

CHAPTER – 8



Gravitation

In the previous chapters 6 and 7, you have learnt about motion and force as the cause of motion of bodies. Besides, you know that a force is always needed to change the speed or direction of motion of a body.

In our daily life we have seen that an object dropped from a height above the earth's surface always falls towards the earth. It is said that a falling apple sparked the idea in the mind of Newton that the earth attracts all the objects towards its centre. Newton generalized this idea and stated that the same force is responsible for the motion of the planets around the sun, of the moon around the earth. This force is called the gravitational force.

In this chapter we shall learn about gravitation and universal law of gravitation and shall discuss the motion of bodies under the influence of gravitational force on the earth. We shall also discuss how the weight of a body varies from place to place.

8.1 Gravitation

We know that any object thrown upwards reaches a certain height after which it falls downward. After observing the fall of an apple on Newton, he thought that if the earth can attract an apple, can it not attract the moon which goes around the earth? Is the force the same in both cases? Newton guessed that the same type of force must be responsible in both cases. He argued that in the case of moon's motion round the earth, the moon maintains a constant distance from the centre of the earth, instead of going off in a straight line along the tangent on the circular orbit. This must be due to the attraction by the earth. But we do not really observe the moon falling towards the earth.

Let us try to understand such motion of the moon through the following activity.

Activity 8.1

Take a piece of thread and tie a small pebble at one end. Holding the other end of the thread try to whirl the pebble in a horizontal circular path as shown in Fig.8.1. Observe and note the motion of the pebble. Now, release the thread when the pebble is at any point in its orbit. Again note the direction of motion of the pebble.

In the activity you must have observed that the pebble moves in a circular path with a certain speed and changes its direction of motion continuously at every point of the path. The change in direction involves a change in velocity which is called acceleration. This acceleration is towards the centre of the orbit and caused by a force which keeps the body moving along the orbit. This force must be acting towards the centre and is called the **centripetal** (meaning 'centre-seeking') **force**.

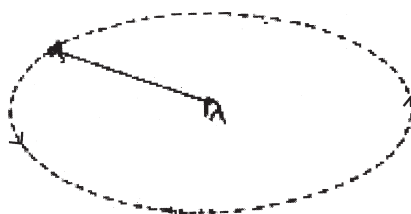


Fig.8.1: A pebble describing a circular orbit/path with a velocity of constant magnitude

In the absence of this force the pebble will fly off along a straight line due to its inertia. Such straight line will be a tangent to the circular path. That is why, whenever you release the thread, the pebble flies off along the tangent to the circular path.

Similarly for the motion of the moon around the earth, centripetal force must be necessary and this centripetal force is provided by the force of attraction between the earth and the moon. Had there been no such force, the moon would not have been seen as at present rather it would pursue a uniform straight line motion.

It is seen that a falling body or apple is attracted towards the earth. We may ask — does the apple also attract the earth? If so, we do not observe the earth moving towards any falling body or falling apple. Why? According to Newton's third law of motion, the apple also does attract the earth, with an equal force. But according to the second law of motion, acceleration is inversely proportional to the mass of the body, for a given force, [Eq. 7.3]. Since, the mass of the earth is too large compared to that of the apple, the acceleration of earth towards the apple is negligible. So, we do not see the earth moving towards the apple. The same reasoning can be given for, why the earth does not move towards the moon.

We have learnt that in our solar system all the planets go round the Sun. On the same reasoning as above we can say that there exists a similar force between the sun and the planets. From the above facts Newton concluded that all objects in this universe attract each other irrespective of the form of the material of the objects. This force of attraction between objects is called the universal gravitational force.

8.1.1 Universal Law of Gravitation

Every object in this universe attracts every other object with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. The force acts along the line joining the centres of the two attracting objects.

Let two objects A and B having masses M and m lying at a distance ' d ' between their centres be considered as shown in Fig.8.2. Let the force of attraction between them be ' F '. According to the above law, the force F is directly proportional to the product of their masses. That is

$$F \propto M \times m \dots\dots\dots(8.1)$$

And the force ' F ' is inversely proportional to the square of the distance between their centres. That is

$$F \propto \frac{1}{d^2} \dots\dots\dots(8.2)$$

Combining Eqs (8.1) and (8.2), we get

$$F \propto \frac{M \times m}{d^2} \dots\dots\dots(8.3)$$

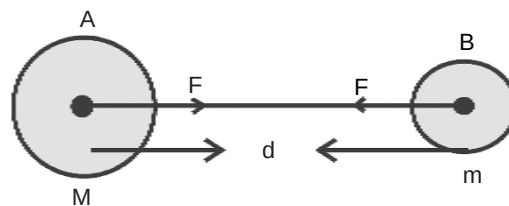


Fig. 8.2: The gravitational force between two objects

$$\text{or, } F = \frac{GM \times m}{d^2} \dots\dots\dots(12.4)$$

Where G is called universal gravitation constant or universal constant of gravitation.

$$\text{From Eq (12.4), } G = \dots\dots\dots(12.5)$$

The SI unit of G is $\text{Nm}^2\text{kg}^{-2}$. The accepted value for G is $6.673 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$, which was determined by Henry Cavendish (1731 – 1810) by using an extra sensitive balance designed for the purpose.

Compute the value of force of attraction between you and the earth when you are on the surface of it. The mass of the earth is about $6 \times 10^{24} \text{ kg}$ and distance of separation between you and centre of the earth, the average radius of the earth, which is about $6400 \times 10^3 \text{ m}$. If you take your mass as 40 kg, the force of attraction will be

$$F =$$

$$= 0.0390996 \times 10^4 = 391 \text{ N.}$$

Compute the value of the force of attraction between you and your friend sitting closely. Conclude how you do not experience this force.

$$\frac{GMm}{Md^2} = \frac{6.673 \times 10^{-11} \times 6 \times 10^{24} \times 40}{(64 \times 10^5)^2}$$

The law of gravitation is universal because it is applicable to all bodies irrespective of material, shape, size and whether they are celestial or terrestrial.

Example 8.1

The mass of the earth is $6 \times 10^{24} \text{ kg}$ and that of moon is $7.4 \times 10^{22} \text{ kg}$. If the distance between the earth and the moon is $3.84 \times 10^5 \text{ km}$, find the force exerted by the earth on the moon and that exerted by the moon on the earth. Take $G = 6.7 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$.

Solution:

The mass of the earth, $M = 6 \times 10^{24} \text{ kg}$

The mass of the moon, $m = 7.4 \times 10^{22} \text{ kg}$

The distance between the earth and the moon,

$$d = 3.84 \times 10^5 \times 1000 \text{ m}$$

$$= 3.84 \times 10^8 \text{ m}$$

$$\text{And } G = 6.7 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

From Eq. (8.4), the force exerted by the earth on the moon

$$F = \frac{6.7 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} \times 6 \times 10^{24} \text{ kg} \times 7.4 \times 10^{22} \text{ kg}}{(3.84 \times 10^8 \text{ m})^2}$$

$$= 2.017 \times 10^{20} \text{ N.}$$

And the force exerted by the moon on the earth is also $2.017 \times 10^{20} \text{ N}$ i.e. of the

same magnitude, since such force of gravitation is mutual.

8.1.2 Importance of the Universal law of gravitation

This law is an important one since this successfully explained several phenomena which were believed not to be related such as (i) force that binds us to the earth (ii) the motion of the moon round the earth (iii) the motion of planets including our earth round the sun (iv) the tides in the ocean .

8.2 Free Fall And Acceleration due to gravity

The meaning of free fall can be understood if you perform the following activity.

Activity8.2

Take a small stone and throw it vertically upwards. It will reach a certain height and then will start falling down due to the gravitational force. The downward journey of the stone under the gravitational force alone is called "Free Fall i.e. the object is in free fall".

While falling there is no change in the direction of motion of the objects. But due to the gravitational force of the earth acting on the freely falling objects, there will be a change in the magnitude of the velocity. A change in velocity involves acceleration. Thus whenever objects fall towards the earth an acceleration is involved. This acceleration is called acceleration due to gravity and is denoted by 'g'. The unit of 'g' is the same as that of acceleration, that is (ms^{-2}).

Let 'm' be the mass of the stone used in the activity (8.2). The gravitational force acting on the stone is given by the product of 'm' and acceleration due to gravity 'g', from Newton's second law of motion. Thus the gravitational force F, acting on the falling stone is

$$F = mg \text{(8.6)}$$

From Eq. (8.4) and (8.6), we get

$$\begin{aligned} mg &= F = \frac{GM \times m}{d^2} \\ \Rightarrow g &= \frac{GM}{d^2} \text{(8.7)} \end{aligned}$$

Here, M is the mass of the earth; 'd' is the distance of the stone from the centre of the earth.

Let the stone be on or close to the surface of the earth. The distance 'd' in Eq. (8.7) will be equal to R, the radius of the earth

$$\text{Thus, } mg = \frac{GMm}{R^2} \text{(8.8)}$$

$$\text{or, } g = \frac{GM}{R^2} \text{(8.9)}$$

We know that the earth is not a perfect sphere. As the radius of the earth increases from the poles to the equator, the value of 'g' is greater at the poles than at the equator. Thus for most calculations, we generally take 'g' to be more or less constant on or at points close to the earth. However, for objects far away from the earth's surface, the acceleration due to gravity of the earth is calculated using Eq.8.7 and 'd' is taken as the distance of the object from the centre of the earth i.e. $d = (R + h)$ where R is the average

radius of the earth and 'h' is the height of the point where the object situates from the earth's surface.

The value of 'g' at the surface of the earth is calculated from Eq.(8.9), by taking $G = 6.7 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$; the mass of the earth $M = 6 \times 10^{24} \text{ kg}$ and radius of the earth $R = 6.4 \times 10^6 \text{ m}$.

$$\begin{aligned}\text{Thus, } g &= \frac{6.7 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} \times 6 \times 10^{24} \text{ kg}}{(6.4 \times 10^6 \text{ m})^2} \\ &= 9.8 \text{ Nkg}^{-1} = 9.8 \text{ ms}^{-2}\end{aligned}$$

Thus the acceleration due to gravity of the earth at its surface is 9.8 ms^{-2} . With the increase of distance of the object from the surface of the earth or as we go upwards from the surface of the earth, the value of 'g' goes on decreasing.

N.B. All bodies whether light or heavy; hollow or solid will fall with equal rapidity.

Example 8.2

A stone is thrown vertically upwards and rises to a height 19.6 m. Calculate (i) the velocity with which the stone was thrown (ii) the time taken to reach the maximum height i.e. the highest point.

Solution:

Distance covered = 19.6 m

Final velocity 'v', at the point of maximum height is zero ms^{-1} , while its initial velocity = $u \text{ ms}^{-1}$

Acceleration due to gravity, $g = 9.8 \text{ ms}^{-2}$

$\therefore$ Acceleration of the stone in the upward direction,

$a = -9.8 \text{ ms}^{-2}$, since the motion is upwards it is against gravity of the earth, ($a = -g$).

From $v^2 = u^2 + 2as$ we get

$$\begin{aligned}0^2 &= u^2 + 2(-9.8) \text{ ms}^{-2} \times 19.6 \text{ m} \\ -u^2 &= -2 \times 9.8 \times 19.6 \text{ m}^2\text{s}^{-2}\end{aligned}$$

$$\therefore u = \sqrt{2 \times 9.8 \times 2 \times 9.8} = 2 \times 9.8 = 19.6 \text{ ms}^{-1}$$

Again, let 't' s be the time taken to reach the highest point.

From $v = u + at$ we get

$$\begin{aligned}\Rightarrow 0 &= 19.6 \text{ ms}^{-1} - 9.8 \text{ ms}^{-2} \times t \\ \therefore t &= \frac{19.6 \text{ ms}^{-1}}{9.8 \text{ ms}^{-2}} = 2 \text{ s}.\end{aligned}$$

Thus the velocity with which the stone was projected is 19.6 m/s and time taken to reach the highest point is 2 second.

8.3 Mass

In the previous chapters we have learnt that the mass of an object is the measure

of its inertia, being larger for greater mass. Inertia of an object remains the same whether it is on the earth, on the moon or even in outerspace. Thus the mass of an object does not change from place to place, i.e. mass remains constant.

8.4 Weight

It is already known to us that the earth attracts every object with a certain force depending on the mass (m) of the object and the acceleration due to gravity (g) for the earth at the point where the object situates. The force with which the earth attracts the body is called the weight of the body on the earth.

Thus, the weight of a body of mass (m) placed on or near the surface of the earth is

$$W = F = \frac{GMm}{R^2} = mg \dots \dots \dots (8.10)$$

Where M is the mass of the earth and R is its radius.

The SI unit of weight is the same as that of force, that is newton (N), and is acting vertically downwards; it has both magnitude and direction.

The weight of a body of mass 1 kg is $w = mg = 1 \text{ kg} \times (9.8 \text{ ms}^{-2})$
 $= 9.8 \text{ N}.$

This weight is called one kilogram weight written in short as 1 kg wt or 1 kgf.

Since the value of 'g' is constant at a given place, at a given place the weight (w) of an object is directly proportional to its mass (m) i.e. $w \propto m$. It is due to this reason that at a given place if the weights of two bodies of masses m_A and m_B are equal then their masses m_A and m_B should be equal, because the mass of an object remains the same everywhere.

8.4.1 Weight of an object on the Moon

From the concept of the weight of an object on the earth, the weight of an object on the moon is the force with which the moon attracts that object when the object is on or very near to the surface of the moon.

Let the mass of the object be (m). By applying the universal law of gravitation, the weight of the object on the moon will be

$$W_m = \frac{G M_m \times m}{R_m^2} \dots \dots \dots (8.11)$$

Where M_m = mass of the moon; R_m = radius of the moon.

Let us now compare the weights of an object of mass (m) when it is on the earth and on the moon, using known values of mass and radius of the celestial bodies given in table (8.1)

Table 8.1

Celestial bodies	Mass (kg)	Radius (m)
Earth	5.98×10^{24}	6.37×10^6
Moon	7.36×10^{22}	1.74×10^6

Weight of object on the earth,

$$W_e = \frac{G \times 5.98 \times 10^{24} \times m}{(6.37 \times 10^6)^2} \text{ N} \dots\dots\dots(8.12)$$

And weight of the same object on the moon,

$$W_m = \frac{G \times 7.36 \times 10^{22} \times m}{(1.74 \times 10^6)^2} \text{ N} \dots\dots\dots(8.13)$$

Dividing Eq (8.13) by Eq (8.12), we get

$$\begin{aligned} \frac{W_m}{W_e} &= \frac{G \times 7.36 \times 10^{22} \times m}{(1.74 \times 10^6)^2} \times \frac{(6.37 \times 10^6)^2}{G \times 5.98 \times 10^{24} \times m} = \frac{298.646 \times 10^{34}}{18.105 \times 10^{36}} \\ &= 16.495 \times 10^{-2} = 0.16495 \approx \dots\dots\dots(8.14) \end{aligned}$$

Thus the weight of the object on the moon = $\frac{1}{6}$ × its weight on the earth.

From Eq. (8.10), we can learn that the acceleration due to gravity of the moon at its surface is about $\frac{9.8}{6} \text{ ms}^{-2}$ i.e. 1.63 ms^{-2}

Example 8.3

Find the weight of a baby whose mass is 3 kg.

Solution:

The weight of the baby is $W = mg$

Where m = mass of the baby = 3 kg

g = acceleration due to gravity of the earth = 9.8 ms^{-2}

= $3 \text{ kg} \times 9.8 \text{ ms}^{-2}$

= $29.4 \text{ kgms}^{-2} = 29.4 \text{ N}$.

Thus the weight of the baby is 29.4 N.

Example 8.4

The weight of an object on the surface of the earth is 18 N. What would be its weight when measured on the surface of the moon?

Solution:

We know that the weight of an object on the moon is always $\frac{1}{6}$ th of its weight on the earth's surface.

Thus, weight of the object on the moon's surface will be

$W_m = \frac{1}{6} \times W_e$ where W_e = weight of the object on the earth's surface and

$$W_e = 18 \text{ N.}$$

$$\therefore W_m = \quad \times 18 = 3 \text{ N.}$$

8.5 Measurement of Mass

Let us consider two bodies (A and B) having masses m_A and m_B respectively. The way to compare the masses is to compare the gravitational force exerted by the earth on two bodies. Using Eq (8.8), we get the gravitational forces F_A and F_B as

$$F_A = \frac{GM \times m_A}{R^2} \quad \text{And} \quad F_B = \frac{GM \times m_B}{R^2}$$

Where M = Mass of the earth; R is its radius.

$$\text{Thus } \frac{F_A}{F_B} = \frac{m_A}{m_B} \dots\dots\dots(8.15)$$

If the gravitational forces on the two bodies are equal then $m_A = m_B$. Thus, if we can be sure that the earth is attracting the two bodies with equal forces, we can conclude that their masses are equal. A common-balance, also known as equal-beam balance does exactly the job for measuring masses of the bodies. A beam balance is shown in Fig.8.3. Two identical pans are suspended from the ends of a uniform beam which is pivoted at its centre. If the earth attracts the two bodies kept on each of the pans with equal forces then only the beam stays at rest in a horizontal position. In the actual process, the beam is made to remain at rest by placing standard masses on left hand side pan A. These standard masses are the copies of those standard kept at International Bureau of Weights and Measures at Sevres, France.

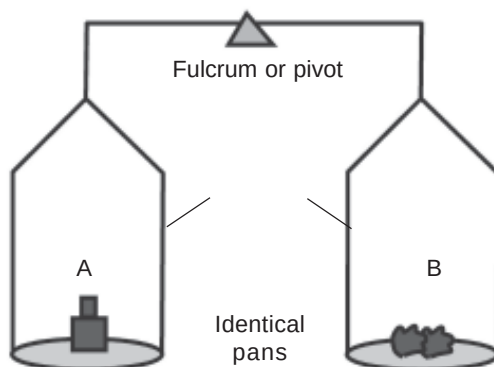


Fig.8.3. Common balance

Once, we have standard masses, we can find the mass of any object by comparing to the standard masses.

8.6 Measurement of weight

We have learnt from the above discussions that mass and weight are two different

quantities. The mass of a body is its inherent property and remains the same everywhere in the universe. But the weight of a body varies with place. A body of mass 1 kg has weight 9.8 N at the surface of the earth and about 1.633 N at the surface of the moon. But the mass 1 kg of the body remains 1 kg in all the situations. It must be noted that mass measured in kilogram whereas weight is measured in newton.

When you stand on a weighing machine, do you get your weight or your mass? The machine measures the apparent weight and converts it to the corresponding mass for earth's surface. For example — if the measured apparent weight is 980 N, the machine will show $980/9.8 = 100$ kg. Generally we say that the weight is 100 kg. If the same weighing machine and the object were taken to the moon, it will show a much smaller reading than on the earth's surface.

Similarly, when we suspend a body from a spring balance, we actually measure the apparent weight. In fact, it is the contact force between the spring and the body that extends the spring and gives the measured weight. The common weighing machine and a simple type of spring balance are shown in Fig.8.4.

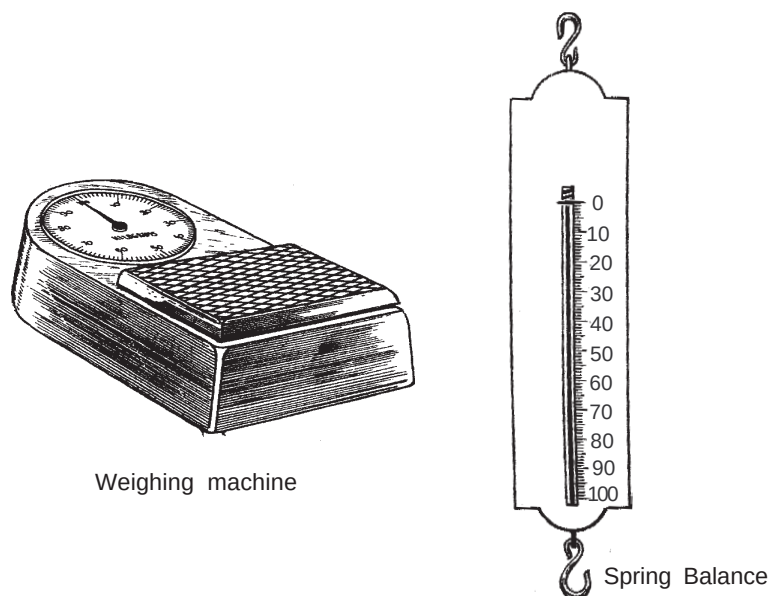


Fig. 8.4: Weighing Machine and simple spring balance

Points to remember

- (i) The gravitational force of attraction between any two bodies or objects is directly proportional to the product of the masses of two bodies and is inversely proportional to the square of the distance between the bodies. The direction of the force is along the line joining their centres and are mutual.
- (ii) Gravitation is a weak force for small masses, unless large masses are involved.

- (iii) The force of gravitation due to the earth is specially called gravity.
- (iv) The force of gravity decreases with the increasing height above the earth's surface and also decreases from the pole towards the equator on the surface.
- (v) The force by which the earth attracts a body is called the weight of the body on the earth. The weight of a body changes with height above the earth's surface but the mass of the body remains the same everywhere.
- (vi) The weight of a body is equal to the product of its mass and acceleration due to gravity at the point where the body is situated.
- (vii) If two bodies have equal weights at a given place then they will have equal weights at any other place.

EXERCISES

1. *In which manner the force of gravitation will change when*
 - (i) the distance between them is made doubled
 - (ii) the distance between them is reduced to half?
2. *We know that the gravitational force acting on all objects is proportional to their masses. Why then a heavier object does not fall faster than a lighter object?*
3. *If each body in the universe attracts the every other body, why don't two books kept on a table come towards each other and collide?*
4. *Distinguish between 'G' and 'g'.*
5. *Starting from Newton's law of gravitation, show that the value of 'g' decreases as one goes above the earth's surface.*
6. *We know that the earth and the moon are attracted to each other by gravitational force. Does the earth attract the moon with a force which is greater or smaller or the same as the force with which the moon attracts the earth? Give reason for your answer.*
7. *From the universal law of gravitation we have learnt that the moon attracts the earth, why does the earth not move towards the moon?*
8. *What will happen to the force between two bodies, if*
 - (i) the mass of one of the them is doubled?
 - (ii) the distance between the bodies is doubled?
 - (iii) the masses of both are doubled?
9. *What do you mean by the weight of a body? State its unit.*
10. *Suggest two methods of comparing masses of two bodies A & B i.e. m_A and m_B .*
11. *When you put an object on a spring balance, do you get the mass of the object or its weight.*

12. A force of 20 N acts on a body whose weight is 9.8 N on the earth. What is the mass of the body and how much is its acceleration? (1 kg; 20 ms⁻²).
13. A stone is released from top of a roof whose height is 19.6 m. Calculate its final velocity just before reaching ground. (19.6 ms⁻¹)
14. A ball which is thrown upwards vertically returns to the earth after 8 s. Find
 (i) the initial velocity with which it is thrown up.
 (ii) the maximum height it reaches.
 (iii) its position after 5 s.
 (i) 39.2 ms⁻¹ (ii) 78.4 m (iii) 73.5 m
15. A stone is dropped from a height of 49 m and simultaneously another ball is thrown upwards from the ground with a speed of 40 m/s. When and where do the two stones meet?
 (1.225 s; 41.647 m above the earth).
16. A particle is taken to a height R above the earth's surface where R is the radius of the earth. The acceleration due to gravity of the earth at that point is
 (A) 2.45 m/s² (B) 3.45 m/s²
 (C) 4.9 m/s² (D) 9.8 m/s² (A)
17. Two bodies A and B of masses 1 kg and 2 kg respectively are dropped near the earth's surface. Let the acceleration of A and B be a_1 and b_1 respectively, then
 (A) $a_1 > b_1$ (B) $a_1 < b_1$
 (C) $a_1 \neq b_1$ (D) $a_1 = b_1$ (D)
18. When a body is thrown up, the force of gravity is
 (A) in the upward direction. (B) in the downward direction
 (C) Zero (D) in the horizontal direction.
 (B)