

Mechanical Properties of Fluids

Module 2 Bernoulli's Principle

This module is designed to help you to:

- describe the concept of streamline and turbulent flow of fluid;
- explain Bernoulli's principle;
- discuss equation of continuity for fluids in streamlined flow through pipes of varying cross-section;
- explain applications of Bernoulli's principle performing various activities;
- use Bernoulli's principle to study the steady flow of non-viscous and incompressible fluids and its applications, in carburetors of an automobile, bunsen burner, spray pump, etc.

Content

- 10.2.1 Introduction
- 10.2.2 Streamlined flow
- 10.2.3 Equation of continuity
- 10.2.4 Types of energy in a fluid
- 10.2.5 Bernoulli's principle
- 10.2.6 Applications of Bernoulli's principle
- 10.2.7 Summary

10.2.1 Introduction

In the earlier module Pressure, we have discussed that pressure is a scalar. We also studied Pascal's law, i.e. undiminished transmission of pressure exerted on an enclosed fluid in all directions. We also learnt how this property of the fluids is used to design hydraulic machines. The above phenomenon and its applications come under the purview of hydrostatics.

In the present module, we will discuss the types of flow such as streamlined and turbulent flow of fluids. This study comes under the purview of hydrodynamics.

Fluid motion plays an important role. A large number of processes in living beings, including plants, are governed by fluids in motion. Food, nutrients and waste products in our body are transported by fluids. The proper flow of blood through arteries and veins is very crucial for normal functioning of the human body as a whole. Therefore, understanding the behavior and properties of fluids in motion is important.

We will discuss the equation of continuity for motion of fluids in pipes of varying cross-section. We will also learn Bernoulli's theorem governing the steady flow of non-viscous and incompressible fluids and its practical applications.

10.2.2 Streamlined flow

Streamlined flow is a highly ordered motion of the liquid molecules[Fig.10.1.1(a)] . At low speeds, the flow of a fluid is steady. Beyond a limiting value, called critical speed, this flow tends to become disordered. Such a disordered flow [Fig.10.1.1(b)] is called turbulent flow. Formation of whirlpools observed when a fast flowing stream encounters rocks is an example of turbulent flow.

The flow of a fluid is said to be streamlined if at any given point, the velocity of each passing fluid particle remains constant in time. The path taken by fluid particles under such a flow is termed a streamline.

The nature of flow departs from streamline and tends to become turbulent when:

- There is an obstacle in the path of fluid flow
- The speed of flow is higher
- The fluid flows through a narrow tube

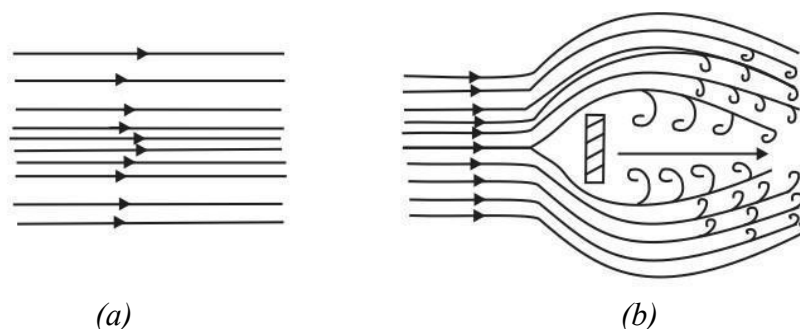


Fig. 10.2.1: (a) Some streamlines for fluid flow, (b) Turbulent flow

In general, fluid flow is quite a complex phenomenon. However, we can study some useful properties for streamlining flows by using the conservation of energy. Such steady flow is observed at low speeds of the fluid. At higher speeds (above a certain critical speed), the flow becomes turbulent.

To understand the phenomenon related to fluid flow, let us perform a few activities.

Activity 10.2.1

- Suspend two table tennis balls from a horizontal rod clamped to a laboratory stand as shown in Fig.10.2.2. The balls should be close to each other and at the same height but not touching each other.

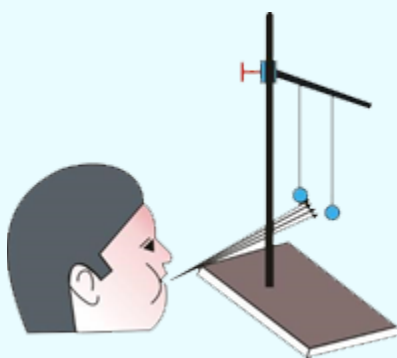


Fig. 10.2.2: Air pressure is reduced between the balls when a high speed wind flows in the space between them

- Let us see what happens if we blow between the two balls.

- Will the balls come closer or move further away from each other? As we see, the balls in fact move closer.
- This indicates reduction of air pressure between the balls when a high speed wind flows in the space between them. So the fluid pressure is reduced in the region where its speed is high.

Activity 10.2.2

- Place a sheet of paper, supported by two books, in the form of a bridge. Let the books be slightly converging (Fig.10.2.3), that is, their separation is larger on the side facing you.

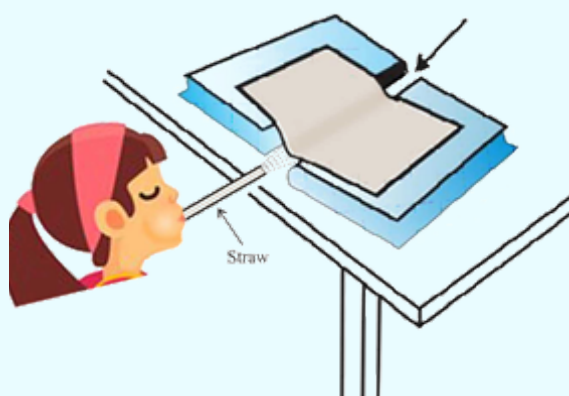


Fig. 10.2.3: *The paper bends downwards on blowing under the paper bridge*

- Blow under the bridge and observe what happens to the paper bridge. (You may use a straw pipe to blow under the bridge. Ensure that the stream of air is not bouncing off the table. It will work better if the air stream is directed straight through the middle.)
- Observe the motion of paper. The paper bends downwards.
- Discuss and draw conclusions.
- What does it indicate about the pressure under the paper?
- This activity also indicates that the pressure under the paper is decreased as the high speed wind is blown under it.

10.2.3. Equation of continuity

The steady flow of an ideal fluid of density ρ in a pipe of varying area of cross-section is governed by the equation of continuity.

For an ideal fluid flowing through a pipe, X and Y are two points in two sections of the pipe. Let A_1 and A_2 be the areas of cross-section at X and Y respectively (Fig. 10.2.4). The corresponding fluid velocities at these points are V_1 and V_2 .

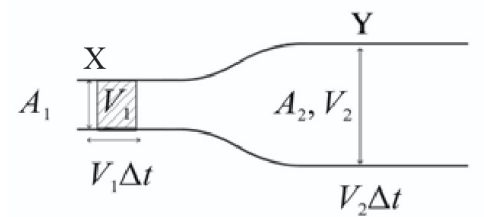


Fig. 10.2.4: Fluid flow in a tube of varying cross-section

Let A_1 be the area of cross-section at X; A_2 , area of cross-section at Y; V_1 , speed of fluid at X and V_2 be the speed of fluid at Y.

In a short interval of time Δt , a liquid column of length $V_1 \Delta t$ travels past X and pushes a volume $A_1 V_1 \Delta t$ across it.

Mass of liquid crossing X in time $\Delta t = \Delta m_1 = A_1 V_1 (\Delta t) \rho_1$

Mass of liquid crossing Y in time $\Delta t = \Delta m_2 = A_2 V_2 (\Delta t) \rho_2$

In a steady flow, the mass of the fluid crossing any cross-section must be the same.

$$\Delta m_1 = \Delta m_2$$

$$A_1 V_1 (\Delta t) \rho_1 = A_2 V_2 (\Delta t) \rho_2$$

As ideal fluids are incompressible, we have

$$\rho_1 = \rho_2$$

$$\text{Therefore, } A_1 V_1 = A_2 V_2$$

In general,

$$A \cdot V = \text{constant}.$$

This is the equation of continuity.

If $A_1 > A_2$, we get $V_1 < V_2$.

Thus the speed of flow of the liquid is faster in the narrower parts of the tube.

This equation is a consequence of the law of conservation of mass.



Let Us Reflect

Q.10.2.1 The radii of cross-section of a pipe at two points are r and $2r$. The speeds of an incompressible fluid in steady flow at these points are in the ratio

- (A) 1:2
- (B) 1:4
- (C) 2:1
- (D) 4:1

Q.10.2.2 Which of the following statements is incorrect?

- (A) A tangent drawn to a streamline at any point gives the direction of velocity of the fluid particles at that point.
- (B) A normal drawn to a streamline at any point gives the direction of velocity of the fluid particles at that point.
- (C) The streamlines are closely packed in the region where the fluid pressure is low.
- (D) The streamlines are closely packed in the region where the speed of flow is more.



Answers

A.10.2.1 (D)

A.10.2.2 (B)

10.2.4. Types of energy in a fluid

A fluid in motion may have either or all of the following three types of energy:

- Gravitational potential energy
- Kinetic energy
- Pressure energy

We have studied the gravitational potential energy and kinetic energy in earlier classes. We will just revisit the expressions for potential energy and kinetic energy.

- **Potential energy-** We know that the gravitational potential energy of a fluid element of mass m is given by mgh , h being the height above a certain reference level.

$$\text{Potential energy per unit mass} = PE/m = gh$$

$$\begin{aligned} \text{Potential energy per unit volume} &= PE/\text{volume} \\ &= \rho gh \end{aligned}$$

- **Kinetic energy** - The energy due to the motion of a fluid is called its kinetic energy

We have KE of an element of fluid of mass $m = 1/2 mv^2$

$$\text{Kinetic energy per unit mass} = KE/m = 1/2 v^2$$

$$\begin{aligned} \text{Kinetic energy per unit volume} &= KE/\text{volume} \\ &= 1/2 \rho v^2 \end{aligned}$$

- **Expression for pressure energy**

Consider a liquid of density ρ in a wide container. The container has a narrow tube T with a uniform area of cross-section A fitted with a piston as shown in (Fig.10.2.5).

Apply a force F on the piston to move it through a small distance Δx . The applied force exerts a pressure P on the piston pushing a small volume $A.\Delta x$ of the fluid.

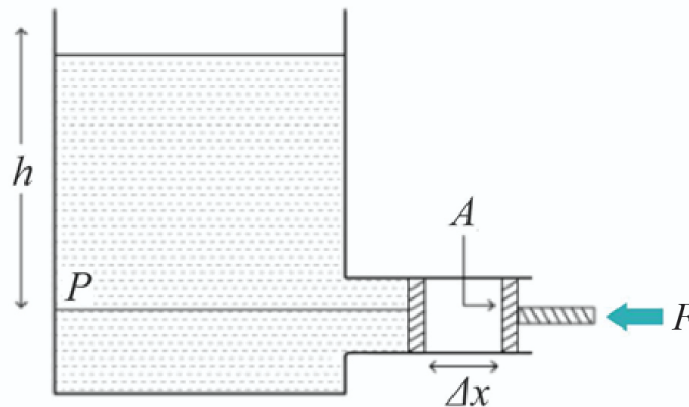


Fig.10.2.5: Pressure energy

The liquid in the container has a height h above the axis of the tube T.

The liquid pushed in does not cause any appreciable change in the level of the liquid in the wide container.

Work done to push the piston against fluid pressure P gets stored in the system as pressure energy. The energy of the fluid by virtue of its pressure is known as pressure energy.

We have,

$$\begin{aligned}\text{Work done to push the piston} &= F \cdot \Delta x \\ &= P.A \cdot \Delta x\end{aligned}$$

$$\therefore \text{Pressure energy} = P.A \cdot \Delta x$$

$$\text{Also volume of the liquid pushed} = A.\Delta x$$

$$\text{Mass of the liquid pushed} = A.\Delta x \cdot \rho$$

$$\begin{aligned}\therefore \text{Pressure energy per unit mass} &= \text{Pressure Energy}/m \\ &= P.A.\Delta x/(A.\Delta x.\rho) \\ &= P/\rho\end{aligned}$$

$$\text{Pressure energy per unit volume} = P.A.\Delta x/(A.\Delta x) = P$$

10.2.5 Bernoulli's principle

The application of the law of conservation of energy to the flow of an ideal fluid gives us a useful result known as Bernoulli's principle.

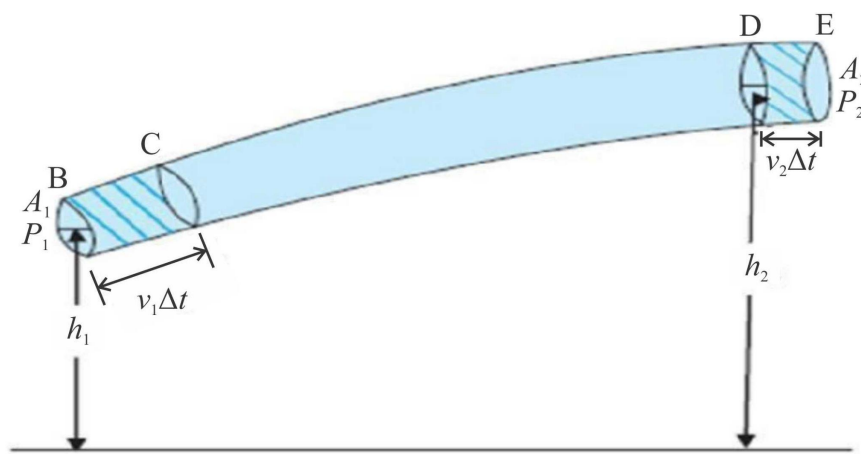


Fig.10.2.6: The flow of an ideal fluid in a pipe of varying cross section. The fluid in a section of length $v_1\Delta t$ moves to the section of length $v_2\Delta t$ in time Δt .

The sum of pressure energy, kinetic energy and potential energy per unit volume of a fluid remains constant at all points of its streamlined flow.

The pressure P_1 , the area of cross-section A_1 , speed V_1 and height h_1 at B of an ideal fluid (Fig.10.2.6) in steady flow are related to the corresponding values at D as $P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$.

In general,

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{Constant}.$$

The result is known as Bernoulli's principle.

For a horizontal pipe(h is constant), Bernoulli's principle reduces to

$$P + \left(\frac{1}{2}\right) \rho v^2 = \text{Constant.}$$

According to the above equation, the region in which the speed of a fluid is high, the pressure is low and vice-versa. The same conclusion was drawn from Activity 10.2.1 and Activity 10.2.2 performed earlier.

Limitations of Bernoulli's principle

Bernoulli's principle does not hold in the presence of a viscous drag on the fluid. For a viscous fluid, the work done against the dissipative viscous force must be taken into account and pressure will be lower than the value given by Bernoulli's equation. Also, Bernoulli's equation does not hold for non-steady or turbulent flows. In such a situation, velocity and pressure would be constantly fluctuating with time. In fact, the real fluids are neither non-viscous nor completely incompressible. There is always some energy loss due to elastic forces as well as viscous forces. Despite these limitations, Bernoulli's equation has wide ranging applications for fluids of low viscosity.



Let Us Reflect

Q.10.2.3 An ideal liquid of density d has a streamlined flow in a horizontal tube of varying cross sections. R_1 and R_2 are radii of the tube at points L and M. P and v denote pressure and speed of flow at L. The pressure at point M is $\frac{3P}{2}$.

$\frac{R_2}{R_1}$ is

(A) $\left[1 - \frac{P}{v^2 d}\right]^{1/2}$

(B) $\left[1 + \frac{P}{v^2 d}\right]^{1/2}$

(C) $\left[\frac{P}{v^2 d} - 1 \right]^{1/4}$

(D) $\left[\frac{P}{v^2 d} + 1 \right]^{1/4}$

Q.10.2.4 A liquid of density d , having a streamlined flow with a speed of v m/s enters the open end of the tube of radius $2R$. The volume of liquid emerging out of the tube at other end of radius $\sqrt{2}R$ per minute is

(A) $2\pi R^2 v$

(B) $4\pi R^2 v$

(C) $120 \pi R^2 v$



Answers

(D) 240π

A.10.2.3 (A)

A.10.2.4 Hint: The volume of liquid entering the tube per second is $4\pi R^2 v$. The rate of flow of liquid in the tube is constant. Therefore the volume of liquid emerging out per minute is

$$60 \times 4\pi R^2 v = 240 \pi R^2 v$$

10.2.6 Applications of Bernoulli principle

In our daily life Bernoulli's equation has a large number of useful applications. It can help us to understand a wide variety of phenomena. The carburetor of an automobile, filter pumps, or aspirators, Bunsenburner, atomisers and sprayers (used for perfumes, or insecticides) work on this principle. In many games, such as cricket, tennis, baseball, or golf, we notice that a spinning ball deviates from its parabolic trajectory as it moves through air. This deviation can be partly explained on the basis of Bernoulli's principle.

1. The spray pump

When the piston of an insecticide/pesticide spray pump is pushed, air is forced out of a narrow hole. The speed of air in the region immediately above the open end of the tube dipped in the tank becomes high (Fig.10.2.7). This increase in speed, as per Bernoulli's equation, results in a decrease in pressure in this region. The upper end of the tube, dipping into liquid, is positioned just below this narrow hole of the piston tube. The opening of the tube is, therefore, in a region of low pressure. The lower end of the tube is, however, at atmospheric pressure. The liquid, therefore, gets pushed up the tube. As it emerges from the upper end, it mixes up with the fast moving air there, and gets sprayed in the surroundings.

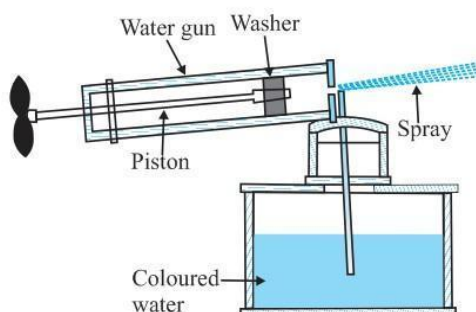


Fig.10.2.7: *The spray pump*

2. Carburetor of an automobile

The carburetor of an automobile also has a nozzle through which air flows with a large speed. As a result, the pressure is lowered at the narrow neck and the petrol (gasoline) gets sucked up in the chamber. This helps to have the correct mixture of air and fuel, necessary for combustion.

3. Bunsen burner

In a Bunsen burner (Fig.10.2.8), the fuel gas emerges out of the jet in the center of the vertical tube with a high speed.

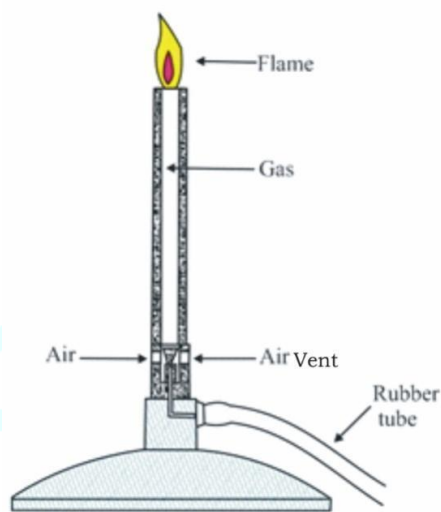


Fig. 10.2.8: *Bunsen burner*

The gas pressure gets reduced in the region of the vent. The wide opening of the vent lets air rush in. This air mixes up with the fuel gas and the gas (mixture) burns with a hot and blue flame. (If the air does not

get mixed with fuel gas at this stage and come into contact with it only at the flame level, the flame will be a bright yellow-orange flame like that of a candle due to an incomplete combustion of the gas. The yellow flame gives comparatively less heat than the blue flame resulting in undesired wastage of energy).

The learner may be encouraged to explore more applications and phenomena which can be explained on the basis of Bernoulli's principle.

Activity 10.2.3

Take a cap of a pen and make a hole in it as shown in Fig. 10.2.9.

Make a small spherical ball from a piece of thermocol. Place the ball on the hole of the cap.

Now blow through the open end of the cap and observe the ball.

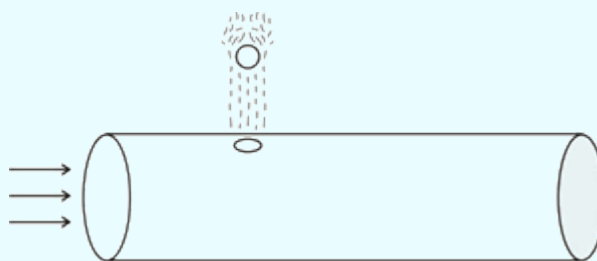


Fig. 10.2.9: *Pen cap with a hole*

The ball lifts up and keeps on spinning in the air without moving to either side (Fig. 10.2.9). The reason is that the speed of air is more near the surface of the spherical ball and this generates a low pressure region around it. The ball, therefore, experiences pressure from either side, keeping it suspended in the air stream.

**Let Us Reflect**

Q.10.2.5 Explain why, during storms (in which high-speed winds blow over the roofs), the tin/thatched roofs, sometimes, get blown off.



Answers

A.10.2.5 The pressure below the roof is equal to the atmospheric pressure which is now larger than pressure above the roof. This difference of pressure causes an upward thrust and the roof is lifted up.

Activity 10.2.4

- Crumple a small piece of paper into a ball of size smaller than the mouth of an empty bottle.
- Hold the empty bottle on its side and place the paper ball just inside its mouth. Now try to blow the ball into the bottle.(Fig.10.2.10).



Fig. 10.2.10: *Blowing over a paper ball placed just inside the bottle*

- Try the activity with bottles of different sizes. Why is it difficult to force the paper ball into the bottle?

10.2.7 Summary

- The flow of a fluid is streamlined if at any given point velocity of each passing fluid particle remains constant in time. Such a flow is also called a steady flow.
- For a steady flow through a pipe of varying cross-section, the mass crossing any cross-section remains constant. This is the equation of continuity.
- For a steady flow of an ideal fluid, through a pipe, the total energy of the fluid per unit volume remains constant. Expressed mathematically,

$$P + \frac{1}{2}\rho v^2 + gh\rho = \text{Constant.}$$

This is known as Bernoulli's principle.

- Bernoulli's principle is applied to devices like spray pumps, bunsen burners and carburetors, etc.
- Damage caused by high speed winds to buildings can be understood in terms of Bernoulli's principle.

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