out of the motions of object you perceive, some objects may move in straight line, others may move along a circular path, some may rotate about an axis and a few others may vibrate. There may also be situations involving a combination of these.

Activity 6.1

Discuss whether the walls of your reading or class room are at rest or in motion.

Activity 6.2

Have you ever experienced that the bus in which you are sitting appears to move while it is actually at rest on a highway? Discuss and share your experience with your friends sitting in the bus.

Think And Act

Some motions of objects around us are dangerous specially when it is erratic and uncontrolled as observed in a flooded river, a storm or sliding rocks from mountains. On the other hand, controlled motions can be a service to human beings such as in the generation of electric power in hydro electric projects and in nuclear power projects. You list some erratic motion of objects and ways to control them.

In this chapter, we shall first learn to describe the motion of objects along a straight line and then to express such motions through simple equations and graphs. Later, we shall also discuss the ways of describing circular motion of objects.

6.1 Describing Motion

In general, we describe the location or position of an object by specifying a reference point. Let us try to understand by some examples. Let us assume that your school is 3 km north of your house. We have specified the position of the school with respect to the house. In this example the house is the reference point. In another example, when you are asked by the teacher to come to the blackboard in your class, you will move from your seat and walk up to the board. In describing the position of the board your seat is the reference point. We could have also chosen other reference points according to our convenience. Thus, to describe the position of an object we need to specify a reference point called the origin. So you have chosen a convenient point i.e. your house, your seat. The position of the school is described by stating

- (i) its distance from the chosen point (your house)
- (ii) in which direction (north), as seen from the chosen point.

6.1.1 Motion Along A Straight Line

The motion of a body along a straight line is the simplest type. Let us take up the following example. Consider the motion of an object moving along a straight path, starting from 'O' which is chosen as its reference point (Fig.6.1). At first the object reaches at Z through X and Y. Then it moves back along the same path and reaches X through Y. The total length of the path covered by the object for the journey is

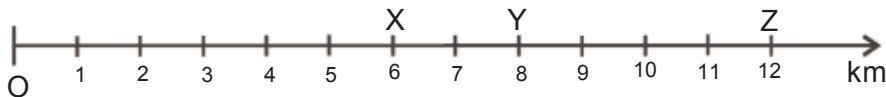


Fig. 6.1: Positions of an object at different instants on a straight path

OZ + ZX i.e. $12 \text{ km} + 6 \text{ km} = 18 \text{ km}$. This is the distance covered by the object for the journey. Thus to find the distance covered we need to specify the numerical value only but not the direction of motion. Like distance there are also other quantities which are described by specifying numerical values only. Now, can you find out the distance of the final position of the object in motion from the initial starting point 'O'? This will give the numerical value of displacement of the object from O to X through Z. In fact this is the shortest distance measured from the initial reference point to the final position of the object, which is known as the displacement.

Considering the example given in (Fig.6.1), can you say that the magnitude of the displacement be equal to the distance travelled by an object? For the motion from O to Z and back to Y, the distance covered = $12 \text{ km} + 4 \text{ km} = 16 \text{ km}$. While the magnitude of the displacement is 8 km. Thus the magnitude of displacement i.e. 8 km does not equal to the path length 16 km covered by the object. In the above example, if the object comes back to the starting point O, the final position of the object coincides with the initial starting position then the displacement is zero. However, the distance covered for the journey is $OZ + ZO = 12 \text{ km} + 12 \text{ km} = 24 \text{ km}$.

Thus the distance and displacement are two physical quantities which are used to describe the overall motion of an object and to locate its final position with reference to its initial position at different instants of time. Hence, the magnitude of displacement from O to Z is 12 km which is also the distance covered by the object.

Activity 6.3

Take a long rope and a metre scale with you walk from one corner to the opposite corner of a rectangular field of sides 15 m and 25m and along the sides of the rectangular field.

Now, measure the magnitude of the displacement from the starting corner to the final corner you arrived at. What difference would you notice between the actual distance covered and magnitude of displacement.

Activity 6.4

In automobiles a device is fitted that shows the distance covered in travelling from one place to another place. That device is called odometer or speedometer. A scooter or car is driven from Imphal to Thoubal. The difference between final reading and initial reading of the odometer is 25 km.

Find out the magnitude of the displacement between Imphal and Thoubal by using the Road Map of Manipur.

Try to answer the Questions

1. In what condition is the distance covered equal to the magnitude of the displacement of an object in motion?
2. Can the distance travelled by an object be smaller than the magnitude of the displacement of the motion?
3. An object covers a distance. Can it have zero displacement? If yes, support your answer with an example.
4. State true or false: (1) Displacement cannot be zero. (2) Magnitude of displacement may be equal to the distance covered (3) Magnitude of displacement is greater than the distance covered.

6.1.2 Uniform and Non-Uniform Motion

Imagine an object moving along a straight line. Let it cover 3m in the first min, 3m in the second min, 3m in the third min, and 3m in the fourth min and so on. For a fast moving object, let it cover 2 m in the first second, 2m in the next second, 2m in the third second, 2m in the fourth second etc. In each of the cases the object covers equal distances in equal intervals of time. As the object covers equal distances in equal and consecutive intervals of time, it is said to be in *uniform motion*. The time interval in such a motion should be small.

On the other hand, in our daily life or day to day life, we come across motions where objects cover unequal distances in equal intervals of time. If you had a car ride sitting next to the driver or during your journey to school by bus, the car or the bus slows down in crowded area and moves fast in uncrowded area. Thus in the above examples the car or

the bus covers unequal distances in equal intervals of time. These are the examples of non-uniform motion.

Activity6.5

Table 6.1 shows the data regarding the motion of two objects or persons A and B. Examine them and try to identify uniform or non-uniform motion.

Table 6.1

Time	Distance covered by	
	A in m	B in m
5.00 am	10	11
5.15 am	20	18
5.30 am	30	24
5.45 am	40	37
6.00 am	50	43

6.2 Speed And Velocity

You know that in a game of cricket there is a signboard showing the bowling speeds of a player as 125 km/hour for example. What do you understand from the signboard?

Different objects in motion may take different amounts of time to cover a given distance. Some of them may move fast while some move slowly. The rate at which objects move can be different or can be same. The method to know the rate of motion of an object is to find out the distance covered by it in unit time. This quantity is referred to as speed. The SI-unit of speed is metre per second (ms^{-1} or m/s). There are other units like centimetre per second (cms^{-1}) or kilometer per hour (km h^{-1}). To specify the speed we need only its magnitude. In most cases the speed of a moving object may not be constant i.e. non-uniform motion. Therefore, we describe the rate of motion of such objects in terms of average speed which is obtained by dividing the total distance covered by the total time taken to cover the distance i.e.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

If an object covers a distance x in time t then its speed $v = \frac{x}{t}$ (6.1).

Let us try to understand the calculation of speed of an object in motion from the following example. A car covers a distance of 70km in 2h. Its average speed for this journey is $\frac{70 \text{ km}}{2 \text{ h}} = 35 \text{ km/h}$. The car might not have travelled at 35km/h for all the time. Sometimes it might have travelled slower and sometimes faster than 35 km/h.

Example 6.1 :

A dog runs 12 m in the first 2 s and 18 m in the next 4 s. What is its average speed?

Solution:

Total distance travelled by the dog = 12m + 18m = 30m

Total time taken = 2s + 4s = 6s

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{30\text{m}}{6\text{s}} = 5\text{ms}^{-1}$$

Therefore the average speed of the dog for the motion is 5m/s or 5 ms⁻¹.

6.2.1 Speed with Direction (Velocity)

The rate of motion of any object can be made more meaningful if we specify its direction of motion along with its speed. Thus we have a new quantity called velocity which tells not only how fast the object is moving but also the definite direction in which it is moving. The velocity of an object can be uniform or variable. It can be changed by changing the speed of the object or direction of motion or both. For the motion of an object along a straight line at variable speed the magnitude of its rate of motion can be expressed in terms of average velocity which can be calculated in the same way as average speed in example 6.1. i.e. displacement divided by time taken.

In some motions the velocity of the object may change at a uniform rate. If so, the average velocity is given by the arithmetic mean of initial velocity and final velocity at the beginning and at the end for a given period of time.

$$\text{Thus, average velocity} = \frac{\text{Initial velocity} + \text{Final velocity}}{2}$$

$$\text{Mathematically, } v_{av} = \frac{u + v}{2} \dots\dots\dots(6.2)$$

Both speed and velocity have the same unit as ms⁻¹ or m/s. Here, 'u' is the initial velocity and 'v' is the final velocity.

Activity6.6. (Estimation of distance):

Measure the time for going from your home to school. If your average speed is 1km/hr, estimate the distance.

Activity 6.7.

Observe the lightning and thundering during the rainy season. The sound of thunder takes some time to perceive after you see the lightning. Try to answer why this happens. Measure this time gap between lightning and hearing of thunder using a stop watch or a digital watch. Estimate the distance of the point of lightning, using the speed of sound in air = 332 m/s.

Example 6.2.

At the start of a trip the speedometer of a car reads 8000km and 8225km at the end. Which takes 5 hours. Calculate the average speed of the car in kmh⁻¹ and also in ms⁻¹.

Solution:

Distance covered by the car = 8225 – 8000km = 225km

Time taken, $t = 5 \text{ h}$

$$\therefore \text{Average speed of the car, } v_{\text{av}} = \frac{s}{t} = \frac{225 \text{ km}}{5 \text{ h}} = 45 \text{ kmh}^{-1}$$

$$\text{Also, } v_{\text{av}} = \frac{15 \cancel{45} \times 1000 \text{ m}}{12 \cancel{3600} \text{ s}} = \frac{5 \cancel{15} \times 10^5}{12 \cancel{4} \cancel{2}} = 12.5 \text{ ms}^{-1}$$

The average speed of the car is 45 kmh^{-1} or 12.5 ms^{-1} .

Example 6.3.

A person goes along a straight road of length 50m and covers 100m by moving from one end to the other end and back along the same road. Find the average speed and average velocity of the person, if he takes 20 minute.

Solution:

Total distance = $50 + 50 = 100 \text{ m}$.

Total time taken = 20 min

$$\begin{aligned} \text{Average speed} &= \frac{\text{Total distance covered}}{\text{Total time taken}} \\ &= \frac{100 \text{ m}}{20 \text{ min}} = \frac{100 \text{ m}}{20} \times \frac{1}{60 \text{ s}} = \frac{1}{12} \text{ ms}^{-1} \end{aligned}$$

Again, total displacement in 20 min = zero m

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Total time taken}} = \frac{0 \text{ m}}{20 \text{ min}} = 0 \text{ ms}^{-1}$$

Thus, his average speed is $\frac{1}{12} \text{ ms}^{-1}$ and his average velocity = 0 ms^{-1} .

6.3 Rate of Change of Velocity

In uniform motion of an object along a straight line, velocity remains constant with time, i.e. the change in velocity of the object for any time interval is zero.

However, when the motion is non-uniform, the velocity of the object varies with time. As a result the velocity has different values at different instants of time and at different points of the straight path. Thus the change in velocity of the object during the motion is not zero, for any time interval. Is it now possible to express the change in velocity of the object discussed above?

In order to answer the above question, we have to introduce another physical quantity called *acceleration*, which is a measure of the change in velocity of an object in motion per unit time. Thus, acceleration of an object is rate of change of velocity. That is acceleration

$$= \frac{\text{change in velocity}}{\text{time taken}}$$

Suppose an object moves along a straight line in a fixed direction. If the velocity of the object increases from an initial value 'u' to the final value 'v' in time 't' then the average acceleration 'a' of the object is,

$$a = \frac{v - u}{t} \dots\dots\dots(6.3)$$

This kind of motion is known as accelerated motion and acceleration is taken to be positive. When the speed of the object decreases i.e. $u > v$, the object is said to be decelerating. Thus, acceleration is taken to be positive if it is in the direction of velocity and negative when it is opposite to the direction of velocity. The SI unit of acceleration or deceleration is metre per second per second (ms^{-2}).

Acceleration may be uniform or non-uniform. When the velocity of an object travelling along a straight line increases or decreases by equal amounts in equal intervals of time, then the acceleration of the object is said to be uniform. The motion of a freely falling object is an example of uniformly accelerated motion. On the other hand, if the velocity of an object changes in a non-uniform rate as in the case of a car travelling along a straight road increases or decreases its speed by unequal amounts in equal intervals of time, depending on the conditions of the road at different positions, then the acceleration of the car is said to be non-uniform.

Activity6.8.

In your day to day life you come across a range of motions in which

- (i) acceleration is in the direction of motion.
- (ii) acceleration is against the direction of motion.
- (iii) acceleration is uniform.
- (iv) acceleration is non-uniform. Try to identify each of the above types of motion with one example for each.

Example 6.4.

Starting from home Mr. Tomba paddles his bicycle to attain a velocity of 10 ms^{-1} in 40s after the start. Then he applies brakes so that the velocity reduces to 6ms^{-1} in the next 10s. Calculate the acceleration of the bicycle in both the cases.

Solution:

In the first case:

Initial velocity $u = 0$; final velocity, $v = 10 \text{ ms}^{-1}$; time taken = 40s

From the relation given in equation 10.3., we get

$$a = \frac{v - u}{t}$$

Substituting the given values of u , v and t in the above equation, we get

$$\begin{aligned} a &= \frac{10\text{ms}^{-1} - 0\text{ms}^{-1}}{40\text{s}} \\ &= \frac{1}{4} \text{ms}^{-2} = 0.25 \text{ ms}^{-2} \end{aligned}$$

In the second case

Initial velocity, $u = 10 \text{ ms}^{-1}$; final velocity, $v = 6\text{ms}^{-1}$; time $t = 10 \text{ s}$.

$$\begin{aligned} \text{Then } a &= \frac{(6\text{ms}^{-1} - 10\text{ms}^{-1})}{10\text{s}} = \frac{-4\text{ms}^{-1}}{10\text{s}} \\ &= -0.4 \text{ ms}^{-2} \end{aligned}$$

Hence, the acceleration of the bicycle in the first case is 0.25 ms^{-2} and in the second case, it is -0.4 ms^{-2} .

Try to answer:

1. When is a motion said to be in (i) uniform acceleration? (ii) non-uniform acceleration?
2. A particle is moving with a uniform velocity. What is its acceleration?

6.4 Graphical Representation of Motion

Graph is a very powerful method of presenting information about a variety of events. In newspapers and magazines, we find a number of graphs to present different information. For example, in the telecast of a fifty over one-day cricket match, the score at the end of each over is shown in the form of line graph which can give an overall idea of batting progress of the team. Thus a straight line graph helps in solving a linear equation having two variables.

To study the motion of an object, we can use graphs showing the dependence of distance or velocity on time.

6.4.1 Distance — Time Graphs

The change in position of an object with time can be shown on the distance — time graph adopting a convenient scale of choice. In plotting the graph, time is taken along the x-axis and distance from the reference point is taken along the y-axis. Such graphs can be employed under various situations where objects remain at rest, move with uniform speed, non-uniform speed etc.

We know that the object is at a constant distance from the origin, from the graph, since the line remains parallel to the time-axis, we can infer that the object is at rest.

We know that if an object travels equal distances in equal intervals of time then it moves with a uniform speed. Thus for a motion with uniform speed, a graph of distance travelled against time is a straight line since the distance covered by the object is directly proportional to time, as shown in Fig.6.3. The line OB shows that the distance covered is increasing with time at a uniform rate. If the motion is along a straight path, you can use the term uniform velocity in place of uniform speed, since for motion along a straight path in a direction the distance travelled is exactly equal to the magnitude of the displacement i.e. along Y-axis.

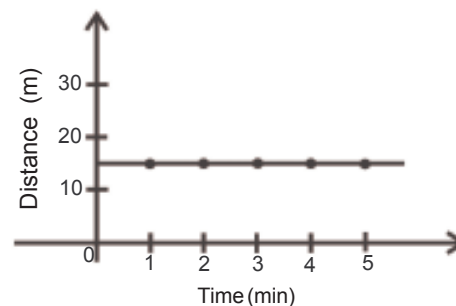


Fig. 6.2. Distance-time graph of an object at rest

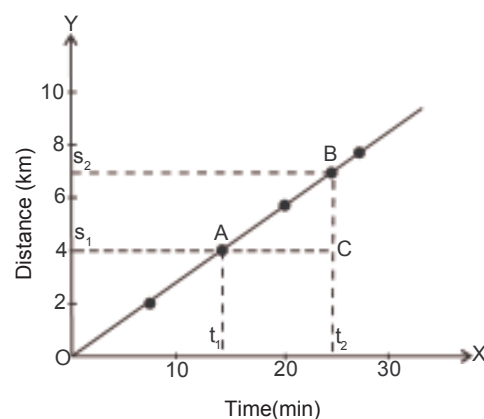


Fig. 6.3. Distance – time graph of an object moving with uniform speed

From the graph we can determine the speed of the object. To obtain the speed, let us consider a small portion AB of the graph in Fig.6.3 and draw a straight line from A parallel to X-axis and from B parallel to Y-axis. These two lines will meet at C. Now AC will denote the time interval ($t_2 - t_1$) taken by the object to cover the distance BC corresponding to the distance ($s_2 - s_1$). The speed 'v' of the object can be expressed as

$$v = \frac{s_2 - s_1}{t_2 - t_1} \dots\dots\dots 6.4.$$

Hence, the nature of the graph in Fig. 6.3 shows a linear variation of the distance covered by the object with time, in uniform motion.

We can also plot the distance – time graph for a uniformly accelerated motion of an object. The following Table 6.2 shows the distance covered by a car in a time interval of two seconds.

Table 6.2: Distance travelled by a car at regular time intervals

Time in second	Distance in metre
0	0
2	2
4	8
6	18
8	32

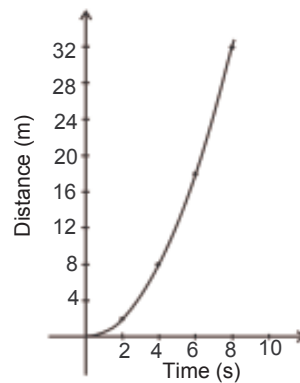


Fig. 6.4: Distance – time graph for motion given in table 6.1

Note that the shapes of the graphs given in Fig.6.3 and Fig.6.4 are different. The nature of the graph in Fig.6.4 shows the non-linear variation between Distance and Time for non-uniform motion.

6.4.2 Velocity — Time graph

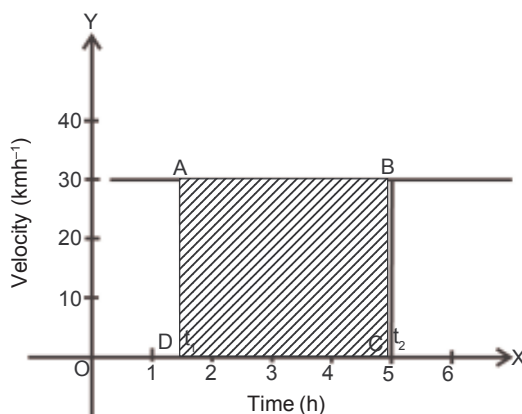


Fig. 6.5: Velocity – time graph for uniform motion of a motor car

The nature of variation of velocity with time for an object moving in a straight line can be represented by a velocity – time graph. In such a graph, time is represented along the X-axis and velocity is represented along the Y-axis. If an object moves with a uniform velocity, its velocity at any instant is the same as shown in (Fig.6.5). It will be a straight line parallel to the X-axis. Fig.6.5 shows the velocity time graph for a motor car moving with a uniform velocity of 30 kmh^{-1} .

We know that the product of velocity and time of any object moving with uniform velocity gives the displacement. Thus the area enclosed by the velocity time graph and time axis, representing the interval of time will be equal to the magnitude of the displacement. To know the distance covered between time t_1 and t_2 from the graph, draw perpendiculars from the points marked A and B corresponding to the time t_1 and t_2 on the graph. Thus the height AD or BC represents the velocity of 30 km/h and length AB represents the time $(t_2 - t_1)$.

Thus the distance 's' moved by the car during $(t_2 - t_1)$ can be expressed as

$$\begin{aligned} s &= AD \times AB = (30 \text{ kmh}^{-1}) \times (t_2 - t_1) \text{ h.} \\ &= 30 \times (t_2 - t_1) \text{ km} \\ &= \text{Area of the rectangle ABCD, which is shaded in Fig.6.5.} \end{aligned}$$

Similar to the above, we can study about the motion of an object with non-uniform velocity but uniformly accelerated motion by plotting its velocity – time graph. Let us consider a person sitting by the side of the driver of a car, who is recording its velocity for every 10 seconds from the speedometer, while the car is accelerated. The velocity of the car at different instants of time is given in table 6.3.

Table 6.3: Velocity of a car at regular intervals of Time during acceleration

Time (s)	Velocity kmh ⁻¹	Velocity (m/s)
0	0	0
10	18	5
20	36	10
30	54	15
40	72	20

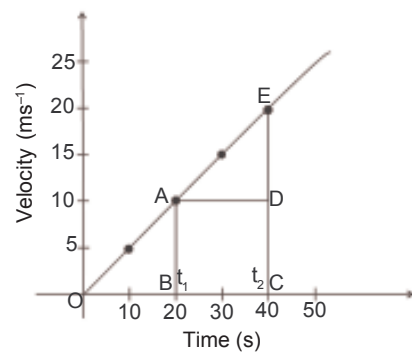


Fig. 6.6: Velocity-time graph for a car, uniformly accelerated

In Fig.6.6, the variation of the velocity with time for a uniformly accelerated motion of a car is shown. The graph shows that velocity changes by equal amounts during equal intervals of time and graph for such motion will always be a straight line.

From the graph we can also determine the distance covered by the car in a given interval of time. If the car was moving with uniform velocity, the distance travelled by it during an interval of time, in Fig.6.5, would be given by the area of the rectangle ABCD. Similarly, for the motion with uniform acceleration, the area ABCDE under the velocity – time graph (Fig.6.6) will give the distance covered by the car during the time interval $(t_2 - t_1)$. That is

$$\begin{aligned} s &= \text{Area ABCDE} \\ &= \text{Area of the rectangle ABCD} + \text{Area of the triangle ADE.} \\ &= AB \times BC + \frac{1}{2} AD \times DE \end{aligned}$$

We have discussed the velocity – time graph in the case of uniformly accelerated

motion in the above. In the case of non-uniformly accelerated motion, the velocity – time graph can have any shape. The velocity – time graphs of an object (i) whose velocity is decreasing uniformly with time (2) whose velocity varies non-uniformly with time are shown in Fig.6.7 and Fig.6.8 respectively. Try to interpret the motions from the graphs.

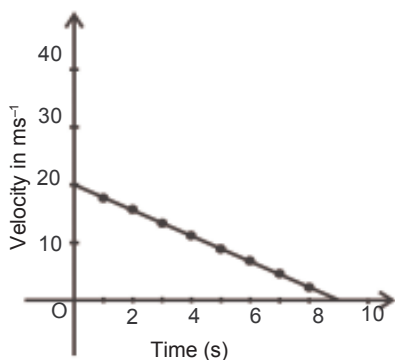


Fig. 6.7: Velocity – time graph of an object, velocity uniformly decreasing with time

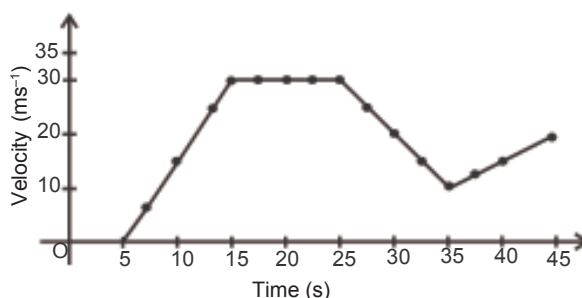


Fig. 6.8: Velocity – time graph of an object with velocity non-uniformly changing with time

Activity 6.8.

The times for arrival and departure of a bus at three stoppages A, B, C along with distances of B and C from A are shown in table 6.4. Plot and interpret the distance – time graph for the bus, assuming that it moves with uniform velocity between any two stations.

Table 6.4: Distances of stoppages B and C from A and times of arrival and departure of the bus

Stoppage	Distance from A (km)	Time of arrival (h)	Time of departure (h)
A	0	6.00	06.30
B	60	08.30	09.00
C	90	10.00	10.30

Activity — 6.9.

Tomba and his sister Mema go to school on their bicycles from their house on the same route. But Tomba could reach the school earlier than his sister. The table 6.5 shows the distance covered by each in different times. Plot the distance – time graph for their motions using the same scale and find the velocity of each.

Table 6.5: Distance covered by Tomba and Mema at different times

Time	Distance covered by Tomba (km)	Distance covered by Mema (km)
9.00 am	0	0
9.10 am	2.0	1.5
9.20 am.	2.8	2.2
9.30 am.	3.5	3.0
9.40 am	4.0	3.6
9.50	—	4.0

Try to answer

1. What can you understand about the motion of an object, if the distance – time graph is a straight line parallel to time axis?
2. What will be the nature of distance – time graphs for uniform and non-uniform motion of an object?
3. Which quantity is measured by the area occupied below the velocity – time graph of an object in motion?

6.5 Equations of Motion from the Graphical Representation

For an object moving along a straight line with uniform acceleration, it is possible to relate its velocity acquired, distance covered by it in a certain interval of time by a set of equations which are known as the equations of motion. They are:

$$v = u + at \dots\dots\dots(6.5)$$

$$s = ut + \frac{1}{2} at^2 \dots\dots\dots(6.6)$$

Squaring equation (6.5) and combining with equation (6.6), the acceleration during the motion is given by

$$2as = v^2 - u^2 \dots\dots\dots(6.7)$$

In the above equations, u is the initial velocity of the object which moves with uniform acceleration ' a ' for a time t , v is the final velocity and ' s ' is the distance covered at the end of time interval ' t '. The above three relations can be obtained by graphical method (Fig.6.9).

6.5.1 Equation For Velocity — Time Relation of a Motion

Let us consider the velocity–time graph of an object moving under uniform acceleration from an initial velocity u . Thus, you can see that velocity at $t = 0$ is u , represented by the point A on the graph. Thereafter it increases to final velocity v at a uniform rate ' a ', shown as E on the same graph. Perpendicular lines DC and DE are drawn from the point D on time and velocity axes, respectively, so that the initial velocity of the motion is represented by OA and final velocity is represented by DC and the time interval t is represented by OC, finally the change in velocity during t interval of time is represented by $DB = DC - BC$, since AB is drawn parallel to OC.

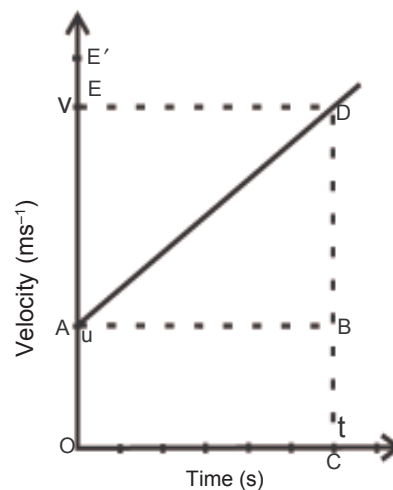


Fig. 6.9: Velocity – time graph to obtain the equations of Motion (with uniform accⁿ)

From the graph we observe that $DC = DB + BC = BD + OA$

Substituting $DC = v$; $OA = u$ in the above relation, we get

$$v = BD + u$$

$$\text{Or, } BD = v - u \dots\dots\dots(6.8)$$

From the graph in Fig.10.9, the acceleration of the object is given by

$$a = \frac{\text{change in velocity}}{\text{time taken}} = \frac{BD}{AB} = \frac{BD}{OC}$$

Substituting $OC = t$ in the above relation, we get ' a ' = $\frac{BD}{t}$

$$\text{Or, } BD = at \dots\dots\dots(6.9)$$

Using equations (6.8) and (6.9) we get

$$v - u = at$$

$$\text{Or, } \boxed{v = u + at}$$

6.5.2 Equation for position — time relation

Let us assume that during the time ' t ', the object has covered a distance ' s ' under uniform acceleration ' a '. From the graph, this distance is obtained by the area enclosed within OADC, under the velocity – time graph AD.

Thus, the distance ' s ' travelled by the object is given by

$$\begin{aligned} s &= \text{area OADC (which is a trapezium)} \\ &= \text{area of rectangle OABC} + \text{area of triangle ADB.} \\ &= OA \times OC + \frac{1}{2} AB \times BD \dots\dots\dots(6.10) \end{aligned}$$

Substituting $OA = u$; $OC = t = AB$ and $BD = at$, we get

$$s = u \times t + \frac{1}{2} (t \times at) = ut + \frac{1}{2} at^2$$

$$\text{Or, } s = \boxed{ut + \frac{1}{2} at^2}$$

6.5.3 Equation for position – velocity relation for the motion

From the above, $s = \text{area of the trapezium OADC}$

$$= \frac{(OA + DC) \times OC}{2}$$

Substituting $OA = u$; $DC = v$ and $OC = t$, we get

$$s = \frac{(u + v) \times t}{2} \dots\dots\dots(6.11)$$

But, from the velocity-time relation (Eq.6.5) we get

$$t = \frac{(v - u)}{a} \dots\dots\dots(6.12)$$

Substituting ' t ' from (Eq.10.12) in (Eq.6.11) we get

$$s = \frac{(u + v)(v - u)}{2a} = \frac{v^2 - u^2}{2a}$$

$$\text{Or, } 2as = v^2 - u^2$$

$$\text{i.e. } v^2 = u^2 + 2as.$$

Example 6.5.

A car starting from rest accelerates uniformly so as to attain velocity of 144 kmh^{-1} in 20 s. Find (a) its acceleration (b) the distance covered by it for attaining that velocity.

Solution:

We have been given the following quantities:

$$u = 0; v = 144 \text{ kmh}^{-1} = \frac{144 \times 10^3}{60 \times 60} \text{ ms}^{-1} = 40 \text{ m/s and time taken } t = 20 \text{ s}$$

$$(a) \text{ From Eq.(6.5), we know that } a = \frac{v - u}{t} = \frac{40 \text{ ms}^{-1} - 0 \text{ ms}^{-1}}{20 \text{ s}} = 2 \text{ ms}^{-2}$$

$$(b) \text{ From Eq.(6.7), we have } 2as = v^2 - u^2$$

$$\text{Thus, } 2 \cdot 2 \cdot s = 40^2 - 0^2$$

$$\Rightarrow 4s = 40 \times 40$$

$$\therefore s = \frac{40 \times 40}{4} = 400 \text{ m}$$

The acceleration of the car is 2 ms^{-2} and distance covered by it is 400m.

Example 6.6.

A car accelerates uniformly from 36 kmh^{-1} to 144 kmh^{-1} in 20s. Calculate (a) the acceleration of the car (b) distance covered by the car in the interval

Solution:

We have been given the following quantities.

$$u = 36 \text{ kmh}^{-1} = \frac{36 \times 1000}{60 \times 60} = 10 \text{ ms}^{-1}$$

$$v = 144 \text{ kmh}^{-1} = \frac{144 \times 1000}{60 \times 60} = 40 \text{ ms}^{-1}$$

And $t = 20 \text{ s}$.

(a) From the relation, $v = u + at$, where 'a' is the acceleration

$$\text{We get } 40 = 10 + a \times 20$$

$$\therefore a = \frac{(40 \text{ ms}^{-1} - 10 \text{ ms}^{-1})}{20 \text{ s}} = \frac{30 \text{ ms}^{-1}}{20 \text{ s}} = 1.5 \text{ m/s}^2 \text{ or } 1.5 \text{ ms}^{-2}$$

(b) From the relation $s = ut + \frac{1}{2} at^2$ we have

$$s = 10 \text{ ms}^{-1} \times 20 \text{ s} + \frac{1}{2} \cdot 1.5 \text{ ms}^{-2} \times (20 \text{ s})^2$$

$$= 200 \text{ m} + \frac{1}{2} \times 1.5 \times 20 \times 20 \text{ m}$$

$$= 200 \text{ m} + 300 \text{ m} = 500 \text{ m}.$$

The acceleration of the car is 1.5 ms^{-2} and distance covered by it is 500m.

Example 6.7

A police van moving along a straight highway is brought to a stop by the application of brakes producing an acceleration of 7ms^{-2} in the opposite direction to the motion. If the van takes 5s to stop after application of brakes, calculate (i) the velocity of the van when the brakes are applied (ii) distance covered before it stops.

Solution:

We have been given, $a = -7\text{ms}^{-2}$; $t = 5\text{s}$ and $v = 0\text{ms}^{-1}$

(i) From equation (6.5) we know that $v = u + at$

$$0 = u + (-7\text{ms}^{-2}) \times 5\text{s}$$

$$\text{or, } u = 7 \times 5 \text{ ms}^{-1} = 35\text{ms}^{-1}$$

(ii) From Equation (6.6), we have $s = ut + \frac{1}{2} at^2$

$$\text{or, } s = 35\text{ms}^{-1} \times 5\text{s} + \frac{1}{2} (-7\text{ms}^{-2}) \cdot (5\text{s})^2$$

$$= 35 \times 5 \text{ m} - \frac{1}{2} \times 7 \times 5\text{m}$$

$$= 175\text{m} - 17.5\text{m}$$

$$= 157.5\text{m}.$$

Thus, the velocity of the van when the brakes are applied is 35ms^{-1} and will move 157.5m before it stops after the application of brakes.

Example 6.8

The driver of a speedy car applied brakes which can produce an acceleration of 10ms^{-2} in a direction opposite to that of the motion. Calculate the distance it travels during 3s which is the time taken by the car before coming to a halt or rest.

Solution:

We have been given, $a = -10\text{ms}^{-2}$; $t = 3\text{s}$ and $v = 0\text{ms}^{-1}$

From Equation $v = u + at$ we get

$$0 = u + (-10\text{ms}^{-2}) \times 3\text{s}$$

$$\text{or, } u = 30\text{ms}^{-1}$$

Again, from equation $s = ut + \frac{1}{2} at^2$ we get

$$s = (30\text{ms}^{-1}) \times 3\text{s} + \frac{1}{2} (-10\text{ms}^{-2}) (3\text{s})^2$$

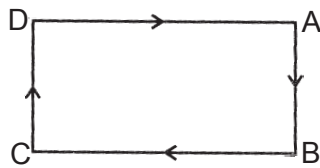
$$= 90\text{m} - \frac{1}{2} \times 10 \times 9\text{m}$$

$$= 90\text{m} - 45\text{m} = 45\text{m}.$$

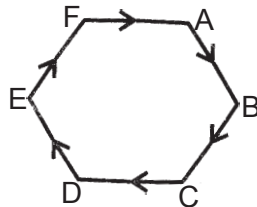
Thus the car will cover a distance of 45m before it stops, after the application of brakes.

6.6 Uniform Circular Motion

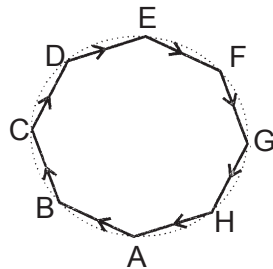
In the above lessons, we say that whenever the velocity of an object changes, the object is accelerating. The change in velocity may be due to change in its magnitude or the direction of the motion or both. Can you imagine about an example when an object does not change its magnitude of velocity but only its direction of motion? You can learn such a motion from the following.



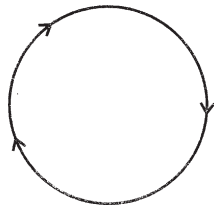
(a) Rectangular Track



(b) Hexagonal track



(c) Octagonal shaped track



(d) A circular track

Fig. 6.10 (a),(b), (c), (d). The motion of an athlete along closed paths or tracks of different shapes

Let us discuss an example of the motion of a body along a closed path. Fig.6.10 (a) shows the path of an athlete racing along a rectangular track ABCDA at a uniform speed on the straight parts. In order to keep himself on track, he will change the direction at each of the corners. How many times will he have to change his direction of motion in one complete round? He has to change his direction of motion four times.

Let us again consider that the athlete is running along a hexagonal track ABCDEFA as shown in Fig.6.10 (b). In this situation, in order to keep himself on the track, he will have to change his direction of motion six times in one complete round.

Let us think, what will happen if the track were a regular octagon with eight equal sides as shown by ABCDEFGHA in Fig.6.10(c)? It is observed that as the number of sides of the track increases, the athlete has to change his direction of motion more and more often. What would happen to the shape of the track as we go on increasing the number of sides indefinitely? If it is so, you will notice that the shape of the track becomes a circle and the length of each of the sides can be imagined as a point. If the athlete runs with a velocity of constant magnitude on the circular track, the only change taking place in his velocity is due to the change in the direction of motion. Such motion of an athlete along a circular track or path at constant speed is exactly an example of an accelerated motion.

If the athlete takes 't' second to go once around the circular track of radius r (say), the magnitude of his velocity 'v' is given by $v = \frac{2\pi r}{t}$ (6.13).

The circumference of the circle of radius r is $2\pi r$ and v is actually the speed of the athlete.

Activity6.10

Take a small stone tied at one end of a piece of string. Rotate the stone on a horizontal circular path by holding the thread at the other end, as shown in Fig.6.11. Now, let the stone go free by releasing the thread. Can you tell the direction in which the stone would move after it has been released.

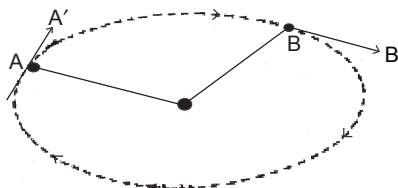


Fig. 6.11: A small stone or any suitable object describing a horizontal circular path with a velocity of constant magnitude

By repeating the above activity and releasing the stone at different points of the circular track, check whether the direction in which the stone moves remains the same or not. If you carefully observe, you will see that when the stone is released at A it will move along AA' and when released at B it will move along BB'. Here AA' and BB' are the tangents to the circular path at A and B. Thus, on being released the stone moves along a straight line tangential to the circular path. From the above you know that when a body moves in a circular path the direction of motion is not the same at any two points. Thus, the motion in a circle with constant speed is an example of accelerated motion. There are many more familiar phenomena of objects moving under uniform circular motion. They are the motion of the moon and the earth, a satellite in a circular orbit around the earth, a cyclist on a circular track or valodrom at constant speed and so on.

Points to remember

- (i) Motion is a change in position and can be described in terms of distance covered or the displacement.
- (ii) Motion may be uniform or non-uniform depending on whether its velocity remains constant or changing.
- (iii) Speed of motion of an object is distance covered per unit time while velocity is the displacement per unit time.
- (iv) The acceleration of an object is the change in velocity per unit time.
- (v) The motions of objects can be studied through graphs.
- (vi) The motion of an object moving with uniform acceleration can be expressed in equations namely

$$v = u + at ; s = ut + \frac{1}{2} at^2 ; 2as = v^2 - u^2$$

where 'u' is the initial velocity of the object which is moving with uniform acceleration 'a' for time 't', 'v' is the final velocity acquired after time 't' or after covering a distance 's'.

- (vii) Uniform motion along a straight line is not accelerated but uniform circular motion is accelerated.

EXERCISES

1. A boy while driving to his school covers at the average speed of 15 km/h. On his return trip along the same route, covers at the average speed of 25 km/h due to less traffic. What is his average speed for the whole trip? (18.75 km/h)
2. A motor boat in Loktak lake starting from rest, accelerates along a straight path at a uniform rate of 2.0 ms^{-2} for 10.0s. How far does the boat travel during this time? (100m)
3. The driver of a car travelling at 72 kmh^{-1} applies the brakes and accelerates uniformly in the opposite direction at the rate of 5 ms^{-2} . How long the car will take to come to rest? How far will the car cover before coming to rest. (4 s ; 40m)
4. The figure below shows the distance – time graph of three motor-cyclists A, B and C. On the basis of this graph answer the following questions:

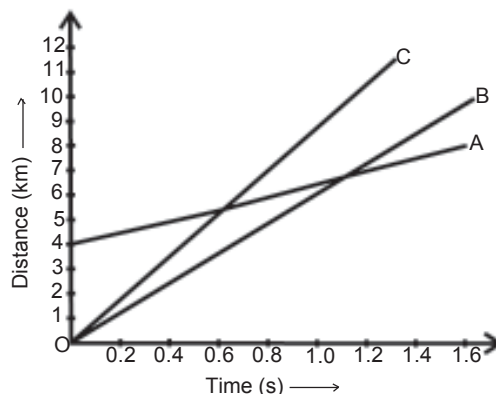


Fig. 6.12

- (i) Which of them is travelling fastest.
 - (ii) Are all the three ever at the same point on the road?
 - (iii) When B passes A where is C?
 - (iv) How far did C travel when B meets A.
5. What type of motion is represented by each of the following graphs?

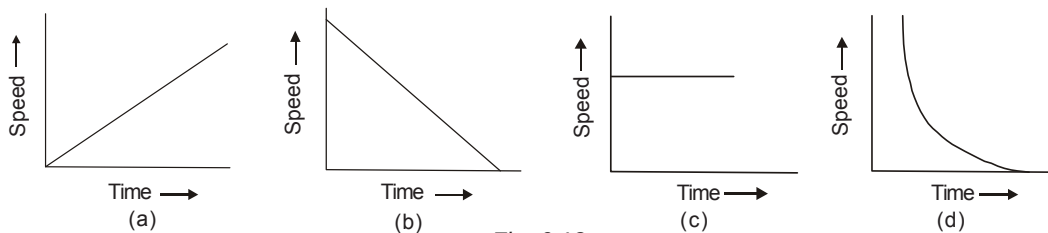


Fig. 6.13