

Thermal Properties of Matter

Module 2 Thermal Expansion

This module is designed to help you to:

- explain thermal expansion of solids, liquids and gases;
- identify thermal expansion of a substance as one of its characteristic properties;
- correlate the coefficients of linear, area and volume expansion;
- apply the concept of thermal expansion in everyday life;
- differentiate between apparent and real expansion of liquids;
- derive the relation between the coefficients of real and apparent expansion of liquids;
- explain the coefficient of volume expansion (α_v) and pressure coefficient of expansion (α_p) for gases.

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11.2.1 Introduction

We know that the solids have definite volume and shape. Any change in temperature leads to a change in its dimensions (length, width, radius, thickness, etc.). An increase in temperature generally causes an increase in the dimensions of the solid. The phenomenon of increase in dimensions of a solid due to an increase in its temperature is referred to as thermal expansion. However, it may so happen that the length of a solid may be large in comparison to its width and thickness, for example, a rod. In such a case we consider only the thermal expansion in the length of the solid or linear expansion. To facilitate comparison between solids of different materials, we take cognizance of their coefficients of thermal expansion.

In solids, the coefficient of linear expansion is defined as the change in length per unit original length for one degree rise in temperature. The coefficients of area and volume expansion of solids are defined in a similar manner.

Liquids are always kept in a container. Therefore, heating a liquid essentially involves heating the container in which it is kept. Thus, the liquid and its container both undergo thermal expansion though it is quite small for the latter. We, therefore, have to take into account the apparent and real expansion of a liquid. The real thermal expansion of a liquid is deduced after accounting for the expansion of the container.

Thermal expansion of gases may occur in two ways unlike in the case of solids and liquids. When the pressure of a gas is kept constant while it expands on heating, its thermal expansion is known as thermal expansion under constant pressure. Volume expansion of a gas may also take place by changing its pressure while keeping its temperature constant. This is known as thermal expansion of the gas under constant temperature. Since the temperature is constant and the expansion in volume of the gas is entirely due to change in pressure, it is not thermal expansion in the true sense. Accordingly, the gases have two coefficients of thermal expansion, one under constant pressure and the other under constant temperature. The relation for coefficient of volume expansion of gases leads us to arrive at the definition of absolute temperature. It is defined as the temperature at which both the coefficient of volume expansion and the coefficient of pressure expansion of a gas becomes zero. This lowest temperature defines the zero on the kelvin scale of temperature.

A change in the temperature of a body results among other things in its dimensions. The phenomenon of increase in the dimensions of a body due to the increase in its temperature is called thermal expansion.

The thermal expansion of solids and liquids has a number of applications in daily life and industry. Some of the applications will be discussed in this module.

11.2.2 Thermal expansion of a solid

(a) Linear expansion

It is the change in any linear dimension of a solid, for example, length, width or thickness. From experiments it is found that change in length Δl depends on the nature of the material, on the original length l and on the small change in its temperature ΔT , we write

$$\Delta l = \alpha_l l \Delta T$$

Here, α_l is called the coefficient of linear expansion and is defined as the change in length per unit original length for one degree rise in temperature. α_l is independent of the unit of length but it depends on the unit of temperature chosen.

The coefficient of linear expansion is a characteristic of a substance, and you will be able to appreciate that by going through the contents of Table 11.2.1, where values of such coefficients for several substances have been listed.

Table 11.2.1: Coefficient of linear expansion for some materials at room temperature

Material	α_l (10^{-5} K^{-1})
Aluminium	2.5
Brass	1.8
Iron	1.2
Copper	1.7
Silver	1.9
Gold	1.4
Glass (pyrex)	0.32
Lead	0.29

(b) Area expansion

Area expansion is the change in any surface of a solid where the coefficient of area expansion α_A is defined as the increase in area per unit area per degree increase in temperature and is given by $\alpha_A = \left(\frac{\Delta A}{A} \right) \frac{1}{\Delta T}$

(c) Volume expansion

It is the change in volume ΔV of a solid for a small change in its temperature, ΔT . The volume expansion of a solid depends on its original volume V , nature of its material and the change in its temperature ΔT . The coefficient of volume expansion α_v is defined as the increase in volume per unit volume ($\Delta V/V$), per unit increase in its temperature ΔT and is given by

$$\alpha_v = \left(\frac{\Delta V}{V} \right) \frac{1}{\Delta T}.$$

The values of the coefficient of volume expansion of some materials are given at Table 11.2.2.

Table 11.2.2: *Coefficients of volume expansion for some materials at room temperature*

Material	α_v (K^{-1})
Aluminium	7×10^{-5}
Brass	6×10^{-5}
Iron	3.55×10^{-5}
Paraffin	58.8×10^{-5}
Glass (ordinary)	2.5×10^{-5}
Glass (pyrex)	1×10^{-5}
Hard Rubber	2.4×10^{-4}
Invar	2×10^{-6}
Mercury	18.2×10^{-5}
Water	20.7×10^{-5}
Alcohol (ethanol)	110×10^{-5}

It is important to note that:

- Units of all the three coefficients α_l , α_A , α_v are reciprocal of temperature, unit in use ($^{\circ}C^{-1}$ or K^{-1}).
- Different solid substances have different values for α_l , α_A , and α_v .
(Table 6.2.1 and Table 6.2.2).
- For an isotropic solid, we have (approximately)

$$\alpha_l = \frac{\alpha_A}{2} = \frac{\alpha_V}{3}$$

- (iv) Values of linear coefficient of some common material in decreasing order are

$$\alpha_{l(\text{Aluminum})} > \alpha_{l(\text{Silver})} > \alpha_{l(\text{brass})} > \alpha_{l(\text{copper})} > \alpha_{l(\text{gold})} > \alpha_{l(\text{iron})} > \alpha_{l(\text{platinum})} > \alpha_{l(\text{glass})} > \alpha_{l(\text{lead})} > \alpha_{l(\text{Invar})},$$

where invar is an alloy of iron and nickel.

- (v) For a crystalline solid, we have the following approximate relation:

$$\alpha_V = \alpha_{l_1} + \alpha_{l_2} + \alpha_{l_3}$$

where $\alpha_{l_1}, \alpha_{l_2}, \alpha_{l_3}$ are linear expansion coefficients in three mutually perpendicular directions.

- (vi) $\alpha_{l(\text{platinum})} = \alpha_{l(\text{glass})} = 8 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$

This is the reason why a platinum wire is sealed in a glass rod for use in flame tests in chemistry laboratories. The platinum wire and the glass rod expand equally.

- (vii) Value of α_l is independent of the unit of length. It depends only on the unit of temperature.

(a) The values are the same in $^\circ\text{C}^{-1}$ and K^{-1}

(b) The value in $^\circ\text{C}^{-1}$ is 1.8 times the value in $^\circ\text{F}^{-1}$.

- (viii) $\alpha_{l(\text{invar})} = 9 \times 10^{-7} \text{ } ^\circ\text{C}^{-1}$ which is very small. So, invar rods are widely used in pendulum clocks to ensure that pendulum rods have no or very small change of their length with change in temperature. This is essential to maintain accuracy in time measurement by pendulum clocks.

11.2.1 Consequences of expansion of solids

- Gaps are left at joints between rails to allow for expansion
- Telephone wires sag more in summer.
- Gaps are left in concrete road surfaces to allow for expansion during summer.
- Expansion of steel bridges — to allow for expansion or contraction one end of a steel bridge is supported on rollers and the other end is fixed.
- Cracking of a glass tumbler — If we pour very hot tea in a tumbler made of thick glass, it cracks because glass is a poor conductor of heat and the inner surface of glass becomes hot and expands while the outer surface being cold does not expand. So the tumbler cracks.
- Pendulum clocks and mechanical watches run slower in summer and
- faster in winters.

11.2.2 Applications of thermal expansion of solids

- (a) Bimetallic strip
- (b) Pendulum clock
- (c) Invariance of differential linear expansion with temperature
- (d) Metallic scale measurement correction
- (e) Density correction
- (f) Temperature and moment of inertia of a body
- (g) Thermal stress

(a) Bimetallic strip

Bimetallic strip is a strip made of two metals (e.g. brass and iron) having different coefficients of linear expansion. These two strips are joined or welded together along their length so that it remains straight at room temperature (Fig. 11.2.1). The strip bends when heated or cooled due to different values of coefficients of linear expansion of the two metals.

If d be the thickness of each component strip, then it can be shown that a straight compound strip will bend into an arc of radius $R = \frac{d}{\Delta T(\alpha_{l_2} - \alpha_{l_1})}$, where ΔT is the change in temperature, from the normal temperature when the strip is straight and if α_{l_1} and α_{l_2} are respective coefficients of linear expansion.

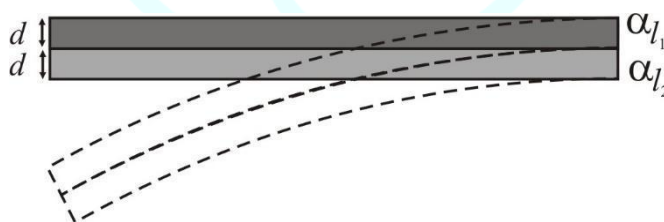


Fig. 11.2.1: Bimetallic strip

The metal with a higher value of α_l will be on the outer side when the strip is heated and on the inner side when the strip is cooled.

Radius of curvature decreases with rise in temperature. Therefore, the strip gets curved more and more with a rise in its temperature. This principle is used to design bimetallic thermometers, thermostats and thermograms.

(b) Pendulum clock

In a pendulum clock, the time period $T = 2\pi\sqrt{\frac{l}{g}}$

With an increase in temperature, there is an increase in length resulting in an increase in T . Therefore, the clock will run slower.

At $t_1^\circ \text{C}$, time period $T_1 = 2\pi\sqrt{\frac{l_1}{g}}$

At $t_2^\circ \text{C}$, time period $T_2 = 2\pi\sqrt{\frac{l_2}{g}}$

$$\begin{aligned}\text{Then } \frac{T_2}{T_1} &= \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{l_1[1+\alpha_l(t_2-t_1)]}{l_1}} \\ &= \sqrt{1 + \alpha_l(t_2 - t_1)} \\ \text{or, } \frac{T_2}{T_1} &= 1 + \frac{1}{2}\alpha_l(t_2 - t_1).\end{aligned}$$

For small values of $\alpha_l(t_2 - t_1)$, the higher order terms can be neglected. We get,

$$\begin{aligned}\frac{T_2 - T_1}{T_1} &= \frac{1}{2}\alpha_l(t_2 - t_1) \\ &= \frac{\Delta T}{T} = \frac{\text{Change in time period}}{\text{Original time period}}\end{aligned}$$

1 day = 86400 seconds. Therefore,

$$\begin{aligned}\text{loss or gain of time by the clock per day} &= \frac{\Delta T}{T} \times 86400 \\ &= [43200 \times \alpha_l \times (t_2 - t_1)] \text{ s.}\end{aligned}$$

(c) Invariance of differential linear expansion with temperature

Condition for two rods of different materials to have same difference in length at any temperature is $\alpha_i l_i = \text{constant}$, ($i = 1, 2$) as shown below.

Let the difference in lengths of two rods of different materials at a particular temperature be $l_2 - l_1$.

When temperature gets raised by $t^\circ \text{C}$, then l_1 becomes $(l_1 + \alpha_{l_1} l_1 t)$ and l_2 becomes $(l_2 + \alpha_{l_2} l_2 t)$.

Difference in lengths of the rods for a rise $t^\circ \text{C}$ in temperature

$$\begin{aligned} &= l_2 + \alpha_{l_2} l_2 t - l_1 - \alpha_{l_1} l_1 t \\ &= (l_2 - l_1) + (\alpha_{l_2} l_2 t - \alpha_{l_1} l_1 t) \end{aligned}$$

For the difference to be independent of temperature, we have

$$(l_2 - l_1) = (l_2 - l_1) + (\alpha_{l_2} l_2 t - \alpha_{l_1} l_1 t)$$

$$\alpha_{l_1} l_1 = \alpha_{l_2} l_2$$

Thus, the length of a rod should be inversely proportional to the coefficients of linear expansion of its material. This forms the basis for the design of gridiron pendulum clocks by British clockmaker John Harrison around 1726 that were later modified by John Ellicott. We have learnt that the time period of the pendulum clocks depends on its length. A change in temperature slows down or increases the time period of the pendulum of a clock leading to inaccurate timekeeping. The grid iron pendulum consists of alternating parallel rods of two metals with different coefficients of thermal expansion, for example, steel and brass. In a gridiron pendulum, rods are connected by a frame in such a way that their differential thermal expansions (or contractions) compensate for each other, so that the overall length of the pendulum, and thus its time period, does not vary with temperature.

(d) Metallic scale measurement correction

Metallic scales are calibrated at a particular temperature for which the scale is correct. When the temperature is raised, the gap between the successive marks increases though the scale looks the same.

When metallic scale is correct at a lower temperature $t_1^\circ \text{C}$ and is being used to measure an object at a higher temperature $t_2^\circ \text{C}$, then

$$l_{\text{true}} = l_{\text{observed}} [1 + \alpha_l (t_2 - t_1)]$$

For a metallic scale, correct at a higher temperature $t_1^\circ \text{C}$ being used at a lower temperature $t_2^\circ \text{C}$, we have

$$l_{\text{true}} = l_{\text{observed}} [1 - \alpha_l (t_2 - t_1)]$$

(e) Density correction

Let density of a solid at 0°C and at $t^\circ \text{C}$ respectively be

$$\rho_0 = \frac{m}{V_0} \quad \text{and} \quad \rho_t = \frac{m}{V_t}$$

$$\text{Then } \frac{\rho_0}{\rho_t} = \frac{V_t}{V_0} = \frac{V_0(1+\alpha_v t)}{V_0} = (1 + \alpha_v t)$$

$$\text{Or, } \rho_0 = \rho_t(1 + \alpha_v t)$$

$$\rho_t = \frac{\rho_0}{(1+\alpha_v t)} \cong \rho_0(1 - \alpha_v t)$$

As $\alpha_v t$ is small, the higher order terms are neglected.

Generally, α_v is a small positive number, therefore, the density of a substance decreases with increase in temperature.

Correction due to variation of density with temperature is essentially used in the pharmaceutical, chemical, petrochemical industry as well as in the food and beverage industry, where a small change in density of ingredients can significantly affect the quality of a product.

(f) Temperature and moment of inertia of a body

Moment of inertia of a body depends on its mass and its dimensions e.g. radius, length, etc. When temperature increases, its dimensions increase (though its mass remains constant) and hence the moment of inertia also increases. We find that in general, for a body of any shape, we have the following approximate relation,

$$I_t = I_0(1 + 2\alpha_l t),$$

where I_t and I_0 are the moments of inertia of the body at $t^\circ \text{C}$ and 0°C respectively

For a rectangular bar of mass M at 0°C , the moment of inertia about its axis of symmetry, i.e; perpendicular to its base and passing through its centre of mass is given by,

$$I_0 = M \left(\frac{l_0^2 + b_0^2}{12} \right)$$

$$\begin{aligned} \text{At } t^\circ \text{C, } I_t &= M \left(\frac{l_t^2 + b_t^2}{12} \right) \\ &= \frac{M}{12} [l_0^2(1 + \alpha_l t)^2 + b_0^2(1 + \alpha_l t)^2] \\ &= \frac{M}{12} [(l_0^2 + b_0^2)(1 + 2\alpha_l t + \alpha_l^2 t^2)] \end{aligned}$$

Since, $\alpha_l t$ is very small, the squared term is neglected, and we get,

$$I_t = \frac{M}{12} \left[(l_0^2 + b_0^2) (1 + 2\alpha_l t) \right] = I_0 (1 + 2\alpha_l t)$$

(g) Thermal stress

Let us suppose that the ends of a rod are rigidly fixed so as to prevent its expansion or contraction. But if the temperature is increased or decreased, there will be a tendency of expansion or contraction and this tendency will be opposed due to the elastic nature of the substance. Thus, tensile or compressive stress will be set up in the rod. The stress developed is called thermal stress. Let us explore it further.

Consider a rod of length L_0 . If its temperature is lowered by t °C, its length will become $(L_0 - \Delta L)$ where $\Delta L = \alpha_l L_0 t$ and α_l is the coefficient of linear expansion of rod material.

If the rod is not allowed to contract by its rigidly fixed ends, it will develop an elastic tensile stress. The

$$\text{strain} = \frac{\Delta L}{L_0}$$

$$\text{stress} = Y \frac{\Delta L}{L_0}$$

$$\text{But } \Delta L = \alpha_l L_0 t, \text{ then}$$

$$\text{Thermal stress} = Y \frac{\alpha_l L_0 t}{L_0} = \alpha_l Y t$$

Thus, the force on supports = Area of cross section of rod $\times$ thermal stress

$$\text{or, } F = A \alpha_l Y t$$

The same expression applies when the rod is heated through t °C but prevented from expanding with stress being compressed.

Some other applications of thermal expansion of solids are like fixing an iron rim on a wooden wheel and removing a tight metal stopper from a glass bottle.



Let Us Reflect

Q.11.2.1 Length of an iron bar at 273 K is 500 mm. The change in length of the bar on raising its temperature to 40°C is (nearly)

Coefficient of linear expansion of iron = $11 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$

- (A) $+ 2.2 \times 10^{-2}$ mm
- (B) $- 2.2 \times 10^{-2}$ mm
- (C) $+ 2.2 \times 10^{-1}$ mm
- (D) $- 2.2 \times 10^{-1}$ mm

Q.11.2.2 A pendulum clock is a second's pendulum at 10°C . Temperature on a hot summer day is 40°C . The coefficient of linear expansion of the material of the pendulum is $3 \times 10^{-5} \text{ K}^{-1}$.

In one day the clock (nearly)

- (A) gains 7.8 s
- (B) loses 7.8 s
- (C) gains 3.9 s
- (D) loses 3.9 s

Q.11.2.3 A wire of radius 2 mm and length 20 m is rigidly attached at its end to two pillars. The wire is taut at 20°C . The temperature of the surroundings decreases to -10°C . Young's modulus and the coefficient of linear expansion of the material of the wire respectively are $1.2 \times 10^{11} \text{ Nm}^{-2}$ and $2 \times 10^{-5} \text{ K}^{-1}$. The stress developed on pillar is

- (A) $72 \times 10^5 \text{ Nm}^{-2}$
- (B) $144 \times 10^5 \text{ Nm}^{-2}$
- (C) $72 \times 10^6 \text{ Nm}^{-2}$
- (D) $144 \times 10^6 \text{ Nm}^{-2}$

Q.11.2.4 R_0 is the radius of a ring and a disc of the same material at 0°C . R_1 and R_2 are the corresponding radii of the ring and disc at 100°C . Coefficient of linear expansion of material $= 2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ $\frac{R_1 - R_2}{R_0}$ is nearly,

- (A) $- 2 \times 10^{-6}$
- (B) $+ 2 \times 10^{-6}$
- (C) $- 2 \times 10^{-7}$
- (D) $+ 2 \times 10^{-7}$



Answers

A.11.2.1 (C)

A.11.2.2 (D)

A.11.2.3 (C)

A.11.2.4 (A)

11.2.3 Thermal expansion of liquids

We have learnt that liquids expand more than solids. Also, liquids do not have a length or area but they have a definite volume (no definite shape) and hence they exhibit expansion only in volume.

11.2.3.1 Apparent expansion and real expansion

A liquid can be heated only when it is taken in a container made of solid material. On heating, both liquid as well as the container expand. This leads us to two different observations of expansion of the liquid -

(a) **apparent expansion**

(b) **real expansion**

(a) **Apparent volume expansion:** Apparent expansion of a liquid is the observed change in the volume of the liquid. It does not take into account the expansion of the container.

(b) **Real volume expansion:** Real expansion of a liquid is the actual change in the volume of the liquid if there has been no change in the volume of the container.

Real expansion = Apparent expansion + Increase in volume of the container.

Real expansion is a characteristic property of the liquid.

For a liquid in a glass container, we have

$$\Delta V_r = \Delta V_a + \Delta V_{glass} \text{ and}$$

$$\alpha_{V_{real}} = \alpha_{V_{apparent}} + \alpha_{V_{glass}}.$$

ACTIVITY 11.2.2**Apparent expansion and real expansion**

Let us perform an activity to study the two different expansions of a liquid.

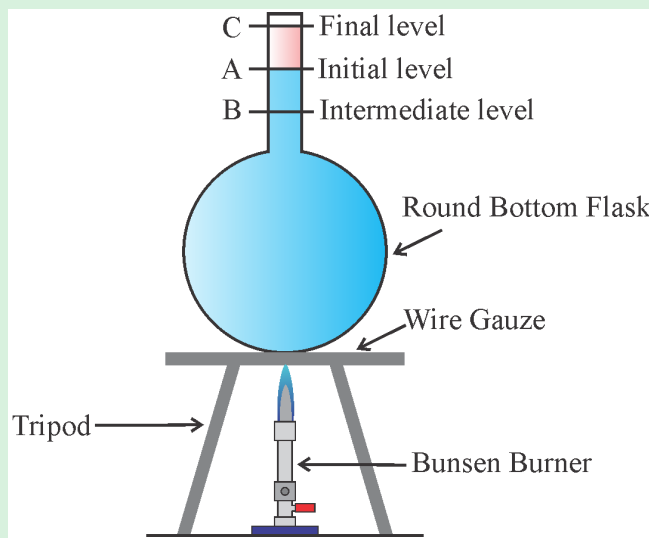


Fig.11.2.2: *Apparent expansion and real expansion*

- Take water or any other liquid in a round-bottom flask with a narrow graduated neck as shown (Fig.11.2.2).
- Pour the liquid in the flask upto the mark A as shown.
- Heat the container and observe carefully.
- The liquid level initially comes down from mark A to mark B and is then seen to rise to the mark C.
- The initial fall in the liquid level is due to the increase in volume of the container.
- Later, the expansion of the liquid raises its level to mark a mark C above A.

The apparent expansion of the liquid is from mark A to mark C. The real expansion is from mark B to mark C. The expansion of the container is represented by the volume between A and B.

We have,

$$BC = AC + AB, \text{ or}$$

Volume between B and C = Volume between A and C + Volume between A and B. Therefore,
Real Expansion of the liquid = Apparent Expansion of the liquid + expansion of the container.

Based on the above, we define coefficients of volume expansion of liquid as follows:

i) Coefficient of apparent volume expansion

$$(\alpha_V)_{\text{apparent}} = \frac{\Delta V_a}{V_a t} \text{ where } \Delta V_a \text{ is the apparent increase in volume of the liquid.}$$

ii) Coefficient of real volume expansion

$$(\alpha_V)_{\text{real}} = \frac{\Delta V_r}{V_0 t} \text{ where } \Delta V_r \text{ is the real increase in volume of the liquid.}$$

$$\text{We have } (\alpha_V)_{\text{real}} = (\alpha_V)_{\text{apparent}} + (\alpha_V)_{\text{container}}.$$

$(\alpha_V)_{\text{container}}$ is the coefficient of volume expansion of the material of the container.



Let Us Reflect

Q11.2.5 α_V is the coefficient of volume expansion of water. Which of the following statements is correct?

- (A) α_V is a positive number from 0 °C to 100 °C
- (B) α_V is a negative number from 0 °C to 4 °C
- (C) α_V is a positive number from 0 °C to 40 °C
- (D) α_V is a negative number from 4 °C to 100 °C



Answers

A.11.2.5 (B)

11.2.3.2 Consequences of expansion of liquid

(a) Change in density of liquid with temperature

Density change is related to the coefficient of real expansion of liquid.

With symbols having their usual meanings, $\rho_1 V_1 = \rho_2 V_2$

$$\therefore \rho_2 = \frac{\rho_1 V_1}{V_2} = \frac{\rho_1 V_1}{V_1 [1 + (\alpha_v)_{real} (t_2 - t_1)]}$$

$$\text{Or, } \rho_2 = \frac{\rho_1}{[1 + (\alpha_v)_{real} (t_2 - t_1)]}$$

$$\begin{aligned} \text{We get, } \rho_t &= \frac{\rho_0}{\left(1 + (\alpha_v)_{real} t\right)} = \left(1 + (\alpha_v)_{real} t\right)^{-1} \\ &\cong \rho_0 \left(1 - (\alpha_v)_{real} t\right) \quad [\text{Using binomial expansion}] \end{aligned}$$

(b) Apparent loss in weight of a solid immersed in a liquid at different temperatures

From Archimedes' principle, apparent loss in weight of a solid immersed in a liquid is equal to the weight of the displaced liquid.

Loss in weight of a solid in a liquid at 0 °C is $W_0 = V_0 \rho_0 g$

where V_0 = volume of solid at 0 °C, ρ_0 = density of liquid at 0 °C and g = acceleration due to gravity

ρ Volume of solid at t °C is

$$V_t = V_0 (1 + \alpha_v t)$$

where α_v = coefficient of volume expansion of solid

Loss in weight of solid in a liquid at t °C is

$$\begin{aligned} W_t &= V_t \rho_t g = V_0 (1 + \alpha_v t) \cdot \rho_0 \left(1 - (\alpha_v)_{real} t\right) g \\ &= V_0 \rho_0 g (1 + \alpha_v t) [1 - (\alpha_v)_{real} t] \\ &= W_0 (1 + \alpha_v t) [1 - (\alpha_v)_{real} t] \end{aligned}$$

$$\therefore \frac{\text{loss of weight at } 0^\circ\text{C}}{\text{Loss of weight at } t^\circ\text{C}} = \frac{W_0}{W_t} = \frac{1}{(1 + \alpha_v t)(1 - (\alpha_v)_{real} t)} = \frac{1}{[1 - \{(\alpha_v)_{real} - \alpha_v\}t]}$$

[Neglecting higher terms of $(\alpha_v)_{real} - \alpha_v$]

$$\therefore W_t = W_0 [1 - \{(\alpha_v)_{real} - \alpha_v\}t]$$

Loss in weight of a solid in a liquid at higher temperature is less than loss of weight at lower temperature.

Since, $(\alpha_v)_{real} > \alpha_v$, the weight of a solid immersed in a liquid will increase with the rise of the temperature

of the liquid.

(c) Correction of the reading of a barometer for expansion of mercury (Hg) and the metallic scale

In a barometer, the pressure is expressed in terms of height of the Hg column measured by a brass scale corrected at 0 °C. Now the observed height is reduced to equivalent height at 0 °C.

(i) Scale correction

For t °C rise in temperature 1 cm of correct scale becomes $(1 + \alpha_l t)$ where α_l is the coefficient of linear expansion of brass. If h_t be the height of Hg column recorded by the increased (elongated) scale, then corresponding correct scale at 0 °C is

$$h_0 = h_t (1 + \alpha_l t), \text{ where } h_0 \text{ is the corrected length at } t \text{ °C.}$$

(ii) Density correction

Since h_0 is the corrected height of Hg column at t °C, if H be the corresponding height at 0 °C, then,

$$H \rho_0 g = h_0 \rho_t g$$

$$\text{or } H = h_0 \frac{\rho_t}{\rho_0}$$

$$\text{or } H = \frac{h_0}{1 + (\alpha_v)_{real} t} \cong h_0 \left(1 - (\alpha_v)_{real} t \right) = h_1 (1 + \alpha_l t) \left(1 - (\alpha_v)_{real} t \right),$$

where α_l is the coefficient of linear expansion of the material (say brass) of the barometer scale concerned. Hence,

$$H = h_1 \left(1 - \left\{ (\alpha_v)_{real} - \alpha_l \right\} t \right)$$

This equation gives the height of barometer after necessary corrections for the brass scale as well as the mercury column, where $(\alpha_v)_{real}$ is the coefficient of real expansion of mercury.

(d) Exposed system correction of a mercury thermometer

A mercury thermometer is placed with its bulb and lower part of the stem in a liquid and indicates a temperature of t °C. The upper portion of the stem containing 'n' divisions of Hg column is in air at 0 °C. The true temperature T °C of the liquid is that which the thermometer would indicate if it is completely immersed in liquid, then 'n' divisions of the Hg column (presently at θ °C) would be at T °C and would

occupy $n[1 + \alpha(T - \theta)]$ divisions where

α = coefficient of expansion of Hg -in- glass.

Here the corrected length of the exposed portion would be greater than the actual length by number of divisions, equal to $n[1 + \alpha(T - \theta)] - n = n(T - \theta)\alpha$.

Hence, true temperature of the liquid is

$$T = t + n(T - \theta)\alpha$$

For stem immersed up to zero mark, $n = t$

Hence, $T = t [1 + (T - \theta)\alpha]$.



Let Us Reflect

Q.11.2.6 The coefficient of linear expansion of a material is $1.33 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$. The percentage change in its density for a $50 \text{ } ^\circ\text{C}$ increase in temperature is (nearly)

- (A) $+ 0.02\%$
- (B) $+ 0.2\%$
- (C) $- 0.02\%$
- (D) $- 0.2\%$

Q.11.2.7 A metallic scale is correctly marked at 20°C . The length of an object measured at a temperature of $45 \text{ } ^\circ\text{C}$ is 202 mm. The coefficient of linear expansion of the material of the scale is $2 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$. The actual length of the object is nearly

- (A) 202 mm
- (B) 201.81 mm
- (C) 202.18 mm
- (D) 202.36 mm



Answers

A.11.2.6 (D)

A.11.2.7 (C)

11.2.4 Thermal expansion of gases

We know that gases are highly compressible and they completely fill any closed vessel in which they are stored. Also, we know that gases expand much more than both solids or liquids. Therefore, thermal expansion of a vessel enclosing a gas is negligible compared to its expansion.

Gases can expand in two ways – on heating as well as on reducing pressure.

For a gas we normally consider

- (a) volume expansion of a given mass of a gas with temperature, keeping the pressure constant.
- (b) volume expansion of a given mass of a gas with pressure, keeping the temperature constant.

Strictly speaking this is not thermal expansion as the temperature is constant.

The coefficient of expansion of a gas, called its volume coefficient of expansion (α_v), is given by

$$\alpha_v = \frac{V_t - V_0}{V_0 t}$$

$$V_t = V_0(1 + \alpha_v t)$$

If we do not allow for volume expansion of gas with temperature and keep the volume fixed, the pressure of gas increases. The coefficient of expansion is called as pressure coefficient of expansion (α_p) given by

$$\alpha_p = \frac{(P_t - P_0)}{P_0 t}$$

$$\therefore P_t = P_0(1 + \alpha_p t)$$

From above, we have

$$V_t = V_0(1 + \alpha_v t) = V_0 \alpha_v \left(\frac{1}{\alpha_v} + t \right) = V_0 \alpha_v T$$

$$\text{where, } T = \left(\frac{1}{\alpha_v} + t \right)$$

The equation $V_t = V_0 \alpha_v T$, defines a new temperature T which implies that the volume of the gas of a given mass is directly proportional to it, V_t is proportional to T .

Also, $V_t = 0$ when $T = 0$

$$\text{Or, } \frac{1}{\alpha_v} + t = 0$$

$$\text{Then } t = -\frac{1}{\alpha_v} = -273.15^\circ\text{C}$$

Hence, the temperature at which the volume of gas becomes zero must be the lowest attainable temperature and is called absolute zero or 0 kelvin or 0 K. And we get $0\text{ K} = -273.15^\circ\text{C}$. This provides us with another definition of absolute zero of the Kelvin scale.

It can also be shown that P_t is proportional to T and at $T = 0$, pressure exerted will also be zero.

$$\text{Where } t = -\frac{1}{\alpha_p} = -273.15^\circ\text{C}$$

$$\text{We get } \alpha_p = \alpha_v = -\frac{1}{273.15}^\circ\text{C}^{-1} = 0.00366^\circ\text{C}^{-1}$$

Pressure coefficient = Volume coefficient

11.2.4.1 Relation between density of a fluid and temperature

According to Charles' law, for a given mass of a gas,

$$V \propto T$$

$$\text{Or, } \frac{1}{\rho} \propto T$$

This implies that the density of a fluid increases when the temperature decreases and it decreases when the temperature increases.

You have read about gas thermometers at constant volume and constant pressure in the previous module. They are a prolific application of thermal expansion of gases (Pressure law and Charles law).



Let Us Reflect

Q.11.2.8 A glass flask has a volume V_0 . A volume $V_1 (< V)$ of mercury is poured into the flask. The volume of air in the flask remains the same at all temperatures.

Coefficient of linear expansion of glass = $9 \times 10^{-6} \text{ K}^{-1}$

Coefficient of cubical expansion of mercury = $18 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$

V_1 expressed as a percentage of V is

- (A) 5 %
- (B) 7.5 %
- (C) 10 %
- (D) 15 %



Answers

A.11.2.8 (D)



11.2.5 Summary

- Solids, liquids and gases expand on heating and contract on cooling. Expansion on heating is known as thermal expansion.
- Thermal expansion is the least in solids and maximum in gases.
- Coefficient of linear expansion for a solid is defined as the increase in length per unit length per unit rise in temperature.
- Coefficient of area expansion for a solid is defined as an increase in area per unit area per unit rise in temperature.
- Coefficient of volume expansion for a solid is defined as increase in volume per unit volume per unit rise in temperature.
- Apparent expansion of a liquid is the observed change in the volume of the liquid. It does not take into account the expansion of the container.
- Real expansion of a liquid is the actual change in the volume of the liquid if there has been no change in the volume of the container.
- Thermal expansion of liquids leads to variation in (i) density, (ii) apparent loss in weight of a body in a liquid (iii) height of mercury column in a barometer etc. with change in temperature.
- The expansion of a gas is studied (i) with change in temperature at constant pressure, (ii) with change in pressure at constant temperature.

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