

Answers

Module 1 System of Particles

A.7.1.1 (C)

$\frac{m_e m_p}{m_e + m_p}$ is less than both m_e and m_p

A.7.1.2 (D)

$$\mu < m_e < m_p = \frac{M_s M_E}{M_s + M_E}, \mu < M_s, \mu < M_E$$

$\mu < m_e < m_p = \left(\frac{1}{M_E} + \frac{1}{M_s}\right)$, which is the harmonic mean between M_s and M_E .

A.7.1.3 (C)

Motion of the centre of mass is affected only by an external unbalanced force.

A.7.1.4 (D)

Let the length of each side of the square hole be x .

$$\therefore x^2 + x^2 = a^2$$

$$\text{Or, } x^2 = \frac{a^2}{2}$$

$$\text{Let } \sigma = \frac{\text{mass}}{\text{area}}$$

$$\therefore \text{Mass of original lamina} = \pi a^2 \sigma$$

∴ Mass of the remaining lamina

$$= \left(\pi a^2 - \frac{a^2}{2} \right) \sigma = \left(\pi - \frac{1}{2} \right) a^2 \sigma$$

The required distance d will be given by:

$$\left(\pi - \frac{1}{2} \right) a^2 \sigma d = \frac{a^2}{2} \sigma \frac{a}{2}$$

$$\therefore d = \frac{a}{4\pi - 2}$$

A.7.1.5 (C)

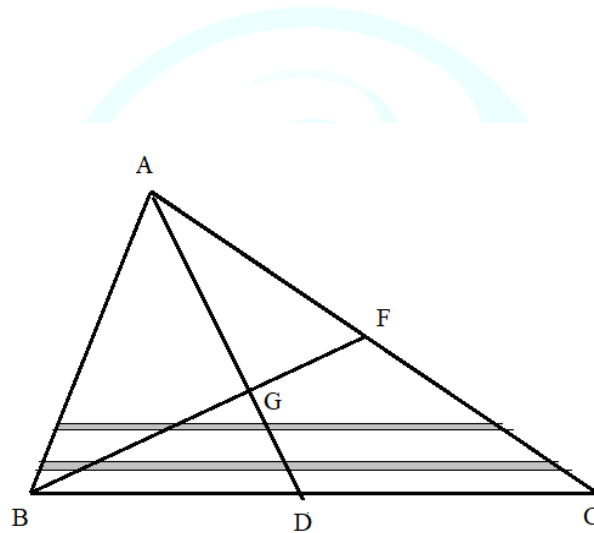


Fig. A.7.1.1

The triangular lamina is cut into multiple infinitesimally thin rectangular strips, each parallel to the base, BC. The centre of mass of each such strip lies on the respective mid-point. So the centre of mass of the triangular lamina which is a combination of such strips lies on the join of the mid-points. So it lies on the median AD. If a similar exercise is done with strips parallel to AC, it will be found that the centre of mass. will be on the median BF. So, the centre of mass will be located at the point of intersection of the medians, that is the centroid.

A.7.1.6 (B) Refer to Fig.A.7.1.2

The coordinates of the centre of mass are

$$\bar{x} = \frac{\int x dm}{\int dm},$$

$$\bar{y} = \frac{\int y dm}{\int dm},$$

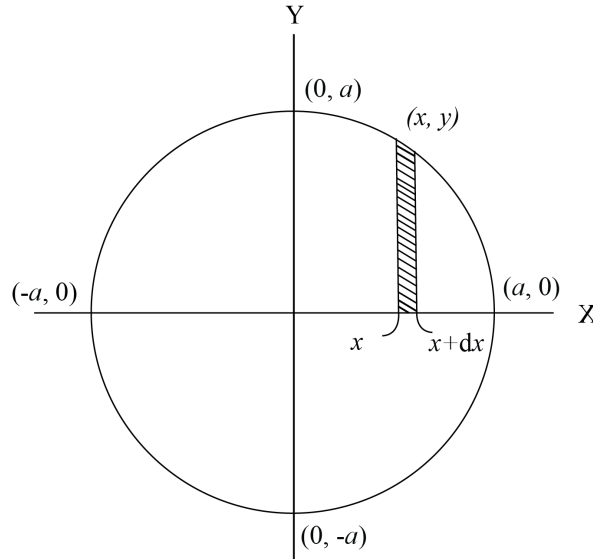


Fig. A.7.1.2

where dm is the mass of the elementary strip contained between x and $x + dx$ about the point (x, y) on the circle. If σ is the mass per unit area of the circular lamina, then $dm = \sigma y dx$.

The integrals extend between the limits of the coordinates of the endpoints of the quadrant.

$$\therefore \bar{x} = \frac{\int xy dx \sigma}{\int y dx \sigma} = \frac{\int xy dx}{\int y dx} = \frac{I_1}{I_2},$$

$$\text{where } I_1 = \int xy dx = \int_a^0 x \sqrt{a^2 - x^2} dx$$

$$\text{And } I_2 = \int y dx = \int_a^0 \sqrt{a^2 - x^2} dx$$

$$x = a \cos \theta, dx = -a \sin \theta d\theta$$

$$\therefore a^2 - x^2 = a^2 - a^2 \cos^2 \theta = a^2 \sin^2 \theta$$

$$\therefore \sqrt{a^2 - x^2} = a \sin \theta$$

$$\text{When, } x = 0, \cos \theta = 0, \theta = \frac{\pi}{2}$$

$$x = a, \cos \theta = 1, \theta = 0$$

$$\therefore I_1 = \int_0^{\pi/2} (a \cos \theta)(a \sin \theta)(-a \sin \theta) d\theta$$

$$= -a^3 \int_0^{\pi/2} \sin^2 \theta \cos \theta d\theta$$

$$\text{Let } \sin \theta = u, \text{ so, } + \cos \theta d\theta = du$$

$$\text{When } \theta = 0, u = 0$$

$$\theta = \frac{\pi}{2}, u = 1$$

$$\therefore I_1 = -a^3 \int_0^1 u^2 du = -a^3 \left(\frac{1}{3} \right) = -\frac{a^3}{3}$$

$$I_2 = \int_a^0 \sqrt{a^2 - x^2} dx = \int_0^{\pi/2} (a \sin \theta)(-a \sin \theta d\theta)$$

$$= -a^2 \int_0^{\pi/2} \sin^2 \theta d\theta = -a^2 \frac{\pi}{4}$$

$$\bar{x} = \frac{I_1}{I_2} = \frac{-\frac{a^3}{3}}{-a^2 \frac{\pi}{4}} = \frac{4a}{3\pi}$$

$$\bar{y} = \frac{\int y \sigma y \, dx}{\int y \, dx \, \sigma} = \frac{\int y^2 \, dx}{\int y \, dx} = \frac{J_1}{J_2},$$

$$\text{Where } J_1 = \int y^2 \, dx = \int_a^0 \sqrt{a^2 - x^2} \, dx$$

$$\text{And } J_2 = \int y \, dx = I_2 = -a^2 \frac{\pi}{4}$$

$$\begin{aligned} J_1 &= - \int_0^a (a^2 - x^2) \, dx = - \left(a^2 \cdot a - \frac{a^3}{3} \right) \\ &= - \frac{2a^3}{3} \end{aligned}$$

$$\bar{y} = \frac{J_1}{J_2} = \frac{-\frac{2a^3}{3}}{-a^2 \frac{\pi}{4}} = \frac{8a}{3\pi}$$

So, the coordinates of C.M. are $(\frac{4a}{3\pi}, \frac{8a}{3\pi})$.

A. 7.1.7 (B)

$$m_1 \vec{V}_1' + m_2 \vec{V}_2' = 0$$

$$\therefore \vec{V}_2' = -\frac{m_1}{m_2} \vec{v}_1 = -\frac{1}{2} \cdot 2\hat{AB} \, \text{ms}^{-1} = 1 \, \text{ms}^{-1} \text{ along } BA$$

A. 7.1.8 (D)

$$K = K' + \frac{1}{2} M v^2 = 15 + \frac{1}{2} (1 + 2) \cdot 2^2 = 15 + 6 = 21 \, \text{J}$$

A.7.1.9 (A)

The original path of the bomb is parabolic. Since there is no external force due to the explosion, the centre of mass will continue to follow the original path of the projectile.

A.7.1.10 (C)

$$\mathbf{r}_{cm} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2}, \mu = \frac{m_1 + m_2}{m_1 + m_2}$$

Let $m_1 \gg m_2$

$$\mathbf{r}_{cm} = \frac{r_1 + \frac{m_2}{m_1} r_2}{1 + \frac{m_2}{m_1}}, \mu = \frac{m_2}{1 + \frac{m_2}{m_1}}$$

As $m_1 \gg m_2$, $\frac{m_2}{m_1} \rightarrow 0$

$\therefore \mathbf{r}_{cm} \rightarrow \mathbf{r}_1$ and $\mu \rightarrow m_2$

Module 2 : Kinematics of Rotational Motion

Ans 7.2.1 (D)

$$\omega_1 = \frac{2\pi}{60} \text{ rad s}^{-1} \quad \omega_2 = \frac{2\pi}{60 \times 60} \text{ rad s}^{-1} \quad \omega_3 = \frac{2\pi}{12 \times 60 \times 60} \text{ rad s}^{-1}$$

$$\therefore \omega_1 : \omega_2 : \omega_3 = 720 : 12 : 1$$

Ans 7.2.2 (C) The minute hand rotates with a uniform angular speed equal to $\frac{2\pi}{60 \times 60} \text{ rad s}^{-1}$ about an axis perpendicular to the plane of the wrist watch and passing through its centre. Every point on the minute hand has the same angular velocity.

Ans 7.2.3 (C)

The axis is fixed, so the direction of angular velocity does not change but its magnitude may change. Hence (C) is correct.

Ans 7.2.4 (B)

$$\frac{\omega - \omega_o}{t} = \alpha \quad \therefore \omega = \omega_o + \alpha t. \quad \text{Hence (q) is correct.}$$

Again, angle described = (Average angular velocity) $\times$ time

$$\therefore \theta = \frac{\omega - \omega_o}{2} t, \quad \therefore \frac{2\theta}{t} = \omega + \omega_o$$

$$\omega = \frac{2\theta}{t} - \omega_o \quad \text{which is (p)}$$

Ans 7.2.5 (A)

Angle subtended by an arc at any point on the circumference of a circle is half the angle subtended at the centre.

Ans 7.2.6 (A)

$$\text{The required angle} = \omega_o \cdot 10 + \frac{1}{2} \alpha \cdot 10^2 - \omega_o \cdot 9 + \frac{1}{2} \alpha \cdot 9^2 = \omega_o + \frac{\alpha}{2} (100 - 81) = \omega_o + (9.5) \alpha$$

Ans 7.2.7 (D)

$$\omega_o \cdot 2 + \frac{1}{2} \alpha \cdot 2^2 = 8\pi$$

$$\omega_o \cdot 6 + \frac{1}{2} \alpha \cdot 6^2 = 30\pi$$

$$\text{i.e. } \omega_o + \alpha = 4\pi$$

$$\omega_o + 3\alpha = 5\pi$$

$$2\alpha = \pi$$

$$\therefore \alpha = \frac{\pi}{2}$$

Ans 7.2.8 (C)

$$\omega_o = 4\pi - \frac{\pi}{2} = \frac{7\pi}{2}$$

$\therefore$ Angle described in 10 seconds

$$= \frac{7\pi}{2} \cdot 10 + \frac{1}{2} \cdot \frac{\pi}{2} \cdot 10^2$$

$$= 35\pi + 25\pi = 60\pi$$

$$\therefore \text{Number of revolutions} = \frac{60\pi}{2\pi} = 30$$

Ans 7.2.9 (B)

Angle described = (Average angular velocity) time

$$\therefore 60\pi = \frac{\omega_o + \omega_1}{2} \cdot 10 \text{ s}$$

$$\therefore 12\pi = \omega_o + \omega_1 = \frac{7\pi}{2} + \omega_1$$

$$\therefore \omega_1 = 12\pi - \frac{7\pi}{2} = \frac{17\pi}{2} = 8.5\pi.$$

Ans 7.2.10 (B)

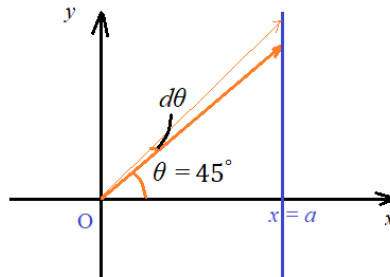


Fig. A.7.2.1

$$\omega = \frac{d\theta}{dt} = \frac{1}{1 + \frac{y^2}{a^2}} \frac{d}{dt} \left(\frac{y}{a} \right)$$

$$\therefore \omega = \frac{a^2}{a^2 + y^2} \frac{1}{a} \frac{dy}{dt}$$

$$= \frac{a}{a^2 + a^2 \theta} u = \frac{u}{a\theta} .$$

Module 3 : Dynamics of Rotational Motion

Ans 7.3.1 (A)

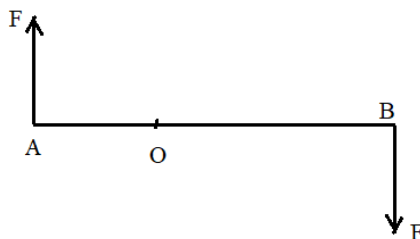


Fig. A.7.3.1

Let O be the point about which the moment of the couple ($\mathbf{F}, \mathbf{F}$) is being considered (Fig. A.7.3.1).

Moment about O is equal to

$$\mathbf{r}_1 \times (-\mathbf{F}) + \mathbf{r}_2 \times (\mathbf{F}) = (-\mathbf{r}_2 - \mathbf{r}_1) \times \mathbf{F} = \mathbf{AB} \times \mathbf{F},$$

that is independent of the choice of origin O.

Ans 7.3.2 (C)

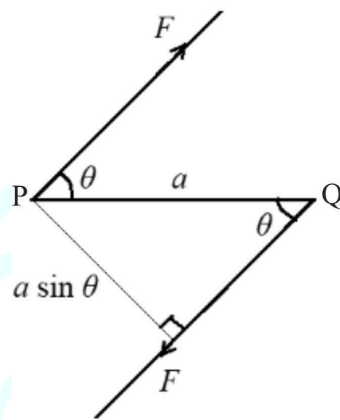


Fig. A.7.3.2

$$A = Fa \sin \theta$$

$$B = Fa \sin(90^\circ - \theta) = Fa \cos \theta$$

$$\therefore A^2 + B^2 = F^2 a^2 (\sin^2 \theta + \cos^2 \theta)$$

$$\therefore A^2 + B^2 = (Fa)^2$$

$$\text{or, } Fa = \sqrt{A^2 + B^2}$$

Ans 7.3.3 (B)

Moment of inertia about each diameter is the same on account of symmetry. Let us take two perpendicular diameters, and let the corresponding MIs be I_1 and I_2

Then,

$$I_1 = I_2$$

And, from the perpendicular axis theorem,

$$I_1 + I_2 = \frac{1}{2}MR^2$$

$$\therefore I_1 = I_2 = \frac{1}{2} \cdot \frac{1}{2}MR^2$$

$$= \frac{1}{4}MR^2$$

Ans 7.3.4 (C)

$$\alpha = \frac{\text{Torque}}{MI} = \frac{FR}{\frac{1}{2}MR^2} = \frac{2F}{MR}$$

Ans 7.3.5 (B)

MI of the child and the seat about the axis of rotation = $(m_1 + m_2)R^2$

$$\therefore \alpha = \frac{FR}{\frac{1}{2}MR^2 + (m_1 + m_2)R^2} = \frac{2F}{(M + 2m_1 + 2m_2)R}$$

Ans 7.3.6 (C)

$$a = \frac{g \sin \theta}{1 + \frac{k^2}{R^2}}$$

$$\therefore 1 + \frac{k^2}{R^2} = \frac{5}{3}$$

$$\text{or } \frac{k^2}{R^2} = \frac{2}{3}$$

Ans 7.3.7 (D)

$$v^2 = \frac{2g \sin \theta \cdot s}{1 + \frac{k^2}{r^2}}$$

The numerator is the same for all.

For a hollow cylinder, a solid cylinder, a hollow sphere, a solid sphere, the value of $\frac{1}{1 + \frac{k^2}{r^2}}$ are

$\frac{1}{2}$ (= 0.5), $\frac{2}{3}$ (= 0.67), $\frac{3}{5}$ (= 0.6), $\frac{5}{7}$ (= 0.71) respectively.

$$\therefore v_4 > v_2 > v_3 > v_1$$

Ans 7.3.8 (D)

Ans 7.3.9 (A)

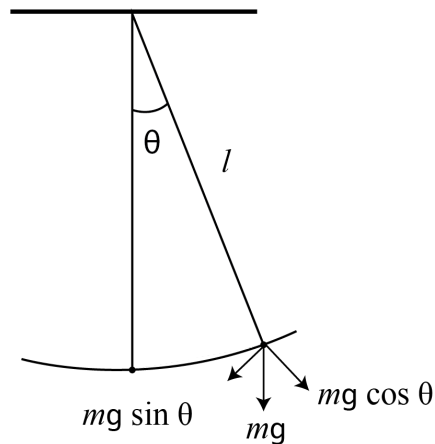


Fig. A.7.3.3

The restoring torque = $mg \sin \theta l$

For a displacement, $d\theta$, the work done = $mg l \sin \theta d\theta$

$\therefore$ **Work done for angular displacement 30° to 60°**

$$= \int_{60^\circ}^{30^\circ} mg l \sin \theta d\theta = - mg l [\cos \theta]_{60^\circ}^{30^\circ} = - mg l (\cos 30^\circ - \cos 60^\circ) = \frac{-mg l}{2} \sqrt{3} - 1 = - (0.366) mg l$$

Ans7.3.10 (C)

$$I_1 \omega_1 = I_2 \omega_2$$

$$\text{or } \omega_2 = \frac{I_1 \omega_1}{I_2}$$

$$I_2 = 0.6 I_1$$

$$\omega_1 = 60 \text{ rev/min} = 1 \text{ rev/s} = 2\pi \text{ rad/s}$$

$$\therefore \omega_2 = \frac{I_1 \times 2\pi}{0.6 I_1} = \frac{10\pi}{3} \text{ rad/s}$$

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