

Module 2 Stationary Waves

This module is designed to help you to:

- distinguish between velocity of a wave and the velocity of an individual particle of the medium in which the wave propagates;
- apply the principle of superposition of waves and to understand the formation of stationary waves;
- derive the equation of the standing waves;
- identify the fundamental overtones and harmonics in closed and open organ pipes.

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15.2.1 Introduction

In Module 1 of Waves, we pointed out the difference between vibrations and waves. We learnt to distinguish between longitudinal and transverse waves. We discussed the mechanism of propagation of a mechanical wave and concluded that a progressive wave is just the propagation of phase.

In this module we shall learn about the velocity of a wave and see how it differs from the velocity of the individual particles of the medium.

You must have realised that progressive waves are a very idealised concept which concerns a single wave. The waves we encounter in real life, most common among them being musical notes produced by an orchestra involving several instruments, arise out of the superposition of waves.

In this module we shall learn about superposition of waves, reflection of waves and formation of stationary or standing waves. In the process we shall be able to outline the differences between a progressive and a standing wave. Finally, we shall illustrate the formation of standing waves in a closed and an open organ pipe and discuss the terms fundamental, overtones and harmonics.

15.2.2 Velocity of waves

We have learnt that propagation of a wave is actually propagation of phase. The velocity with which the phase of a wave propagates is the velocity of the wave. Here we shall restrict ourselves to mechanical waves only which require a medium for its propagation and factors responsible are inertia and elasticity of the medium.

We have seen that a wave can either be a transverse wave or a longitudinal wave. Sound waves are longitudinal waves. The velocity of sound waves, in a gas, is given by $v = \sqrt{\frac{\gamma P}{\rho}}$. Here γ is the ratio of the two specific heat capacities (C_p and C_v) of the gas. P is the pressure and ρ is the density of the gas. The numerator represents the elasticity of the medium, whereas the denominator represents the inertia.

How would changes in pressure and density of a gas, such as air, affect the velocity of sound? We note that pressure P , volume V and temperature T of a gas are related as

$$\text{Thus, } PV = Nk_B T.$$

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma N k_B T}{\rho V}} = \sqrt{\frac{\gamma k_B T}{\rho(V/N)}} = \sqrt{\frac{\gamma k_B T}{m}} \quad (15.2.1)$$

We observe that the ratio P/ρ depends only on the temperature of the gas. It follows that, it is the temperature that can affect the velocity of sound in a gas. For an (ideal) gas $\frac{P}{\rho}$ remains unchanged as long as temperature is constant. So, by only changing P the velocity of the wave in a gas does not get altered. However, if we take another gas, with a different value of ρ , the velocity gets changed even though P and T may remain the same..

We notice from Eq(15.2.1) that the velocity of sound also depends on m , the molecular weight of the gas as $\frac{1}{\sqrt{m}}$.

This implies that (at a given temperature) lighter the gas, higher the velocity of sound in that gas.

For example, if we take the case of oxygen and hydrogen, we would have

$$\frac{v_o}{v_H} = \sqrt{\frac{m_H}{m_o}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$

i.e. $v_o = \frac{1}{4}v_H$

The velocity of sound in oxygen is one-fourth of the velocity of sound in hydrogen, under the same external conditions.

When water vapour is present in air, the density of air gets slightly reduced. This is because the molecular weight of a water molecule is smaller than the (average) molecular weight of air. The presence of water vapour (humidity), therefore leads to an increase in the speed of sound.

15.2.3 Velocity of waves in solids and liquids

velocity of transverse waves in a stretched string is given by, $v = \sqrt{\frac{T}{\mu}}$ where T is the tension in the string and μ is the mass per unit length of the string. It means that velocity of transverse waves is more in a thinner string than in a thicker one (of the same material) under equal tension.

velocity of longitudinal waves, in an elastic medium, is given by $v = \sqrt{\frac{E}{\rho}}$, where E is the elasticity of the medium. Since elasticity of solids is higher, the velocity of longitudinal waves (sound waves) is greater in solids than in gases or liquids.

$$\text{i.e. } v_s > v_L > v_G$$

Let us now see how we can determine the velocity of a particle in a progressive wave.

15.2.4 Particle velocity in a progressive wave

Equation of a plane progressive harmonic wave with symbols having their usual meanings, is given by

$$y = A \sin \frac{2\pi}{\lambda}(vt - x) \quad (15.2.2)$$

$$\text{particle velocity} = \frac{\partial y}{\partial t} = U = \frac{2\pi v}{\lambda} A \cos \frac{2\pi}{\lambda}(vt - x) \quad (15.2.3)$$

U = particle velocity, v = wave velocity

slope of the displacement curve of the wave is

$$\frac{\partial y}{\partial x} = -\frac{2\pi}{\lambda} A \cos \frac{2\pi}{\lambda}(vt - x) \quad (15.2.4)$$

From Eqs(15.2.3) and (15.2.4) we get

$$U = \frac{\partial y}{\partial t} = -v \frac{\partial y}{\partial x} \quad (15.2.5)$$

$\therefore$ particle velocity at a point = – (wave velocity) $\times$ (slope of displacement curve at that point) let us now study the superposition of waves.

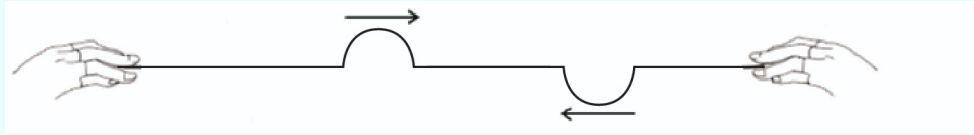
15.2.5 Superposition of waves

We shall study the superposition of waves through some activities

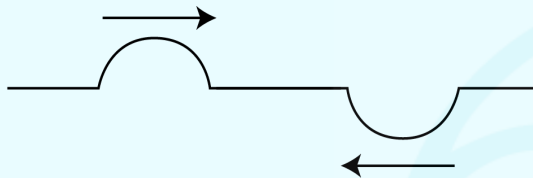
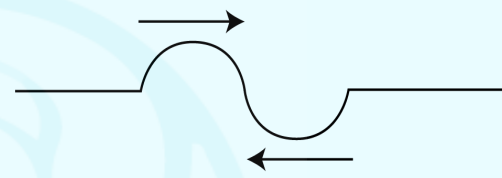
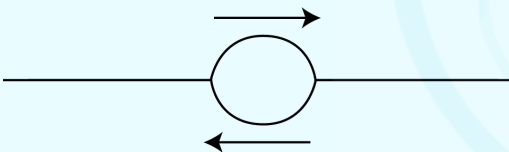
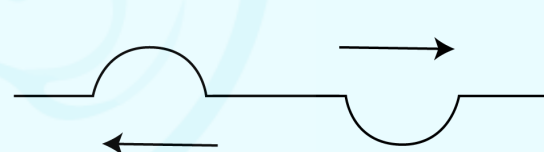
Activity 15.2.1

Refer to Figs. 15.2.1(a), (b), (c), (d) and (e)

Help students generate one pulse in a string in each direction, nearly of the same amplitude.

**Fig.15.2.1(a)**

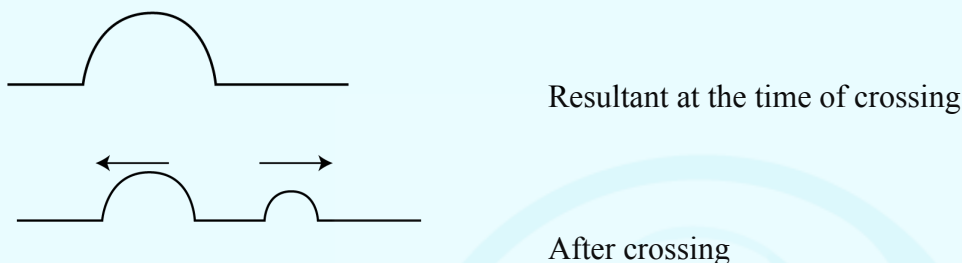
Observe how these two pulses, propagating in opposite directions, cross each other. Do they retain their identities after they have crossed? During the time they overlap, is the wave pattern different from that corresponding to either of the pulses?

**Fig.15.2.1(b)****Fig. 15.2.1(c)****Fig. 15.2.1 (d)****Fig. 15.2.1 (e)**

When the pulses overlap, the resultant displacement is the vector sum of the displacement due to each pulse. This is the principle of superposition. We find that the instantaneous displacement, due to each of the two pulses, can be positive as well as negative. Net displacement is the vector sum of the two. In Fig. 15.2.1(d) we observe that the displacement due to two pulses exactly cancel each other and the net displacement (due to their superposition) is zero at that instant.

Activity: 15.2.2

Let us consider two pulses in the same phase, propagating on a string, in opposite directions as shown in Fig. 15.2.2.

**Fig.15.2.2**

Before and after crossing each other, they act completely independently. At the time of crossing, the resultant displacement is the algebraic sum of the individual displacements.

Using the principle of superposition of waves, we can analyse a complicated wave motion through a set of simple waves.

If two or more independent waves are propagating through a medium, at the same time, then the resultant of any physical quantity (for example, displacement), at any point, is the vector sum of the quantities, due to each individual wave, it is given by

$$y = y_1 + y_2 + y_3 + \text{-----} \quad (15.2.6)$$

If we consider two waves, both propagating along say, x-axis, the resultant displacement, at any point, is the sum of the displacements due to individual waves. It means that two or more waves can traverse the same medium independently of one another. You might have noticed that you can hear the individual notes of various instruments, constituting an orchestra, even though the resultant sound wave reaching our ears is a complex one.

Let $y_1(x, t)$ and $y_2(x, t)$ be the displacements due to the two waves in the medium. If these two waves arrive in a region simultaneously, and overlap, the net displacement is given by:

$$y(x, t) = y_1(x, t) + y_2(x, t) \quad (15.2.7)$$

If a large number of waves are simultaneously present in the medium, then

$$y = \sum_{i=1}^n y_i(x, t) \quad (15.2.8)$$

Consider two waves propagating along the same line. Under certain specific conditions, the superposition of these waves can lead to phenomena like stationary waves, interference, beats, etc. We shall study about stationary waves in this module and about beats in Module 3.

- i. If two identical progressive waves simultaneously pass through a medium, in opposite directions, the resulting wave pattern does not propagate with time, then such a resulting wave is called a stationary wave.
- ii. Two sound waves, of nearly equal frequencies (difference ≤ 10 Hz), propagating simultaneously through a medium, resulting in alternately waxing and waning of sound intensity. This phenomenon is known as 'beats'. The beat frequency is equal to the frequency difference between the two waves.
- iii. Two waves, of the same frequency propagating simultaneously through a medium, in the same direction can result in a redistribution of energy through the medium. This phenomenon is called Interference.

Activity: 15.2.3 Waves in a bucket full of water

This is a very simple activity which illustrates several concepts about waves. You can perform these activities even at home.

Fill a bucket with water up to about 5 cm below the top. When the water surface is calm, let water from the tap fall in it drop by drop; say one drop in about 5 seconds to 10 seconds. As each drop strikes the water surface, ripples are produced on the water surface.

Drop a match stick on the surface of the water. Observe that as the waves propagate, the match stick moves only up and down at its own place. Next, drop two short pieces of a matchstick, a few millimetres long, at points about 1 cm or 2 cm apart, along a radius of the ripples, starting from the point where the drops strike the water surface. How does their motion differ?

Position the bucket such that a drop strikes at the centre of the circular surface of water and produces a pulse moving outwards. This can be checked by seeing that the reflected pulse arrives at the same point where it originated. Estimate by fast counting the time taken by the pulse to travel a distance $2r$, i.e. to reach the boundary, gets reflected and arrives at the origin. If you have an electronic watch with a stopwatch facility, you may be able to find the speed of the pulse quite accurately. If you allow drops of water to fall at regular intervals, the waves produced on the surface of water will also have the same speed. Repeat this activity with shallow water in a dish. Is the speed of the waves in this dish different?

Now let the drops strike the water surface, at a point, a few centimetres away from the centre of the surface. Note the position of the point where the waves come to a 'focus' after reflection from the boundary. How are the three points related: (i) the centre of the surface, (ii) the point where waves start, and (iii) the point where they come to a focus after reflection? Is this behaviour similar to that of light rays and do these waves seem to obey the laws of reflection?

Let us study more about these in the next section.

15.2.6: Reflection of waves

When a wave hits an obstacle, the whole or part of the wave is sent back in the same medium. This is called reflection of waves. The direction of propagation of the wave changes after reflection.

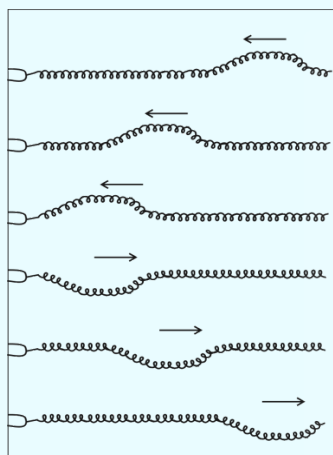
When a wave gets reflected at the boundary of two different media, the reflected wave undergoes a phase change by π if the wave is propagating from a rarer to a denser medium. No change in phase takes place for

reflection of waves going from a denser to a rarer medium. You will examine these with the help of some activities.

15.2.6.1 Reflection and transmission of waves at the boundary of two media

Activity:15.2.4

Stretch a slinky on a smooth floor or suspend it from a rigid support.



(a)



(b)



(c)

Fig. 15.2.3 (a,b,c)

Fig.15.2.3 (a): *Reflection of a pulse*

Fig.15.2.3 (b): *Reflection and transmission of a pulse moving from a rarer to a denser medium*

Fig. 15.2.3(c): *Reflection and transmission of a pulse moving from a denser to a rarer medium*

Keeping one end fixed, send a transverse pulse from the other end. Note the size, and direction of displacement, of the pulse after it gets reflected at the fixed end. Note that the reflected pulse is ‘upside down’, with little change in size, in comparison to the incident pulse Fig. 15.2.3(a).

Next join the coil spring (slinky), end to end, with another long helical spring of heavier mass Fig.15.2.3(b). Stretch them by holding the free end of each spring and produce a pulse at the free end of the lighter spring (slinky). Observe what happens when the pulse arrives at the joint of two springs. In

what way (i.e. with respect to size and direction of displacement) does the reflected pulse undergo a change? Does the pulse, transmitted into the heavier spring, also undergo any change?

Repeat the demonstration by sending the pulse from the free end of the heavier spring. Note how the reflected and transmitted pulse at the boundary of the two springs undergoes a change compared to the incident pulse in Fig. 15.2.3 (c).

How do these changes differ from those in case of incident pulse going from lighter to heavier spring?

You may try another simple activity. Stretch a thread tightly between two reasonably rigid ends. Now collect some sipping straws and fix them on the string in such a way that straws on the left half of the thread are separated by 1 cm each and straws on the right half of the thread are separated by 0.5 cm each as shown in Fig. 15.2.4.

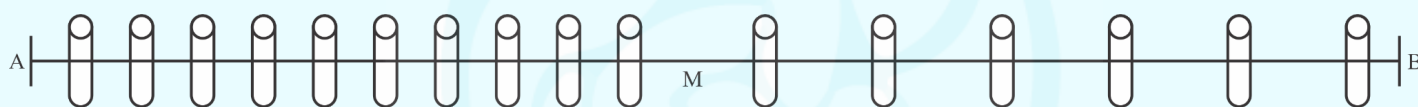


Fig. 15.2.4 A simple model to understand wave propagation

Now make the following observations in respect of travelling of wave pulses, their reflection and transmission while travelling from a rarer medium to a denser medium and vice versa. Note that the closely fixed straws represent a denser medium and the straws with larger distance between them represent a rarer medium.

1. Hit the upper end of the first straw from the right gently with your finger and observe :
 - a) What happens with all other straws?
 - b) Do you observe any change in the nature of movement of the straws on the right and left of M?
2. Repeat the above activity by hitting the upper end of the first straw from the left end in the similar manner.

15.2.6.2 Reflection of longitudinal waves

We will discuss the concept of reflection of waves by observing the motion of a longitudinal wave produced in a spring. Is the behaviour of longitudinal waves on reflection the same as that of the transverse waves, at the fixed end, and at the open end? How about their behaviour when they propagate from a rarer to a denser medium and from a denser to rarer medium?

To explain this, we take some objects (A, B, C, D, and E) of very small mass connected to each other with soft springs, fixed at the end F (Fig.15.2.5).

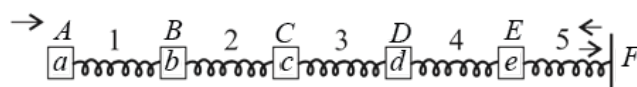


Fig.15.2.5 Longitudinal waves are reflected from a denser medium with same phase but with a reversal of direction of propagation

When we give a small push to A towards the right, the spring 1 gets compressed and pushes B to the right. It then regains its original shape. As spring 1 expands, spring 2 gets compressed. As spring 2 expands spring 3 gets compressed. In this way the compression moves to the last spring, i.e. No.5, which is fixed at the point F. As the spring 5 expands to regain its shape, the object E moves to the left. As a result, spring 4 gets compressed. This process continues till the compression reaches back to the spring 1. We, thus, observe that a compression comes back as a compression after reflection from a rigid support.

Thus, when reflection takes place at a fixed point, or from a denser medium, the longitudinal waves are reflected in the same phase but move in opposite directions. However, on reflection from an open end (rarer medium), a longitudinal wave is reflected back with a change in phase.

Now, how is the amplitude of a wave related to its speed? Let us study that with the help of an activity.

Activity: 15.2.5

Attach a slinky to the wall and generate a pulse of certain amplitude as shown in Fig. 15.2.6.

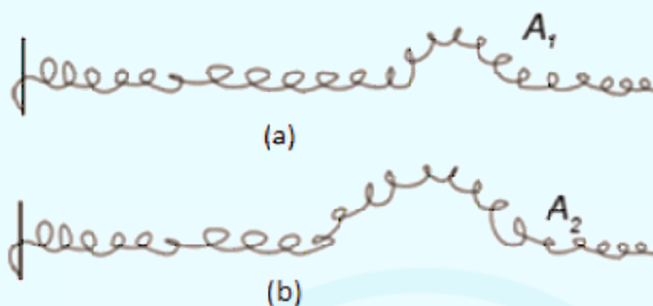


Fig 15.2.6 (a,b) waves on a slinky

Observe a pulse of amplitude A_1 . Now generate another pulse of a larger amplitude A_2 in the same slinky. Now observe the motion of this pulse of amplitude A_2 . Which of these two pulses reach the fixed end earlier? Do they reach the fixed end at the same time? Let students perform this activity to help them arrive at the conclusion.

It can be observed that the amplitude of a wave does not affect its speed.

We shall now study about the formation of stationary or standing waves.

15.2.7 Formation of standing waves

What happens when two exactly identical progressive waves, travelling in opposite directions, superpose on each other? We shall understand this by studying the formation of standing waves in a string. We are showing this in Fig. 15.2.7 step by step.

At time $t = 0$, the incident (solid line) and reflected wave (dotted line) are out of phase by π . The superposition of these results in zero amplitude, making the string straight. After one quarter of a time period ($t = T/4$), the incident wave has advanced in phase by $\frac{\pi}{2}$. The reflected wave has also advanced by the same phase in the

opposite direction. The two waves are now in phase, the result is that the amplitudes of two waves get added and the string takes the shape as shown by the red line (Fig. 15.2.7). At time $t = T/2$, the incident and reflected waves are again out of phase and the string is straight again. At time $t = 3T/4$, the waves have advanced, in phase, by another $\frac{\pi}{2}$ in opposite directions. They are, therefore, again in phase and there is a constructive interference. The shape of the string is shown by the red line. At $t = T$, the waves are again out of phase and the string is again a straight line. Once such a stationary wave gets formed, each segment of the string oscillates as a whole. A and N are the positions of antinodes (maximum amplitude) and nodes (minimum amplitude) respectively.



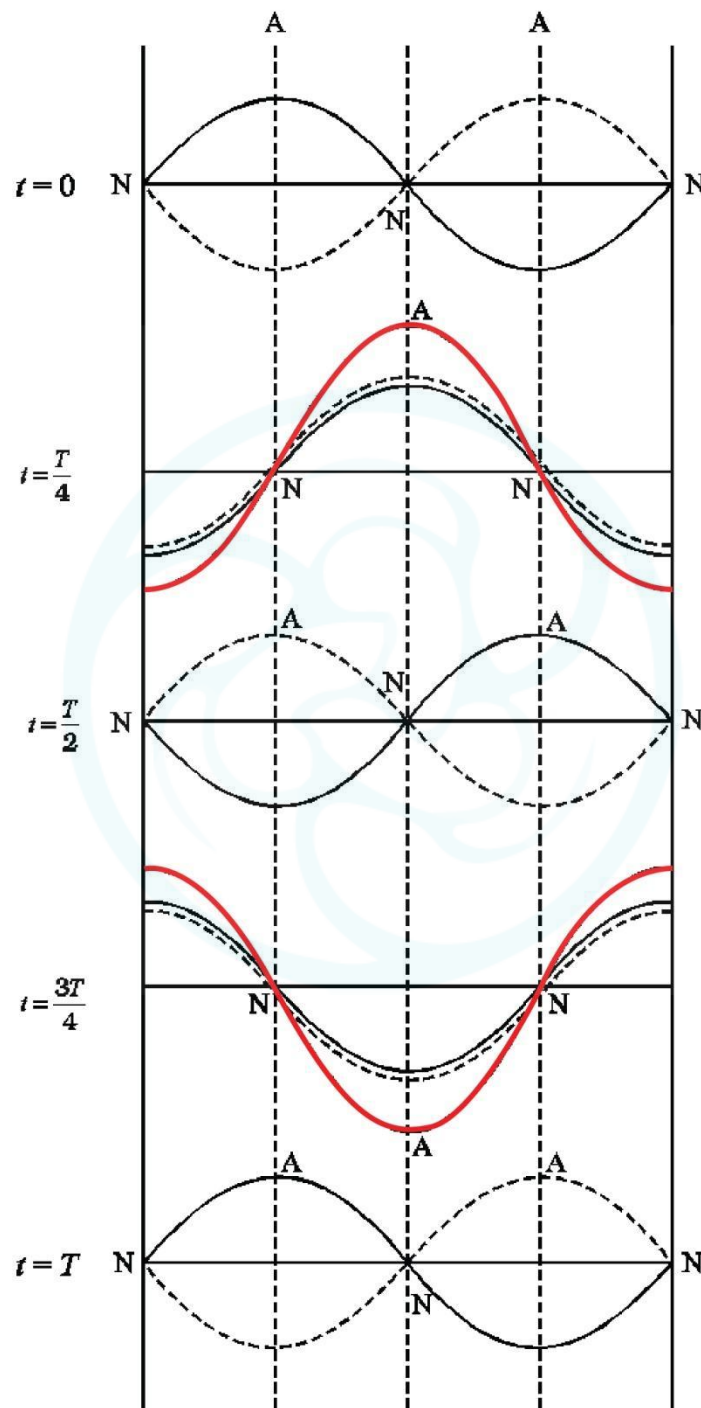


Fig. 15.2.7 Formation of standing waves

15.2.7.1 Derivation of equation of a standing wave

The equation of a plane progressive harmonic wave propagating along the positive direction of x -axis is

$$\text{given by } y_1 = a \sin \frac{2\pi}{\lambda} (t - x) \quad (15.2.9)$$

If this wave gets reflected from a boundary such that the reflected wave retains all the characteristics of the original wave, then its equation will be

$$y_2 = a \sin \frac{2\pi}{\lambda} (vt - x) \quad (15.2.10)$$

The two waves superpose and so the displacement of the resulting wave will be given by

$$\begin{aligned} y &= y_1 + y_2 = 2a \sin \frac{2\pi vt}{\lambda} \cos \frac{2\pi x}{\lambda} \\ \frac{2\pi x}{\lambda} &= (2n + 1) \frac{\pi}{2}, \quad \therefore y = 2a \cos \frac{2\pi x}{\lambda} \sin 2\pi vt \\ &= 2a \cos kx \sin \omega t \end{aligned} \quad (15.2.11)$$

It can be expressed as,

$y = A \sin \omega t$, where A can be looked upon as the amplitude which is independent of ' t ' and dependent on ' x ' only. So, at a fixed point of time, the particle at a particular position will vibrate with an amplitude ranging from a minimum ($-2a$) to a maximum ($2a$).

$$A = 2a \cos \frac{2\pi x}{\lambda}$$

If $A = 0$, the position x is called a Node.

$$\text{Then, } \cos \frac{2\pi x}{\lambda} = 0 \quad (15.2.12)$$

or where $n = 0, 1, 2, 3, \dots$

$$\therefore x = (2n + 1) \frac{\lambda}{4} \quad (15.2.13)$$

The distance between two consecutive nodes is, thus,

$$\{2(n + 1) + 1\} \frac{\lambda}{4} - (2n + 1) \frac{\lambda}{4} = 2 \cdot \frac{\lambda}{4} = \frac{\lambda}{2} \quad (15.2.14)$$

Amplitude A is maximum at 'Antinodes',

$$\text{When } \cos \frac{2\pi x}{\lambda} = \pm 1 \quad (15.2.15)$$

$$\text{i.e. } \cos \frac{2\pi x}{\lambda} = \cos n\pi \quad (n = 0, 1, 2, \dots)$$

$$\therefore \frac{2\pi x}{\lambda} = n\pi$$

$$\text{Or } x = \frac{n\lambda}{2} \quad (15.2.16)$$

The distance between two consecutive antinodes is equal to

$$(n + 1)\frac{\lambda}{2} - \frac{n\lambda}{2} = \frac{\lambda}{2} \quad (15.2.17)$$

Thus, the position of 'Nodes' and 'Antinodes' are respectively. Given by

$$\text{Nodes: } x = (2n+1)\frac{\lambda}{4}$$

$$\text{Antinodes: } x = \frac{n\lambda}{2}$$

The distance between two consecutive nodes or between two consecutive antinodes, is each equal to $\frac{\lambda}{2}$.

We shall next present the difference between a progressive wave and a stationary wave in a Tabular form. You may verify the entries using the equations of progressive and stationary waves.

15.2.7.2 Differences between progressive waves and standing waves

Table 15.2.2. Characteristics of progressive Waves and standing Waves

progressive Waves	standing Waves
1) All particles vibrate with the same amplitude.	1) Amplitude of vibration of particles is zero at nodes and maximum at antinodes.
2) Density of the medium is the same everywhere	2) No change of density at antinodes but maximum change of density at the nodes.
3) Medium particles pass through their mean positions with maximum velocity	3) Velocity of particles is zero at nodes and maximum at antinodes.
4) Continuous transfer of energy takes place from particle to particle.	4) Oscillation of energy in a segment between two nodes but no transfer of energy across the 'node' points.
5) Each region undergoes equal strain one after the other	5) Maximum strain at nodes and zero at antinodes

6) Particles of the medium attain their maximum velocity one after the other.

6) Different points have different maximum velocity values. At nodes, the velocity is zero and at antinodes, it is maximum. However, all particles reach their maximum velocity at the same instant.

Example: 15.2.1

Plucking a sitar or a guitar string generates standing waves. Let us try to find out the frequency of vibration of a musical instrument having a string of length 2 m and mass 5×10^{-4} kg, fixed at both ends, and stretched under a tension of 25 N. When it is plucked at a point situated 50 cm from one end, this point becomes an antinode. Since the ends of the string are fixed, each end has to be a node. Nodes and antinodes are separated by equal distances in a stationary wave. Let us draw a diagram of this situation (Fig. 15.2.8).

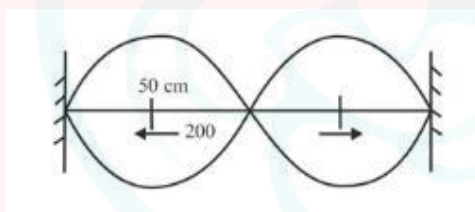


Fig. 15.2.8: The string vibrates in two segments.

$$\text{Now } v = \frac{n}{2L} \sqrt{\frac{T}{m}}$$

where n is the number of loops, T is the tension in the string, L is the total length of the string and m is the mass per unit length.

$$\begin{aligned} v &= \frac{2}{4} \times 5 \times \sqrt{\frac{25}{5 \times 10^{-4}}} \text{ Hz} \\ &= 2.5 \times \sqrt{4000} \text{ Hz} \\ &= 155 \text{ Hz} \end{aligned}$$

Standing waves play an important role in producing musical sounds. Here, we observe that when a wave is incident normally on a boundary of the transmitting medium, it is reflected wholly, or partially. The superposition of the incident and the reflected waves creates a pattern of nodes and antinodes.

When reflection of waves takes place from a fixed boundary, the displacement $y(x, t)$ remains zero there. Also, the reflected wave changes its sign because of the phase change of π . When reflection of a wave takes place from a free boundary (e.g. open end of an organ pipe), the reflected wave has the same sign as the incident wave, i.e., there is no change in its phase.

Example.15.2.2 A transverse harmonic wave, on a string, is described by

$$y(x, t) = 3.0 \sin(36t + 0.018x + \frac{\pi}{4})$$

Here x and y are in cm and t is in s.

The positive direction of x is from left to right.

(a) Is this a progressive wave or a stationary wave?

If it is a progressive wave, find (i) speed and (ii) direction of its propagation?

(b) State the values of its amplitude and frequency.

(c) Give the value of its initial phase at the origin.

(d) Find the distance between two successive crests in this wave.

Solution: The Equation of a harmonic wave (propagating along the negative x -axis) can be written as

$$y(x, t) = a \sin \left[\frac{2\pi}{\lambda} (vt + x + \phi) \right] \quad (15.2.18)$$

The given equation can be rearranged as

$$y(x, t) = 3.0 \sin \left[0.018 \left(\frac{36}{0.018} \cdot t + x + \frac{1}{0.018} \frac{\pi}{4} \right) \right] \quad (15.2.19)$$

(a) It is, therefore, a progressive wave, moving from right to left, i.e along the negative x - axis.

Let us compare Eq(15.2.18) and Eq(15.2.19)

(b) We get: $\frac{36}{0.018} = v$ or $v = 2000 \text{ cms}^{-1} = 20 \text{ ms}^{-1}$

The amplitude (a) of the wave is 3.0 cm.

$$\text{Again, } \frac{2\pi}{\lambda} = 0.018 \quad \therefore \lambda = \frac{2\pi}{0.018} \text{ cm}$$

$$\therefore \lambda = 348.8 \text{ cm} = 3.49 \text{ m}$$

$$\text{Also } = \frac{v}{\lambda} = \frac{20}{3.49} = 5.7 \text{ Hz}$$

(c) To find initial phase, at the origin, let us put

$x = 0$ and $t = 0$ in Eqn(B),

$$\text{We get } \phi = \frac{\pi}{4} \times \frac{1}{0.018} \text{ radian} \cong 13.9\pi$$

(d) The least distance between two crests equals the wavelength of the wave i.e $\lambda = 3.49 \text{ m}$.

Example. 15.2.3

Decide whether the functions

(a) $y(x, t) = 2 \cos(3x) \sin(10t)$

and

(b) $y(x, t) = 0.060 \sin\left(\frac{2\pi}{3}x\right) \cdot \cos(120\pi t)$,

represent a progressive, or, a stationary wave?

Solution:

(a) $y(x, t) = 2 \cos(3x) \sin(10t)$

This function has two harmonic functions, that is, $\cos 3x$ and $\sin 10t$, one only a function of x and the other only a function of t .

This is the form of the equation characteristics of a stationary wave.

(b) $y(x, t) = 0.060 \sin\left(\frac{2\pi}{3}x\right) \cos(120\pi t)$

The function $y(x, t)$ has two distinct harmonic terms i.e. $\sin\left(\frac{2\pi x}{3}\right)$

a harmonic in x . The other, $\cos(120\pi t)$, a harmonic in time t . Therefore, the given function, again, represents a stationary wave.

Following are some examples of progressive harmonic waves:

$$1. y = 10\sin\pi(0.01x - 2.00t)$$

$$2. y = a\cos 2\pi\left(\frac{t}{2.0} - \frac{x}{0.5}\right)$$

$$3. y = 2.2\cos(300t - 0.24x)$$

$$4. y(x, t) = 5\sin(6\pi t - 4x)$$

Students can compare the above equations with the standard equation for progressive wave

$$y = a\cos 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$$

to arrive at the correct conclusion.

15.2.8 Standing waves in air columns

We can have standing waves in air-columns with

- (i) One end closed and the other open – referred to as ‘closed pipes’
- (ii) Both ends open – referred to as ‘open pipes’

To find the permitted or normal modes of oscillation, we note that the

- (i) Closed end is always a **node** where the displacement is minimum (zero), while the pressure changes are maximum.
- (ii) Open end is always an **antinode** where the amplitude of displacement is maximum, while the pressure changes are minimum.

We have to also note that in any of the permitted modes of oscillation, there has to be a **node** between two successive **antinodes** and an **antinode** between two successive **nodes**. Further, the separation between two successive antinodes, or between two successive nodes, equals $\frac{\lambda}{2}$. It follows that the separation between an antinode and its adjacent node, would be $\frac{\lambda}{4}$.

Using these facts, we can see that the first six permitted modes of oscillation, of the air column in a **closed pipe**, have the forms shown in Fig. 15.2.9.

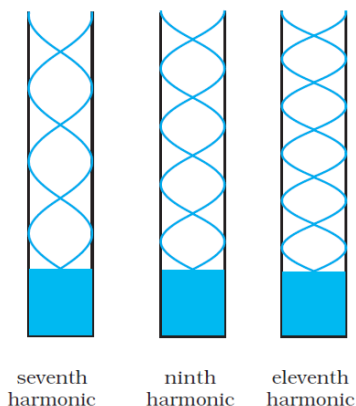


Fig. 15.2.9: Some of the normal modes of vibration of an air column open at one end only

The relation between the length L of the closed pipe, and the wave length λ of any permitted mode of vibration (of the air-column enclosed by it) would be

$$L = \left(n + \frac{1}{2}\right) \frac{\lambda}{2} = (2n + 1) \frac{\lambda}{4}$$

where $n = 0, 1, 2, 3, \dots$

$$\therefore \lambda = \frac{4L}{(2n+1)}$$

The permitted frequencies of vibration are, therefore, given by

$$v = \frac{v}{\lambda} \quad (v \text{ is velocity of waves})$$

$$v = \frac{v}{4L} (2n + 1)$$

The least, or the fundamental frequency of vibration is, therefore,

$$v_0 = \frac{v}{4L} \quad (n = 0)$$

The other permitted frequencies are

$$v_1 = 3v_0, v_2 = 5v_0, v_3 = 7v_0, \dots \text{and so on.}$$

Thus, a closed air column can vibrate only with frequencies that are **odd** integral multiples of its least (or fundamental) frequency of vibration.

For an **open pipe**, open at both ends, the first four permitted modes of oscillation of its air-column, have the forms shown in Fig. 15.2.10.

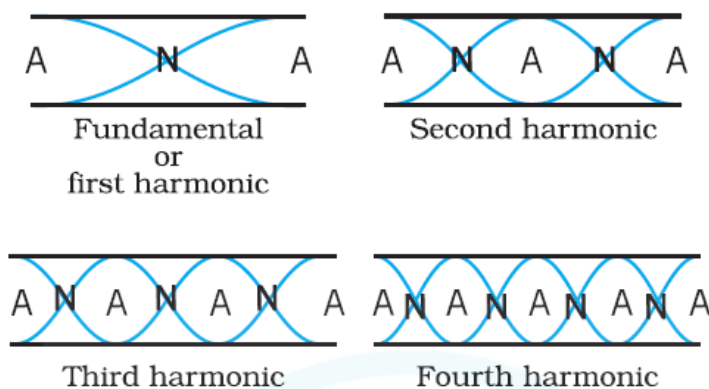


Fig. 15.2.10: Some of the normal modes of vibration of an air column open at both ends

The relation between the length L of the open-pipe and the wavelength λ of any permitted mode of vibration (of the air-column enclosed by it) would be

$$L = (n + 1) \frac{\lambda}{2}$$

where $n = 0, 1, 2, 3, \dots$

$$\therefore \lambda = \frac{2L}{(n+1)}$$

The permitted frequencies of vibration are, therefore, given by

$$\begin{aligned} \nu &= \frac{v}{\lambda} \\ &= \frac{v}{2L} (n + 1) \end{aligned}$$

The least, or fundamental frequency of vibration, is therefore,

$$\nu_0 = \frac{v}{2L} (n = 0).$$

The other permitted frequencies are

$$v_1 = 2v_0, v_2 = 3v_0, v_3 = 4v_0, \dots \text{and so on}$$

Thus, an open air-column can vibrate with frequencies that are (all) integral multiples of its least (or fundamental) frequency of vibration.

We notice that

- (i) For an open and a closed pipe of the same length, the fundamental frequency of vibration (of the air-column), in the open pipe, is twice the fundamental frequency of vibration of the closed pipe.
- (ii) Compared to a closed pipe, the 'open pipe' has a 'richer collection' of its permitted frequencies of oscillation. In a closed pipe one can have only **odd** integral multiples of its fundamental frequency; in an open pipe, on the other hand, all integral multiples of the fundamental frequency, that is, both odd and even harmonics, are permitted.

Example:15.2.4 The third harmonic ($n=2$) in a closed pipe of length L_1 has the same frequency as the fifth harmonic ($n=4$) in an open pipe of length L_2 . Find the ratio($\frac{L_1}{L_2}$).

Solution:

For the closed pipe:

$$\text{Fundamental frequency} = \frac{v}{4L_1}$$

$$\therefore \text{Frequency of the third harmonic} = 5 \cdot \frac{v}{4L_1}$$

For the open pipe:

$$\text{Fundamental frequency} = \frac{v}{2L_2}$$

$$\therefore \text{Frequency of the fifth harmonic} = 4 \cdot \frac{v}{2L_2}$$

We, therefore, have

$$5 \cdot \left(\frac{v}{4L_1} \right) = 4 \cdot \frac{v}{2L_2} = \frac{2v}{L_2}$$

$$\therefore \frac{5}{4} \times \frac{1}{2} = \frac{L_1}{L_2}$$

$$\therefore \frac{L_1}{L_2} = \frac{5}{8}$$



Let Us Reflect

Q.15.2.1 Find the first three permitted frequencies of oscillation of the air column in a closed pipe of length 17 cm. Take velocity of sound as 340 ms^{-1}

Q.15.2.2 The frequency of vibration of the third harmonic ($n = 1$, third multiple of ν_0), in a given closed pipe equals 2500 Hz. Find its length. Take velocity of sound as 340 ms^{-1}

Q.15.2.3 Find the length of an open pipe for which the frequency of vibration of its fifth harmonic is 2500 Hz. Take velocity of sound as 340 ms^{-1} .



Answers

A.15.2.1 500 Hz, 1500 Hz, 2500 Hz

For a closed pipe the frequencies are given by $\nu = \frac{v}{4L} (2n + 1)$. Using the value of the velocity of sound, the length of the pipe and $n = 0, 1, 2$ we get frequencies as 500 Hz, 1500 Hz and 2500 Hz.

A.15.2.2 10.2 cm.

For a closed pipe the frequencies are given by $\nu = \frac{v}{4L} (2n + 1)$. Using the value of the velocity of sound, $n = 1$ (third harmonic) and the given frequency, we find the length of the pipe as 10.2 cm.

A.15.2.3 34 cm

For an open pipe the frequencies are given by $\nu = \frac{v}{2L} (n + 1)$. Using the given frequency, velocity of

sound and $n = 4$ (fifth harmonic), we find the length of the pipe as 34 cm.

15.2.9 Fundamental, harmonics and overtones

The sounds that we normally hear contain a large number of frequencies. The lowest among these frequencies is called the fundamental frequency or first harmonic. The other frequencies present, may, or may not be, integral multiple of this fundamental. If the higher one has twice the fundamental frequency, it is called the second harmonic; if it is three times the fundamental frequency, it is called the third harmonic, and so on. All harmonics, second, third and so on, are called overtones of the fundamental frequency. Harmonics may be thought of as a special group of overtones.

If a sound contains waves with frequencies that are 'harmonics' with respect to the fundamental, the sound is pleasant for the ear and may be termed as a musical sound. On the other hand, if the sound contains non-harmonic frequencies along with the fundamental, then the sound is more like a noise; it is (usually) not a pleasant sound.

Any single frequency, present in a sound, is called a tone; a 'tone' therefore, has only one single frequency. On the other hand, a musical 'note' refers to a sound made up of the fundamental frequency and its harmonics.

The Fig. 15.2.11 shows the formation of different permissible modes in a stretched string.

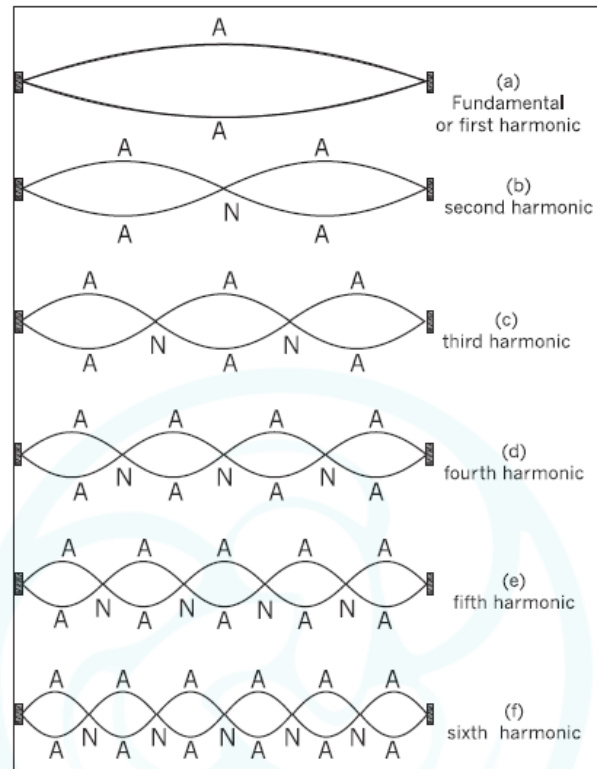


Fig.15.2.11: Stationary waves in a stretched string fixed at both ends. Various modes of vibration are shown.

15.2.10 Summary

- Velocity of sound in a gas is given by

$$v = \sqrt{\frac{\gamma P}{\rho}},$$

where γ is the ratio of the two specific heats (C_p and C_v) of the gas, P is the pressure and ρ is the density of the gas.

- Velocity of sound is highest in solids and lowest in gases. Mathematically, $v_{\text{solid}} > v_{\text{liquid}} > v_{\text{gas}}$
- If two or more independent waves propagate simultaneously through a medium, then the displacement of the resultant of two waves at any point is the vector sum of the individual displacements of the two waves.
- When reflection of a longitudinal wave takes place at a fixed point, from a denser to a rarer medium, it moves in the same phase but in opposite direction. In the case of reflections from a rarer to a denser medium, the wave is reflected back with a change of phase equal to π .
- When two exactly identical progressive waves travelling in opposite directions superpose, standing waves with points of minimum (nodes) and maximum (antinodes) displacements are formed.
- Harmonics are overtones whose frequencies are integral multiples of the fundamental tone (smallest frequency)
- A closed organ pipe produces only odd harmonics.
- An open organ pipe produces all harmonics.

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