

Module 3 Laws of Motion Part- 3

This module is designed to help you to:

- explain the concept of friction and use it as an important phenomenon associated with motion in order to solve real life problems;
- explain different types of friction;
- conceptually describe static friction and use this to solve problems in mechanics which involve unbalanced forces acting on particle and its effect on motion;
- explain the concept of dynamic friction and its impact on motion of particles executing uniform and non-uniform motion;
- discuss types of dynamic friction, i.e. kinetic friction and rolling friction which deals with two different types of motion, namely translational and rotational motion;
- apply the concepts of different types of forces acting on an object executing circular motion or rotational motion to solve the problems on angle of banking and stability of object executing such motion;
- elaborate on the concept of relative motion, frame of reference and the relation between them.

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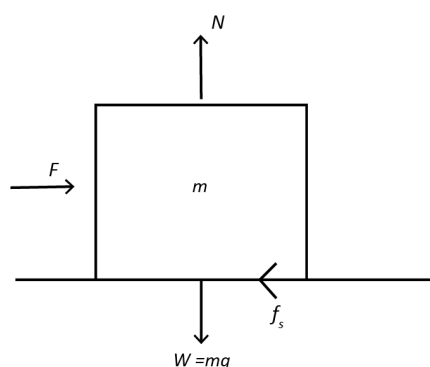
5.3.1 Introduction

In order to study and solve problems in mechanics, especially relating to different types of motion, viz. translational motion, circular motion, rotational motion, etc., it is important to take into account different types of hidden forces like friction which act on the body and hence impact its motion. The concepts of braking, acceleration, uniform motion, deceleration all involve frictional forces. Have you ever wondered why spikes are put underneath the shoes of athletes or sprinters, cricketers and footballers? Why do even ordinary shoes have grooves and not smooth surfaces or why do tyres of vehicles and cycles have treads and are not made smooth? All this is done to provide friction for stability and to ensure that when we suddenly start from rest or stop our motion, then we do not skip or skid on the surface. The banking along circular tracks or sharp turns uses frictional forces to prevent the vehicles from overturning or skidding.

In motion involving an object in a frame of reference or when motion of two or more objects are being studied, then the concept of relative motion is critical towards developing an understanding of such a motion.

5.3.2 Friction: Frictional force opposes (impending or actual) relative motion between two surfaces in contact. It is the component of the contact force along the common tangent to the surfaces in contact. Static friction f_s opposes impending relative motion; kinetic friction f_k opposes actual relative motion. They are independent of the area of contact.

5.3.2.1 Static Friction: When there is no net external force acting on a body kept on the surface, there is no static frictional force acting on the body. For example, a book lying stationary on the table feels no force of friction.



Now from Fig 5.3.1,

$$f_s \leq \mu_s N$$

$$(f)_{max} = \mu_s N$$

μ_s = coefficient of static friction

Fig. 5.3.1 Static friction

static friction opposes impending motion. The term impending means motion that would take place (but does not actually take place) under the applied force, if friction were absent.

5.3.2.2 Kinetic Friction

Frictional force that opposes relative motion between surfaces in contact is called kinetic or sliding friction and is denoted by f_k .

- (i) In general $f_k < (f_s)_{\max}$ where $(f_s)_{\max}$ is the maximum value of static friction.
- (ii) It is independent of the area of contact, like static friction.
- (iii) It is nearly independent of the velocity of motion.

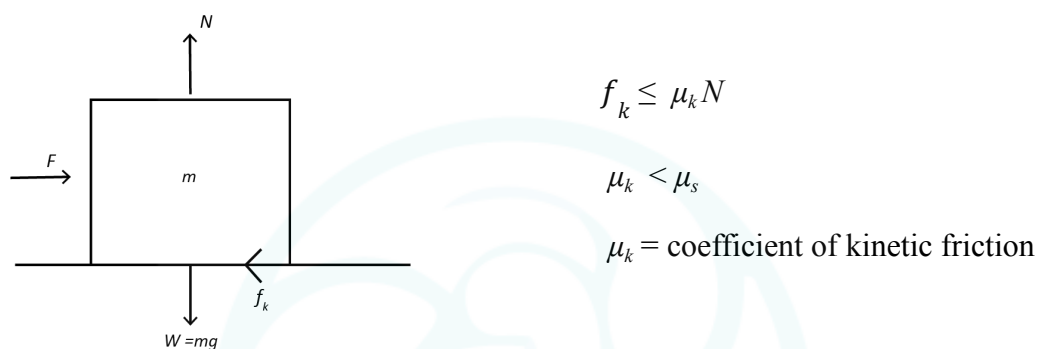


Fig. 5.3.2 Kinetic friction

Fig. 5.3.3 shows how friction adjusts itself against applied force.

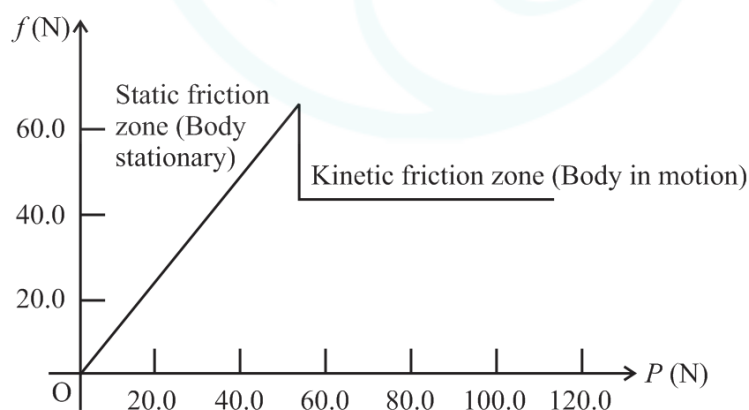


Fig. 5.3.3 Variation of frictional force (f) with applied force (P) up to the force of limiting friction.

The force of static friction is a self-adjusting one with the applied force, and is initially equal to the applied force. So, the graph of frictional force versus applied force at the beginning is a straight line passing through the origin making an angle 45 degree with the applied force axis. Once the applied force reaches the force of limiting friction the body starts moving and there is a sudden drop in the force of friction, and

thereafter it becomes constant, which is shown by the graph becoming parallel to the applied force axis. Then it manifests itself as kinetic friction.

5.3.2.3 Rolling Friction: It is defined as the frictional force resisting the motion of a body that is rolling on a surface (like a wheel or ball). This is caused normally because the body and roadbed or surface on which the body is rolling are not completely rigid and the weight of the body causes some deformation at the point of contact both in the body and the road bed or surface on which the body is rolling (Fig 5.3.4).

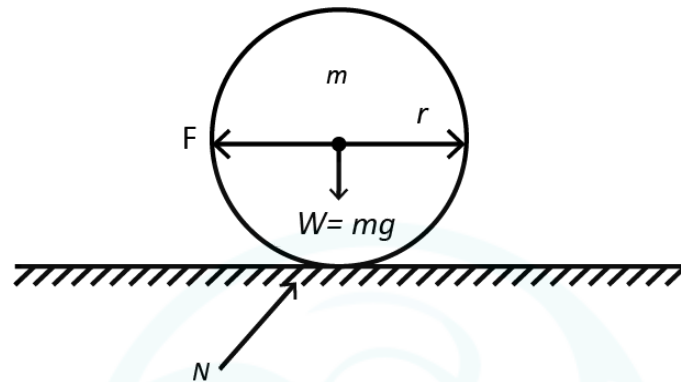


Fig. 5.3.4 Rolling friction

When a body rolls on a surface there is deformation in the surface and the body at the point of contact, thereby increasing the surface area of contact between the two instead of contact being at a point. Here W is the total weight of the rolling object, F is the forward towing force, r is the radius of the rolling object and N is normal to the surface on which the object is rolling but is inclined with its direction opposing the rolling of the wheel as can be seen from Fig 5.3.4.

Activity 5.3.1

Let us understand the concept of friction using the following experiment.

For static, dynamic and rolling friction

Following experiment (Fig 5.3.5) may first be performed in order to (i) observe self-adjusting nature of force of static friction, (ii) discuss dynamic friction, and (iii) discuss rolling friction and distinguish among static, dynamic and rolling friction. It may be done at a qualitative level in a short time.

Apparatus: Wooden block, inclined plane apparatus with glass top and pulley (or a pulley which can be fixed at the edge of the table), pan, weight box, spirit level, spring balance, a set of 5×200 g slotted weights.

(i) To demonstrate static friction, instead of adding weights in the pan, gradually add some dry sand (Fig. 5.3.5). Let the block be stationary and do not tap the horizontal surface. As soon as the weight of the pan and sand in it just exceeds the upper limit of static friction, the block moves and rapidly accelerates. Quickly hold the block or the pan or both to arrest the motion. Measure this weight of pan and sand by a spring balance. Discuss the self-adjusting nature of force of static friction with the students. For slow and continuous motion of the block, significantly less sand will be needed in the pan. Repeat the process at a few spots on the horizontal glass surface. At a spot that is not quite clean, the limiting force of static friction will be much higher than that at other spots, which may be neglected.

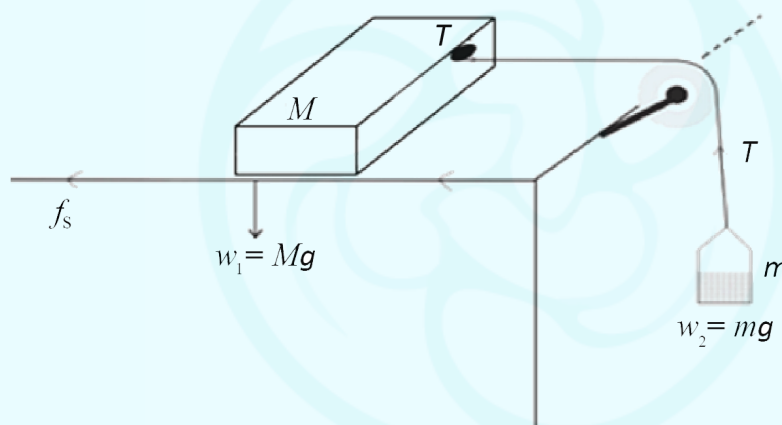


Fig. 5.3.5

(ii) To study the relationship between force of dynamic friction and normal reaction, between a block and a horizontal surface

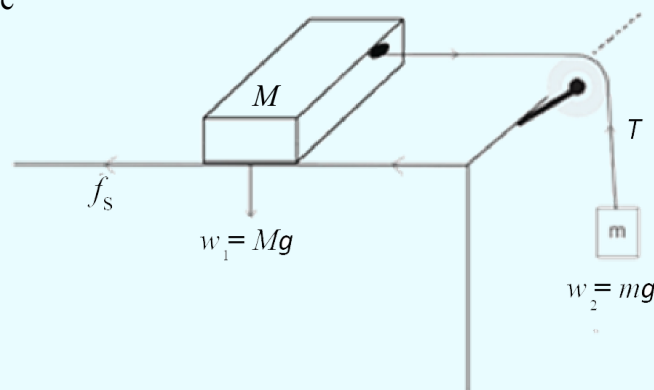


Fig. 5.3.6

Procedure: Setting the apparatus as in Fig. 5.3.6.

Find the force of dynamic friction first with the block alone sliding on the horizontal surface. Also find the weight of the block with the help of a spring balance, which is equal to normal reaction between the block and the glass top. Next, place one slotted weight on the block and measure the force of dynamic friction as well as normal reaction. In this manner make these two measurements with various weights placed on the block. Then plot a graph between force of dynamic friction and normal reaction between the block and the horizontal surface. Draw a straight line with a ruler which best fits the points marked on the graph paper to record your experimental data because measurement of force of dynamic friction is not a precise measurement.

Observations

Weight of the block (W_1) =

Weight of each slotted weight (w) =

Total Weight of the pan (W_2) =

S.No.	No. of slotted Weights on the block (n)	Total Weight of the pan (W_2)	Normal reaction ($W_1 + nw$)	Force of dynamic friction ($W_2 + nw$)

Questions for discussion

- Does your graph pass through the origin? Should the graph pass through the origin, which represents zero force of friction for zero normal reaction and experimental data? Give reasons for your answer.

2. Can you explain the process of walking? What are the forces involved when you walk? What would happen if the friction between your feet and the ground were to become zero?
3. You are somehow stuck on the surface of ice, where friction is almost zero. How will you attempt to come out of it? (You can give more than one answer.)

Note: 1. You may have to make a judicious choice of the size of the block and the set of slotted weights for this experiment. If the block is too light, its force of dynamic friction may be even less than the weight of an empty pan and you may not be able to take an observation with the block alone. Similarly, the maximum normal reaction, with which you work by placing the entire set of slotted weights on the block, should not be so large that its force of dynamic friction is larger than the force obtained by placing all the weights of the weight box on the pan.

2. If the slow continuous motion of the block on the glass surface is too jerky, clean the glass surface by rubbing it briskly with a clean, soft, dry cloth. If oil spots are seen on it clean with soap solution. If some extraneous material is stuck on the block, clean it with sandpaper.

(iii) Study of rolling friction : A steel ball-bearing rolling on a hard surface is an example of motion with low friction as shown in Fig. 5.3.7 below. You can use an equipment – a double inclined track, which can be used to actually measure this friction. Keep both the arms of the double inclined track equally inclined to horizontal at an angle θ . The ball, then, moves to and fro with gradually decreasing amplitude. Note the distance S_0 from the centre at which you release the ball and S_1, S_2, S_3, S_4 , the successive distances up to which the ball ascends on the arm on which it started. Total distance moved by the ball, as explained in Fig. 5.3.7 is approximately equal to:

$$2(S_0 + S_1) + 2(S_1 + S_2) + 2(S_2 + S_3) + 2(S_3 + S_4)$$

Also PE lost due to friction = $W(S_0 - S_4) \sin\theta$, where W is the weight of the ball.

Average resistive force (which is mostly the force of rolling friction) on the ball during these oscillations

$$= \frac{\text{Loss of PE}}{\text{Distance moved by the ball}} = \frac{W(S_0 - S_4) \sin\theta}{2(S_0 + S_1) + 2(S_1 + S_2) + 2(S_2 + S_3) + 2(S_3 + S_4)}$$

Vary the starting point of the ball (i.e, S_0), inclination of the tracks (θ), material size of the ball, etc. Try to explain the dependence of the average resistive force on these factors. Magnitude of rolling friction will depend on the nature of the surfaces.

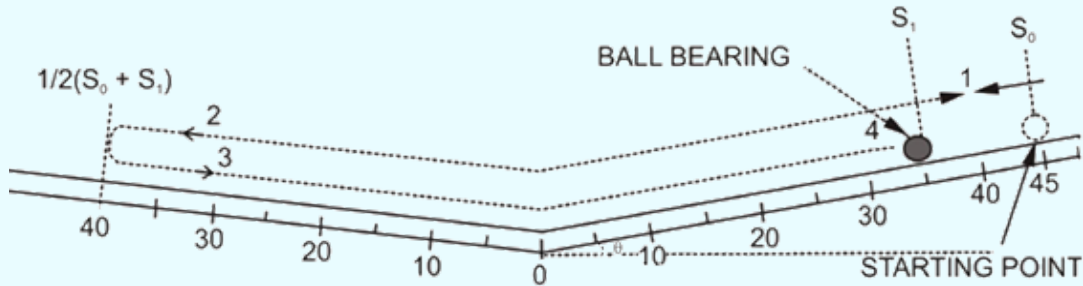


Fig. 5.3.7

Lengths of motion 2 and 3 on the left-hand side are each equal to $\sqrt{S_0 S_1}$. If damping is small, the arithmetic mean $\frac{1}{2}(S_0 + S_1)$ can be used and total distance moved in four motions is $2(S_0 + S_1)$.

Discuss the distinction among static, dynamic and rolling friction.

Example 5.3.1 Determine the maximum acceleration of the train in which a box lying on its floor will remain stationary, given that the coefficient of static friction between the box and the train floor is 0.15.

Solution: The acceleration of the box due to the static frictional force is,

$$ma = f_s \leq \mu_s N = \mu_s mg$$

$$\text{i.e. } a \leq \mu_s g$$

$$\begin{aligned} \therefore a_{\max} &= \mu_s g = 0.15 \times 10 \text{ m s}^{-2} \\ &= 1.5 \text{ m s}^{-2} \end{aligned}$$

where $a_{\max}$ is the maximum acceleration of the train for which the box lying on the floor remains stationary.

Example 5.3.2 See Fig. 5.3.8. A mass of 4 kg rests on a horizontal plane. The plane is gradually inclined until at an angle $\theta = 15^\circ$ with the horizontal, the mass just begins to slide. What is the coefficient of static friction between the block and the surface?

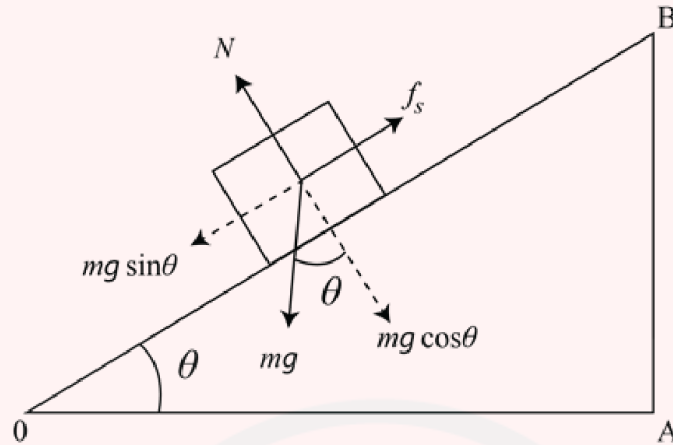


Fig. 5.3.8

Solution: The forces acting on a block of mass m at rest on an inclined plane are (i) the weight mg acting vertically downwards (ii) the normal force N of the plane on the block, and (iii) the static frictional force f_s opposing the impending motion. In equilibrium, the resultant of these forces must be zero. Resolving the weight mg along the two directions shown, we have

$$mg \sin \theta = f_s, \quad mg \cos \theta = N$$

As θ increases, the self-adjusting frictional force f_s increases until at $\theta = \theta_{max}$, f_s achieves its maximum value, $(f_s)_{max} = \mu_s N$

Therefore,

$$(\tan \theta)_{max} = \mu_s \quad \text{or} \quad \theta_{max} = \tan^{-1} \mu_s$$

When θ becomes just a little more than θ_{max} , there is a small net force on the block and it begins to slide. Note that θ_{max} depends only on μ_s and is independent of the mass of the block.

For $\theta_{max} = 15^\circ$

$$\begin{aligned} \mu_s &= \tan 15^\circ \\ &= 0.27. \end{aligned}$$

Example 5.3.3

Mass m_1 moves on a slope making an angle θ with the horizontal and is attached to mass m_2 by a string passing over a frictionless pulley as shown in Fig 5.3.9. The coefficient of friction between m_1 and the sloping surface is μ .

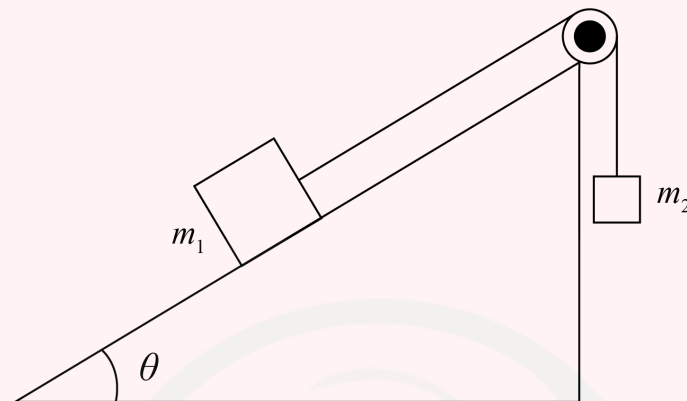


Fig .5.3.9

Which of the following statements are true?

- (i) If $m_2 > m_1 \sin\theta$, the body will move up the plane.
- (ii) If $m_2 > m_1 (\sin\theta + \mu \cos\theta)$, the body will move up the plane.
- (iii) If $m_2 < m_1 (\sin\theta + \mu \cos\theta)$, the body will move up the plane.
- (iv) If $m_2 < m_1 (\sin\theta - \mu \cos\theta)$, the body will move down the plane.

Solution: (ii) and (iv).

Example 5.3.4 In Fig. 5.3.10, a body A of mass m slides on plane inclined at angle θ_1 to the horizontal and is the coefficient of friction μ between A and the plane. A is connected by a light string passing over a frictionless pulley to another body B, also of mass m , sliding on a frictionless plane inclined at angle θ_2 to the horizontal. Which of the following statements are true?

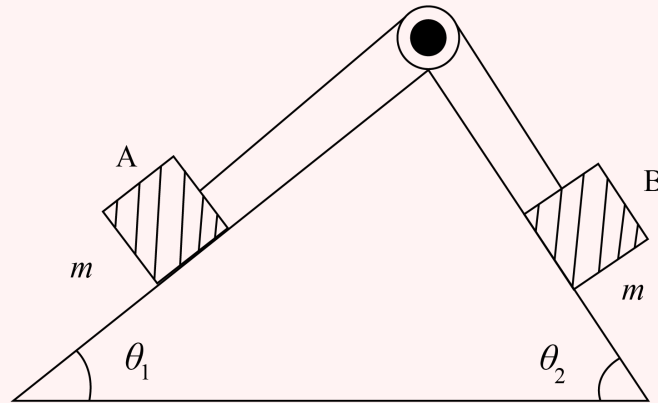


Fig. 5.3.10

- (i) A will never move up the plane.
- (ii) A will just start moving up the plane when

$$\mu = \frac{\sin \theta_2 - \sin \theta_1}{\cos \theta_1}$$

- (iii) For A to move up the plane, θ_2 must always be greater than θ_1 ,
- (iv) B will always slide down with constant speed.

Solution: (ii) and (iii)

Discussion on the Example 5.3.4

This problem presents some interesting aspects. Both the masses are equal but A is resting on a rough inclined plane while B is on a smooth inclined plane. The system may show movement or may remain stationary depending on the relative values of the angles of inclination (θ_1 and θ_2) and the coefficient of friction μ .

Now let us assume that both the inclined planes are rough with the coefficient of friction (μ_1 for the plane of A and μ_2 for the plane of B). We shall also try to look at what will happen if the masses are unequal. Let us work out the problem under these conditions.

Let us assume that the mass m on the right-hand incline will go down with an acceleration a . Then we must have that the following conditions satisfied

$$T - mg \sin \theta_1 - \mu_1 mg \cos \theta_1 = ma \quad (5.3.1)$$

$$mg \sin \theta_2 - \mu_2 mg \cos \theta_2 - T = ma \quad (5.3.2)$$

Adding Eq. (5.3.1) and (5.3.2), we get

$$mg \sin \theta_2 - \mu_2 mg \cos \theta_2 - mg \sin \theta_1 - \mu_1 mg \cos \theta_1 = 2 ma \quad (5.3.3)$$

For a to be positive, i.e. for the mass to slide down the right hand inclined plane we should have $\sin \theta_2 - \mu_2 \cos \theta_2 - \sin \theta_1 - \mu_1 \cos \theta_1 > 0$

If the angles of inclination are equal i.e. if $\theta_1 = \theta_2$, then the left hand side is always negative and there cannot be this movement irrespective of the values of the coefficient of friction. In fact, one can find that it is quite expected as the two masses and the angles of inclination are the same there cannot be any movement of the masses whether there is any friction or not. If, however, we assume that the mass on the right-hand incline (m_2) is greater than the mass on the left incline (m_1) the equation (3) will look like this (with $m_2 > m_1$)

$$m_2 g \sin \theta_2 - \mu_2 m_2 g \cos \theta_2 - m_1 g \sin \theta_1 - \mu_1 m_1 g \cos \theta_1 = (m_1 + m_2) a$$

Once again for a to be positive, i.e. for the mass m_2 to slide down the right hand inclined plane we should have

$$m_2 \sin \theta_2 - \mu_2 m_2 \cos \theta_2 - m_1 \sin \theta_1 - \mu_1 m_1 \cos \theta_1 > 0$$

$$\text{or, } m_2 \sin \theta_2 > (\mu_2 m_2 \cos \theta_2 + m_1 \sin \theta_1 + \mu_1 m_1 \cos \theta_1)$$

This is possible by suitable adjustment of the values of the masses, angles of inclination and the coefficients of friction to have the LHS greater than the RHS and a positive.

Let us now see the way the problem of the inclined plane with pulley and mass system changes when we consider the surface of the inclined plane to be frictionless and when we consider it to be rough having some coefficient of kinetic friction.

Example 5.3.5. The mass on the inclined plane is 3 kg and the other mass is 2 kg.

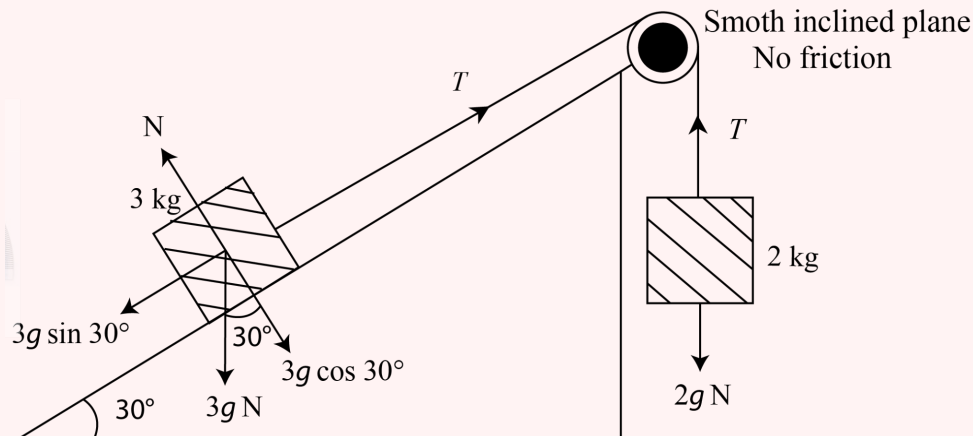


Fig. 5.3.11 (a)

Let us assume the 3 kg mass goes up along the inclined plane and the 2 kg mass moves down. We can write down the equation using a free body diagram. Let T be the tension on the string.

FBD equation for 3 kg mass

$$T - 3g \sin 30^\circ = 3a$$

FBD equation for 2 kg mass

$$2g - T = 2a$$

Adding the two equations we have

$$2g - 1.5g = 5a$$

$$a = \frac{0.5}{5}g$$

$$\therefore a = 0.1g$$

However, if there is friction the situation is quite different. In the case above, let us assume the inclined plane is rough with a coefficient of kinetic friction equal to $\mu_k = 0.1$.

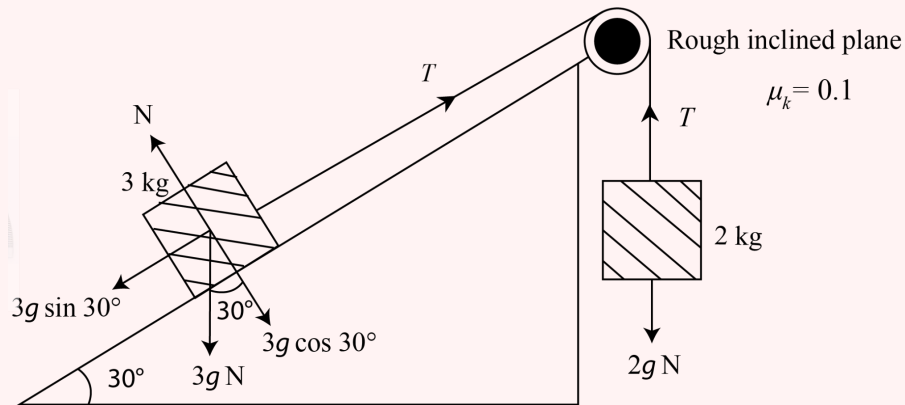


Fig. 5.3.11 (b)

The same problem may now be dealt with the introduction of the rough inclined plane [Fig.5.3.11 (b)].

$$T - 3g \sin 30^\circ - \mu_k 3g \cos 30^\circ = 3a$$

$$2g - T = 2a$$

Adding the two equations we have

$$2g - 1.5g - 0.26g = 5a$$

$$a = \frac{0.24}{5} g$$

$$\therefore a = 0.048 g$$

This shows the 3 kg mass is still moving up the inclined plane with a reduced acceleration.

Now if $\mu_k = 0.4$ we get [Fig.5.3.11 (c)],

$$T - 3g \sin 30^\circ - \mu_k 3g \cos 30^\circ = 3a$$

$$2g - T = 2a$$

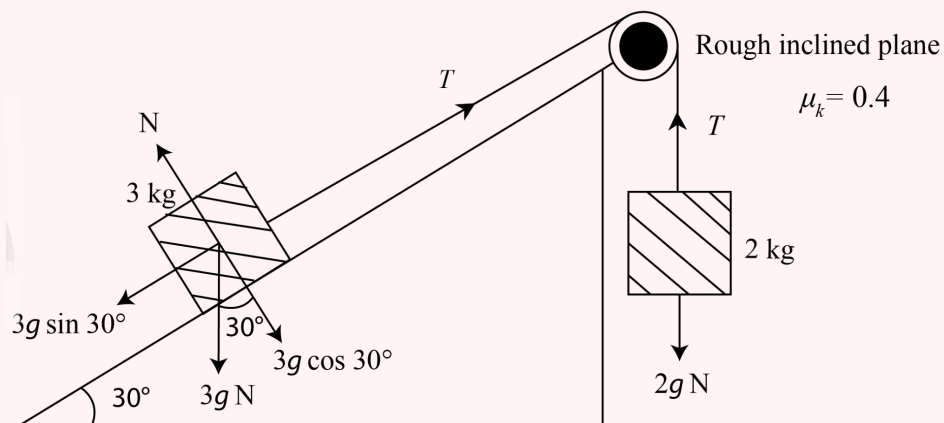


Fig .5.3.11 (c)

Adding the two equations we have

$$2g - 1.5g - 1.04g = 5a$$

$$a = -\frac{0.54}{5}g$$

$$\therefore a = -0.108g$$

Does the negative acceleration mean that the 3 kg mass on the inclined plane would start moving downward when the friction is increased? (μ_k has changed from 0.1 to 0.4).

Let us check with $\mu_k = 0.4$ and the direction of motion of the 3 kg mass lying on the inclined plane and 2 kg mass hanging from the pulley is reversed.

$$3g \sin 30^\circ - T - \mu_k 3g \cos 30^\circ = 3a$$

$$\text{or, } T - 2g = 2a$$

Adding the two equations we have

$$1.5g - 1.04g - 2g = 5a$$

$$\text{or, } a = -\frac{1.54}{5}g$$

$$\therefore a = -0.308g$$

This shows that the -ve sign of acceleration denotes that there is a deceleration in case the blocks are already in motion but if they are stationary then there will be no motion.



Let Us Reflect

Q. 5.3.1 A block of mass 15 kg is placed on a long trolley. The coefficient of static friction between the block and the trolley is 0.18. The trolley accelerates from rest with 0.5 m s^{-2} for 20 s and then moves with uniform velocity. Discuss the motion of the block as viewed by (a) a stationary observer on the ground, (b) an observer moving with the trolley.

Q. 5.3.2 The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in Fig. 5.3.12. The coefficient of friction between the box and the surface below it is 0.15. On a

straight road, the truck starts from rest and accelerates with 2 ms^{-2} . At what distance from the starting point does the box fall off the truck? (Ignore the size of the box).

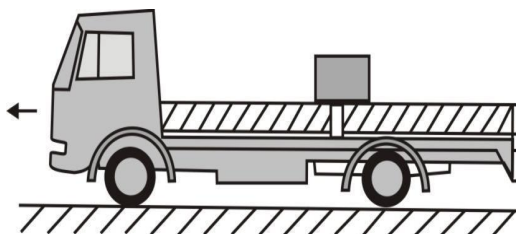


Fig. 5.3.12

Q. 5.3.3 Design two problems: (a) based on the second and the third law of motion and (b) an MCQ based on the concept of momentum and third law of motion.



Answers

A.5.3.1

(a) Maximum frictional force possible for opposing impending relative motion between the block and the trolley $= 150 \times 0.18 = 27 \text{ N}$, which is more than the frictional force of $15 \times 0.5 = 7.5 \text{ N}$ needed to accelerate the box with the trolley. When the trolley moves with uniform velocity, there is no force of friction acting on the block.

(b) For the accelerated (non-inertial) observer, the frictional force is opposed by the pseudo force of the same magnitude, keeping the box at rest relative to the observer. When the trolley moves with uniform velocity there is no pseudo-force for the moving (inertial) observer and no friction.

A.5.3.2 Acceleration of the box due to friction $= \mu g = 0.15 \times 10 = 1.5 \text{ m s}^{-2}$. But the acceleration of the truck is greater. The acceleration of the box relative to the truck is 0.5 m s^{-2} towards the rear end. The time taken for the box to fall off the truck $= \sqrt{\frac{2 \times 5}{0.5}} = \sqrt{20} \text{ s}$. During this time, the truck covers a distance $= \frac{1}{2} \times 2 \times 20 = 20 \text{ m}$.

A.5.3.3 Teacher can facilitate the students to design the problems.

5.3.4 Circular Motion

We shall next take up an interesting example of Motion Along the Turn in a Road. Before we discuss that, it will be worthwhile to discuss a special kind of acceleration and force associated with angular motion. We shall discuss the particular case of uniform circular motion.

5.3.4.1 Centripetal acceleration and centripetal force

Let us consider a body of mass m executing uniform circular motion with a uniform angular velocity ω . Let the radius of the circle be R . The motion has been schematically represented by Fig 5.3.13. Let OA along the x-axis be the reference line about which the body has described an angle ωt in time t , when it has arrived at P. The coordinates of P are $(R \cos \omega t, R \sin \omega t)$. So, The radius vector OP is given by,

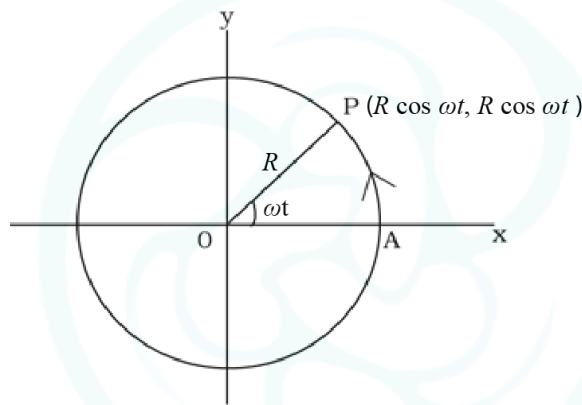


Fig. 5.3.13

$$\vec{R} = R \cos \omega t \hat{i} + R \sin \omega t \hat{j} \quad (5.3.4)$$

$$\therefore \vec{v} = \frac{d\vec{R}}{dt} = \omega R (-\sin \omega t \hat{i} + \cos \omega t \hat{j}) \quad (5.3.5)$$

$$= \omega R \left\{ \cos\left(\omega t + \frac{\pi}{2}\right) \hat{i} + \sin\left(\omega t + \frac{\pi}{2}\right) \hat{j} \right\} \quad (5.3.6)$$

Eq.(5.3.6) indicates that the body has a linear velocity, $R\omega$ making an angle $(\omega t + \frac{\pi}{2})$ with the +ve direction of the x-axis, i.e along the tangent at P. In other words, at every point this velocity is along the tangent and

thus it keeps changing the direction though its magnitude remains same. Since the velocity vector undergoes a continuous change, there should be some acceleration. What is that?

From Eq. (5.3.5)

$$a = \frac{dv}{dt} = -\omega^2 R (\cos \omega t \hat{i} + \sin \omega t \hat{j}) \quad (5.3.7)$$

$\therefore$ Using Eq.(5.3.4), we get

$$a = -\omega^2 R \quad (5.3.8)$$

Eq.(5.3.8) indicates that the body has an acceleration of magnitude $\omega^2 R$ directed opposite to the radius vector. So it keeps changing direction and always acts towards the centre. Hence, it is called the centripetal acceleration.

Since, $v = R\omega$ so, centripetal acceleration a can also be expressed as $\frac{v^2}{R} \cdot R = \frac{v^2}{R}$

$$\text{The corresponding force} = m\omega^2 R = \frac{mv^2}{R} \text{ and,} \quad (5.3.9)$$

it is called the centripetal force .

The centripetal force should not be regarded as yet another kind of force. It is simply a name given to the force that provides inward radial acceleration to a body in circular motion. We should always look for some material force like tension, gravitational force, electrical force, friction, etc as providing the centripetal force in any circular motion.

Quite an illustrative example is that of a stone tied in a string and rotated in a circle. The string is taut and so it is under tension. It is this force of tension T that provides the centripetal force. Refer to Fig 5.3.14.

$$T = \frac{mv^2}{R} \quad (5.3.10a)$$

$$\text{or } T = m\omega^2 R \quad (5.3.10b)$$

Eq. (5.3.10a) and (5.3.10b) follow from Newton's second law of motion.

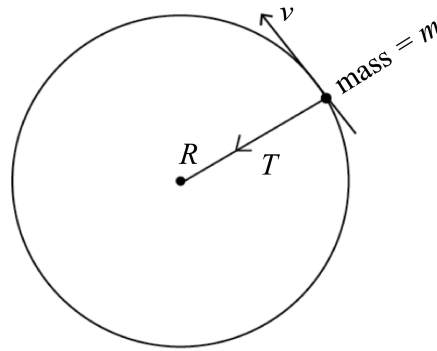
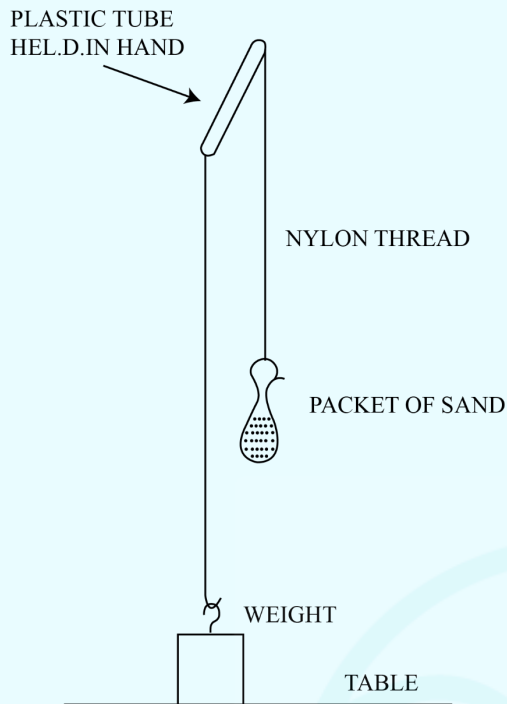


Fig. 5.3.14

Activity 5.3.2 : To observe that a centripetal force is necessary for moving a body uniformly in a circle, and that the magnitude of this force increases with angular speed of the circular motion.

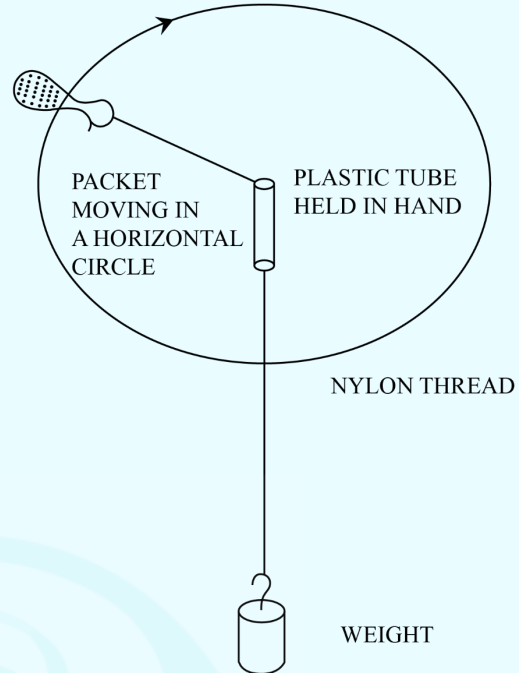
- a. Using a hard plastic tube and slotted weights, a plastic tube about 15 cm long and 10 mm outside diameter is taken. A strong nylon thread about 1.5 m long is passed through it. At one end of the thread is tied a packet of sand or a rubber stopper and at the other a weight (about three to four times the weight of the sand) is tied. It is first demonstrated that on lifting the plastic tube, the weight stays on the table and the packet of sand lifts up (Fig. 5.3.15).
- b. Now rotate the packet of sand in a horizontal circle (Fig. 5.3.16). If the motion is made faster, the weight (W) even moves up and vice versa, why? (One can have this plastic tube taken out of a commonly available ball or gel pen with its caps at the two ends and refill removed.)



The weight tied at the end passing down the plastic tube is much heavier than the packet of sand.

Fig. 5.3.15

Moves up and vice versa. why?



On revolving the packet of sand at a suitable speed the weight lifts off the table; its weight is just enough to provide the necessary centripetal force.

Fig. 5.3.16

c. Now by holding the plastic tube firmly in one hand and the weight (W) in the other, the packet of sand is rotated in a horizontal circle. When the motion is sufficiently fast, the weight (W) can hang freely without the support of your hand. Adjust the speed of rotation such that the position of the weight (W) does not change. In this situation, weight (W) provides the centripetal force necessary to keep the packet in motion in a circular orbit

5.3.3.2 Centrifugal force

Refer back to Fig 5.3.14.

Let us take a frame of reference attached to the stone. With reference to that frame of reference, the stone is at rest. It may sound somewhat puzzling. Then think of our earth moving round the Sun (for the sake of

simplicity let us consider the orbit to be circular with the Sun at the centre). We on the earth feel at rest. But someone watching us, say from the moon, will see us moving with the earth.

The same happens with a very tiny insect sitting on the stone. Let us think that it is not looking towards the outside wall, or any such other thing, then it will feel itself to be at rest in the rotating frame of reference of the stone. But it will also see that the string is taut, that is, it is under tension. So, it will ask as to why it is not drawn towards the centre of the circle. So, for a stone of mass m , frame of reference of the insect, it will have to consider the presence of a fictitious force (also called pseudo force), that will balance T . This fictitious force is called the centrifugal force. In other words,

$$T = \text{centrifugal force}$$

But we know from Eq. (5.3.10a) and (5.3.10b), that

$$T = m \omega^2 R = \frac{mv^2}{R}$$

So, the expression of centrifugal force is $m\omega^2 R$ or $\frac{mv^2}{R}$

And, in the frame of reference of the rotating stone,

$$T = \frac{mv^2}{R} \quad (5.3.11)$$

Eq. (5.3.11) is the same as Eq. (5.3.10b) mathematically, but (5.3.10a) is an equation of motion of the type,

Force = mass $\times$ acceleration, or, $\mathbf{F} = m\mathbf{a}$, where the quantities on the two sides of the equation are in the same direction, while Eq. (5.3.11) is a condition of equilibrium, where the quantities on the two sides of the equation are oppositely directed. Again, centrifugal force is no new force. It exists only in the frame of reference of the rotating object concerned. It is called fictitious, because neither does it arise by way of action-at-a-distance, nor by way of touch or contact. Moreover, its magnitude is the same as centripetal force, but they do not exist together. Their magnitudes are same and directions opposite, but their domains being different, the question of their nullifying each other does not arise at all.

Centripetal force exists in an inertial frame in the form of tension, gravitational attraction, etc. (see, the former is a force by way of contact and the latter by way of action-at-a-distance). And the centrifugal force exists in a non-inertial frame.

You may consider a relook at all the examples provided for centripetal force by handling the same using the concept of centrifugal force. However, for studying motion along the turn of a road we shall use the concept of centripetal force, which we shall do now.

5.3.4 Motion along the turn in a road

We know that acceleration of a body moving in a circle of radius R with uniform speed v is $\frac{v^2}{R}$ directed towards the centre. According to the second law, the force f_c providing this acceleration is: $f_c = \frac{mv^2}{R}$ where m is the mass of the body. This force directed towards the centre is called the centripetal force.

Motion in the turning of a road:

A car on a level road will execute a circular bend without slipping essentially due to static friction between the tyres of the car and the road provided the speed of the car is less than or equal to the following value (see Fig 5.3.17)

$$a_c = \frac{v^2}{R}$$

$$f_c = m \frac{v^2}{R}$$

$$f_s = f_c$$

$$f_s \leq \mu_s N \Rightarrow v^2 \leq \frac{\mu_s NR}{m}$$

$$\Rightarrow v \leq \sqrt{\frac{\mu_s NR}{m}}$$

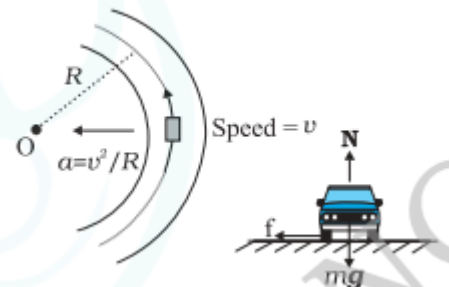


Fig 5.3.17

If ω is the angular velocity of a disc rotating with frequency ν rot/sec then

$$\omega = 2\pi\nu \quad \text{and} \quad v = \omega R$$

where v is the instantaneous speed of the disc with direction tangential to the point on the disc being considered.

Example 5.3.4 A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The coefficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn?

Solution: On an unbanked road, frictional force alone can provide the centripetal force needed to keep the cyclist moving on a circular turn without slipping. If the speed is too large, or if the turn is too sharp (i.e. of too small a radius) or both, the frictional force is not sufficient to provide the necessary centripetal force, and the cyclist slips. The condition for the cyclist not to slip is given by the following equation.

$$v^2 = \mu_s Rg$$

Now, $R = 3 \text{ m}$, $g = 9.8 \text{ m s}^{-2}$, $\mu_s = 0.1$. That is, $\mu_s Rg = 2.94 \text{ m}^2 \text{ s}^{-2}$, $v = 18 \text{ kmh}^{-1} = 5 \text{ m s}^{-1}$; i.e., $v^2 = 25 \text{ m}^2 \text{ s}^{-2}$.

The condition is not obeyed. The cyclist will slip while taking the circular turn.



Let Us Reflect

Q. 5.3.4 Discuss (i) motion of a car on a level road and (ii) motion of a car on a banked road.

Q. 5.3.5 A record disc revolves with a speed of $33\frac{1}{3} \text{ rev / min}$, and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the coefficient of friction between the coins and the record is 0.15, which of the coins will revolve with the record?

Q. 5.3.6 You may have seen in a circus a motorcyclist driving in vertical loops inside a ‘death-well’ (a hollow spherical chamber with holes, so the spectators can watch from outside). Explain clearly why the motorcyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m?

Q. 5.3.7 A 70 kg man stands in contact against the inner wall of a hollow cylindrical drum of radius 3 m rotating about its vertical axis with 200 rev/min. The coefficient of friction between the wall and his clothing

is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed?

Q. 5.3.8 Design two problems: (a) a MCQ based on motion of a body in a vertical circle, and (b) based on motion of a vehicle on a banked road.

Q.5.3.9 Two billiard balls each of mass 0.05 kg moving in opposite directions with speed 6 ms^{-1} collide and rebound with the same speed. What is the impulse imparted to each ball due to the other?

Q. 5.3.10 A shell of mass 0.020 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 m s^{-1} , what is the recoil speed of the gun?

Q. 5.3.11 A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 54 km/h . What is the impulse imparted to the ball? (Mass of the ball is 0.15 kg.)

Q. 5.3.12 Explain why

- (a) A horse cannot pull a cart and run in empty space,
- (b) Passengers are thrown forward from their seats when a speeding bus stops suddenly,
- (c) It is easier to pull a lawnmower than to push it,
- (d) A cricketer moves his hands backward while holding a catch.

Q. 5.3.13 A thin circular loop of radius R rotates about its vertical diameter with an angular frequency ω . Show that a small bead on the wire loop remains at its lowermost point for $\omega = \sqrt{g/R}$. What is the angle made by the radius vector joining the centre to the bead with the vertically downward direction for

$$\omega = \sqrt{2g/R} ?$$



Answers

A.5.3.5 For the coin to revolve with the record disc, the force of friction should be enough to provide the necessary centripetal force, i.e. $\frac{mv^2}{R} \leq mg$. Now $v = r\omega$, where $\omega = 2\pi\nu$ is the angular frequency of the disc. For a given μ and ω , the condition is $r \leq \mu g / \omega^2$. The condition is satisfied by the nearer coin (4 cm from the centre)

A.5.3.6 At the uppermost point, $N + mg = \frac{mv^2}{R}$ where N is the normal force (downwards) on the motorcyclist by the ceiling of the chamber. The minimum possible speed at the upper most point corresponds to $N = 0$.

$$\text{i.e. } v_{\min} = \sqrt{Rg} = \sqrt{25 \times 10} = 15.81 \text{ m s}^{-1}$$

A.5.3.7 The horizontal normal force N by the wall on the man provides the needed centripetal force: $N = mR\omega^2$. The frictional force f (vertically upwards) opposes the weight mg . The man remains stuck to the wall after the floor is removed if

$$mg = f < \mu N \text{ i.e. } mg < \mu mR\omega^2. \text{ The minimum angular speed of rotation of the cylinder is i.e. } \omega_{\min} = \sqrt{g/R} = 4.71 \text{ s}^{-1}.$$

A.5.3.8 Teacher can facilitate the students to design the problem.

A.5.3.9 Impulse on each ball = $0.05 \times 12 = 0.6 \text{ kg m s}^{-1}$ in magnitude. The two impulses are opposite in direction.

A.5.3.10 Use law of conservation of momentum : $100 v = 0.02 \times 80 \text{ m s}^{-1}$

$$v = 0.016 \text{ m s}^{-1} = 1.6 \text{ cm s}^{-1}$$

A.5.3.11 Impulse is directed along the bisector of the initial and final directions. Its magnitude is $0.15 \times 2 \times 15 \times \cos 22.5^\circ = 4.16 \text{ kg m s}^{-1}$

A.5.3.12 (a) The horse-cart system has no external force in empty space. The mutual forces between the horse and the cart cancel (third law of motion). On the ground, the contact force between the system and the ground (friction) causes their motion from rest.

(b) Due to inertia of the body not directly in contact with the seat.

(c) A lawn mower is pulled or pushed by applying force at an angle. When you push, the normal force (N) must be more than its weight, for equilibrium in the vertical direction. This results in greater friction f ($f \propto N$) and, therefore, a greater applied force to move. Just the opposite happens while pulling.

(d) To reduce the rate of change of momentum and hence to reduce the force necessary to stop the ball.

A.5.3.13 Consider the free-body diagram of the bead at the lowest point of the thin circular loop when the radius vector joining the centre of thin circular loop to the center of the bead is R . Then the inward centripetal force acting on the bead is given by

$$mR\omega^2 \cos\theta = mR\omega^2 \text{ i.e. } \theta = 0^\circ.$$

For the bead to remain at the lowermost point this centripetal force should balance the gravitational force acting downward. Thus,

$$mR\omega^2 = mg$$

$$R\omega^2 = g.$$

$$\omega = \sqrt{\frac{g}{R}}$$

When θ is not zero, these equations give $\cos\theta = R\omega^2$. Since $\cos\theta \leq 1$ the bead makes an angle θ with the vertical downward direction for angular frequency ω .

For $\omega = \sqrt{\frac{2g}{R}}$, $\cos\theta = \frac{1}{2}$ i.e. $\theta = 60^\circ$

5.3.5 Relative Velocity

Suppose you are standing on a railway platform of a small station. An express train passes by with a velocity of 50 km/h. You can possibly see the faces of some of the passengers on the train. Now suppose you are travelling in a local train moving with a velocity of 40 km/h and as a passenger of the train you observe the same express train moving in the opposite direction with a velocity of 50 km/h. What will you be feeling now? You will feel the other train is moving much faster now. But its velocity is still the same. Then why do you feel it is moving much faster? The simple answer that comes up is that now you are no longer a

stationary observer on the platform but an observer on a train moving in the opposite direction. So, now the relative velocity of the express train with respect to you has gone up. In fact, for a passenger in the express train your relative velocity is the same but in the opposite direction if we assume that the two railway tracks are parallel. To have the relative velocity of the express train with respect to the local train we need to simply calculate vector addition of the two velocities i.e. $(\mathbf{v}_{\text{expr}} - \mathbf{v}_{\text{local}})$. If we keep in mind that $\mathbf{v}_{\text{local}}$ is actually in the direction opposite to the $\mathbf{v}_{\text{expr}}$ then this turns out to be a simple addition of these two velocities. That makes the relative velocity of the express train 90 km/h with respect to you who is a passenger in the local train and the express train appears to be moving quite fast. But do not get disheartened. A passenger in the express train also sees that your local train is moving with a velocity of 90 km/h but in the opposite direction.

The basic situation remains the same even when two bodies are not moving in parallel tracks but their tracks make a certain angle with each other. Once again, we need to go for vector addition this time by applying the principle of parallelogram law of vectors. But we can actually describe the principle in an even simpler language when we are trying to find the velocity of say A with respect to B and their direction of velocities make an angle θ with each other. It involves two steps

1. Make the observer; say A effectively stationary by imposing an equal and opposite velocity on it.
2. Find out the resultant of the imposed velocity and the velocity of the other body, say B by applying the principle of parallelogram law of vectors.

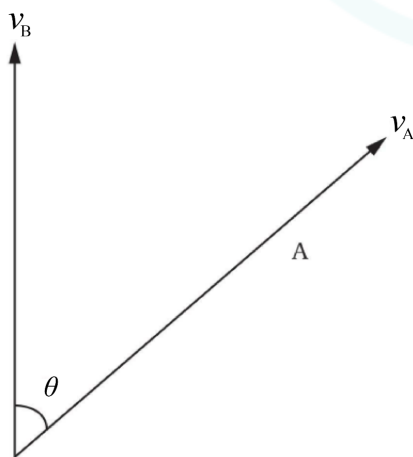


Fig. 5.3.18

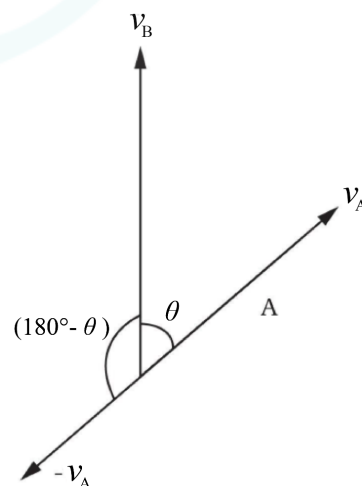


Fig. 5.3.19

The resultant of the velocities $-v_A$ and v_B making an angle of $(180^\circ - \theta)$ between their directions will give us the relative velocity of B w.r.t. A.

Example 5.3.5 Cyclist Ajay is moving towards east with a velocity of 15 km/h. His friend Pawan is walking in the north-east direction with a velocity of 5 km/h. Find the relative velocity of Pawan with respect to Ajay. Also calculate the relative velocity of Ajay with respect to Pawan.

Solution:

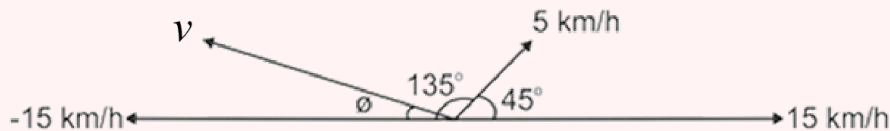


Fig. 5.3.20

The north-east direction implies that the path makes an equal angle of 45° both with the east and with north. So, for finding the relative velocity of Pawan w.r.t Ajay we have to bring Ajay 'effectively' to rest.

So, we need to find out the resultant of two velocities 15 km/h towards west (we have called it -15 km/h as this is an imposed velocity to bring Ajay to rest) and 5 km/h (actual velocity of Pawan) in the north-east direction. That makes the angle between the velocities 135° .

So the magnitude of the resultant velocity comes out from $v^2 = 15^2 + 5^2 + 2 \cdot 15 \cdot 5 \cdot \cos 135^\circ$

We get $v = 12$ km/h (approximately). This is the magnitude of relative velocity of Pawan

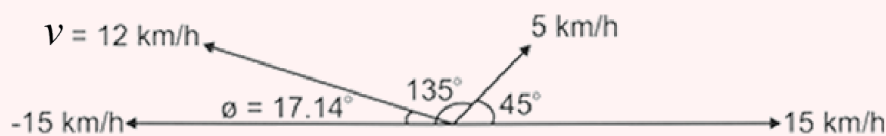


Fig. 5.3.21

with respect to Ajay

If this velocity makes an angle ϕ with the direction of imposed velocity to bring Ajay to rest then we can write

$$\tan \phi = \frac{5 \sin 135^\circ}{15 + 5 \cos 135^\circ}$$

$$\text{or, } \tan \phi = 0.308 \quad \text{so, } \phi = 17.139^\circ$$

Pawan will be moving with a velocity of 12 km/h making an angle of 17.139° with west direction or the direction of imposed velocity to bring Ajay to rest. Hence this is the relative velocity of Pawan with respect to Ajay. Note that this velocity is much higher than the actual velocity of Pawan and is in a different direction too.

If we now want to calculate the relative velocity of Ajay with respect to Pawan then we take a similar approach and bring Pawan effectively at rest by imposing an equal and opposite velocity on him.

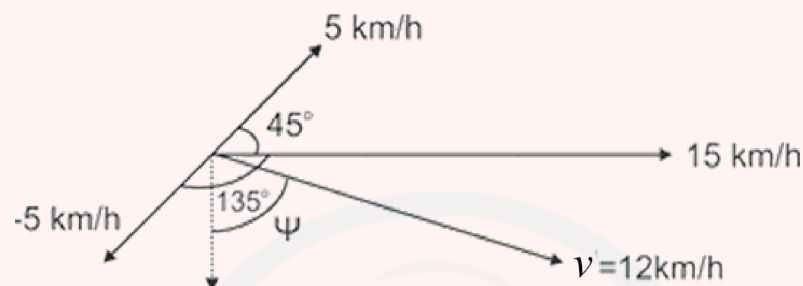


Fig. 5.3.22

Now the magnitude of relative velocity of Ajay w.r.t. Pawan is v

$$= \sqrt{15^2 + 5^2 + 2 \times 15 \times 5 \cos 135^\circ} = 12 \text{ (km/h)}$$

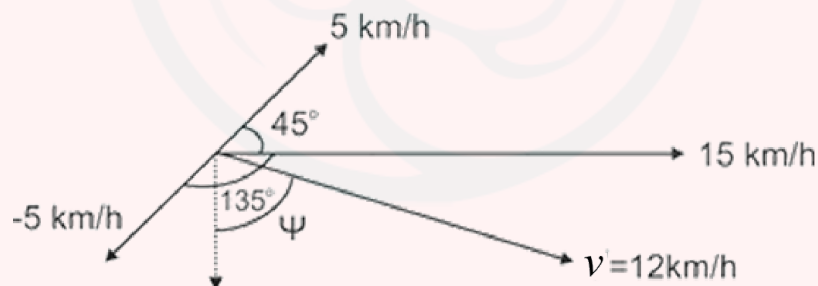


Fig. 5.3.23

If Ψ is the angle made by the direction of relative velocity of Ajay with the direction of Pawan's velocity then

$$\tan \Psi = \frac{15 \sin 135^\circ}{5 + 15 \cos 135^\circ} = \frac{15 \times \frac{1}{\sqrt{2}}}{5 - 15 \times \frac{1}{\sqrt{2}}} = \frac{15}{5\sqrt{2} - 15}$$

$$\text{Therefore, } \Psi = \tan^{-1} \left(\frac{15}{5\sqrt{2} - 15} \right) = -62.14^\circ$$

So Pawan sees Ajay moving in a direction south of east.

5.3.6 Summary

- Friction: Frictional force opposes (impending or actual) relative motion between two surfaces in contact. It is the component of the contact force along the common tangent to the surface in contact. Static friction f_s opposes impending relative motion; kinetic friction f_k opposes actual relative motion. They are independent of the area of contact and satisfy the following approximate law:

$$f_s \leq (f_s)_{max} = \mu_s N$$

$$f_k = \mu_k N$$

μ_s (coefficient of static friction) and μ_k (coefficient of kinetic friction) are constants characteristic of the pair of surfaces in contact. It is found experimentally that $\mu_s < \mu_k$.

- Friction is of great use in our daily life as it is the reason why we are able to walk or run without slipping and why vehicles stop without slipping while braking.
- Understanding of relative motion is put to use in our daily life in a number of ways. We apply brakes to our vehicle to avoid an accident with a vehicle in front of us by judging the relative motion of the vehicle in front of us with respect to our vehicle. In cricket a fielder is able to catch the ball by judging the relative motion of the ball with respect to himself/herself. A sprinter is able to win a race by deciding his pace relative to other sprinters.

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