

Module 3 Beats and Doppler Effect

This module is designed to help you to:

- explain the occurrence of beats;
- derive mathematical expression for beats;
- derive the equation of a plane progressive wave;
- explain the Doppler effect;
- compute the change in frequency at the instant of crossing.

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15.3.1 Introduction

In the previous two modules you have studied the propagation of waves. You have learnt to distinguish between progressive and stationary waves. You have also applied these concepts to practical situations, most prominent among them being vibrations in closed and open organ pipes and strings.

Nature presents us with several interesting phenomena related to vibrations and waves. Two of them are 'Beats' and 'Doppler effect'. The latter can be applied even for understanding the concept of expanding universe.

We shall discuss these phenomena in this module.

15.3.2 Phenomenon of beats

When two waves of slightly different frequencies are simultaneously present in a given medium, they superpose each other to produce waxing and waning of amplitude. This is called the phenomenon of 'beats'. The amplitude of the resultant wave increases and decreases, periodically, with a frequency equal to the difference between the frequencies of the two individual waves.

Activity: 15.3.1

Demonstration of the phenomenon of beats due to superposition of waves produced by two tuning forks of slightly different frequencies.

Take two tuning forks of the same frequency. Attach a small piece of plasticine, or wax, to the prongs of one of the tuning forks. This will slightly lower the frequency of this tuning fork. Now holding the tuning forks, one in each hand, strike both the tuning forks simultaneously on two rubber pads.

Hold them close to each other in calm, undisturbed surroundings. Carefully listen to the combined sound produced by the two tuning forks. A gradual increase and decrease in the intensity of sound will be heard.

This waxing and waning of sound is called beats. Beats are produced by the superposition of two waves of slightly different frequencies. You can also count the number of beats produced per second if the beat frequency does not exceed a few beats per second. To measure the frequency of beats one of the persons can give a (silent) signal at each minimum intensity (or maximum intensity), to the other person. Using a stopwatch the second person either finds the time taken by 10 beats or counts the number of beats in 5 seconds.

If two tuning forks, of the same frequency, mounted on (resonating) wooden boxes are available, all the students in the class may be able to listen to the beats. Place the tuning forks with the open sides of boxes

facing each other on a desk somewhere in the middle of the classroom.. Make their frequencies slightly different by loading one with plasticine or wax or by tightly attaching a small load with adhesive tape. Let there be pin-drop silence in the classroom. Strike the tuning forks, with a rubber hammer in quick succession, with roughly equal force. Both tuning forks must be of good quality and must give audible sound for about 8 to 10 seconds. The whole class can then listen to the resulting 'beats'.

15.3.2.1 Mathematical derivation for beats

Let us consider two simple harmonic vibrations of very nearly equal frequencies, ν_1 and ν_2 , such that

$$\nu_1 - \nu_2 = \Delta \nu \text{ and } \nu_1, \nu_2 \gg \Delta \nu.$$

The equations of motion of the individual vibrations, assumed to be of equal amplitude, are respectively,

$$x_1 = a \cos 2\pi\nu_1 t \quad (15.3.1)$$

$$x_2 = a \cos 2\pi\nu_2 t \quad (15.3.2)$$

By the principle of superposition, the resulting vibration is given by,

$$\begin{aligned} x &= x_1 + x_2 \\ \therefore x &= a(\cos 2\pi\nu_1 t + \cos 2\pi\nu_2 t) \\ &= 2a \cos \left\{ 2\pi \left(\frac{\nu_1 + \nu_2}{2} \right) t \right\} \cos \left\{ 2\pi \left(\frac{\nu_1 - \nu_2}{2} \right) t \right\} \\ &= 2a \cos \left\{ 2\pi \left(\frac{2\nu_1 + \Delta\nu}{2} \right) t \right\} \cos (\pi\Delta\nu t) \end{aligned}$$

$$\text{The first cosine term} = \cos 2\pi \left(\nu_1 + \frac{\Delta\nu}{2} \right) t$$

$$\text{But } \nu_1 + \frac{\Delta\nu}{2} \sim \nu_1$$

$\therefore$ The first term of the phase part of resultant displacement = $\cos (2\pi\nu_1 t)$.

So, after rearranging we write,

$$x = 2a \cos (\pi \Delta v t) \cos (2\pi v_1 t) \quad (15.3.3)$$

In Eq(15.3.3), we have two cosine terms on the RHS. The second term is almost equivalent to the phase part of RHS of Eq(15.3.1). Since, $v_1 \simeq v_2$, it could as well be $\cos (2\pi v_2 t)$ and there is nothing much to distinguish between the phase parts of RHS of Eq(15.3.1) and Eq(15.3.2). And since the frequencies of the two component vibrations are very nearly equal, it is expected that their phase terms would be identical. So the second term is the phase term. The first cosine term thus becomes a part of the amplitude. We can now rewrite Eq(15.3.3), as

$$x = A(t) \cos 2\pi v_1 t \quad (15.3.4)$$

Here $A(t)$ is the amplitude which is now a function of time.

$|A(t)|$ is maximum, when $\cos (\pi \Delta v t) = \pm 1$

$$\therefore \pi \Delta v t = n\pi, n \text{ is an integer}$$

$$\therefore t = \frac{n}{\Delta v} \quad (15.3.5)$$

$$\therefore t = \frac{n}{v_2 - v_1}$$

In other words, t now takes values $t = \frac{1}{v_2 - v_1}, \frac{2}{v_2 - v_1}, \frac{3}{v_2 - v_1}, \dots$ (15.3.5A)

Again $|A(t)| = 0$, when

$$\cos (\pi \Delta v t) = 0$$

Or when, $\pi \Delta v t = (2n + 1)\frac{\pi}{2}$, n is an integer

$$\text{or } t = \frac{2n+1}{2\Delta v} \quad (15.3.6)$$

$$\therefore t = \frac{2n+1}{2(v_2 - v_1)}$$

In other words, t can take values

$$t = \frac{3}{2(v_2 - v_1)}, \frac{5}{2(v_2 - v_1)}, \frac{7}{2(v_2 - v_1)}, \dots \quad (15.3.6A)$$

From Eqs. (15.3.5A) and (15.3.6A), we note that the minimum and maximum of amplitude take place at regular intervals of

$$t = \frac{1}{(\nu_2 - \nu_1)}$$

In other words, the waxing and waning of the amplitude of vibration takes place at a frequency of $(\nu_2 - \nu_1)$, i.e., the difference of the component frequencies. This is the phenomenon of beats, the difference, $(\nu_2 - \nu_1)$ is called the beat frequency.

15.3.2.2 Calculations related to the formation of beats

Two waves having the same velocity and of nearly equal frequencies, when superposed, can give rise to a situation known as beats. In this process the amplitude of the resultant wave becomes time dependent. If the frequency difference is small

(< 10 Hz) then this results in an observable waxing and waning of the resultant sound. The situation may be mathematically represented as follows.

Let there be two waves with angular frequencies ω_1 and ω_2 , where corresponding linear frequencies are respectively ν_1 and ν_2 , and the difference between them i.e. $(\nu_2 - \nu_1)$ is < 10 Hz.

Now, the equations of the two waves with symbols having their usual meanings are

$$x_1 = a \sin \omega_1 (t - x/v) \text{ and}$$

$$x_2 = a \sin \omega_2 (t - x/v)$$

Where a is the amplitude of both the waves and v is their velocity.

We can write

$$x_1 = a \sin \omega_1 \tau,$$

$$x_2 = a \sin \omega_2 \tau, \text{ where, } \tau = t - \frac{x}{v}$$

Let, $\omega_2 - \omega_1 = \Delta\omega$.

Then we have

$$\begin{aligned} x &= x_1 + x_2 = 2a \sin \left\{ \left(\omega_1 + \omega_2 \right) \frac{\tau}{2} \right\} \cos \left\{ \left(\omega_2 - \omega_1 \right) \frac{\tau}{2} \right\} \\ &\simeq 2a \sin \omega_1 \tau \cos \left\{ 2\pi(\nu_2 - \nu_1) \frac{\tau}{2} \right\} \\ &= 2a \sin \omega_1 \tau \cos (\pi \Delta \nu \tau) \end{aligned}$$

$$2a \sin \omega_1 \left(t - \frac{x}{v} \right) \cos (\pi \Delta v \tau) = A \sin \omega_1 \left(t - \frac{x}{v} \right),$$

$$\text{Where } A = 2a \cos (\pi \Delta v \tau)$$

We see that the phase part is retained in the later part of expression of the resulting displacement, whereas the amplitude term is time-dependent. By applying the knowledge of minima and maxima of cosine function, we can arrive at the conclusion that the time difference between two consecutive minima and two consecutive maxima is $\frac{1}{v_2 - v_1}$.

Our ears cannot differentiate between sounds that arrive within 1/10 second of each other. If the amplitude of sound changes, say from maximum to the next maximum, within 1/10 second, our ears will not be able to detect the in-between minimum. And that is why the condition for the detection of beats is that the difference between two frequencies be within 10 Hz.



Let Us Reflect

Q.15.3.1 When a sonometer wire was sounded along with a tuning fork of frequency 256 Hz, four beats were heard every second. When the tension in the string was increased slightly, the beat frequency increased. Calculate the (original) frequency of the sonometer wire.



Answers

A.15.3.1 Since the number of beats increases with the tension, the frequency of the sonometer is higher. Therefore, the frequency of the sonometer = (256+4) Hz = 260 Hz.

15.3.3 Phenomenon of transient beats

In all cases of forced vibrations, there are two systems- a driving system and a driven system. The latter has a natural frequency of oscillation, say ν_o and the driver forces its vibration on it say having frequency ν .

The driven system initially continues to vibrate with its own natural frequency, which dies down slowly, and it picks up the frequency of vibration of the driver. In the process of this pick up the frequency of the driven system approaches the frequency of the driver, and there is a threshold value (ν_{th}) which is close enough to ν so

that beats of frequency $(\nu - \nu_{th})$ is formed. But the frequency of the driver does not settle at ν_{th} as it moves further closer to ν , with say frequencies $\nu_1, \nu_2, \nu_3, \dots$ and finally settling at ν . In the process, beats of frequencies- $(\nu - \nu_{th}), (\nu - \nu_1), (\nu - \nu_3), (\nu - \nu_2), \dots$ are formed until the difference between the frequencies of the driver and the driver becomes zero.

Since the beat frequencies vary during the last stage of the driven frequency approaching that of the driver, the event is called the phenomenon of transient beats.

15.3.4 Doppler effect

If there is a relative motion between the source of sound and the observer, the frequency of the sound appears to change for the observer. This apparent change in the frequency of sound due to the relative motion between the source of sound and the observer, is known as the Doppler Effect. This effect can be observed both for the mechanical waves (for example, sound waves) and for the electromagnetic waves (for example, light waves). However, our discussion in this Section will remain confined to sound waves only.

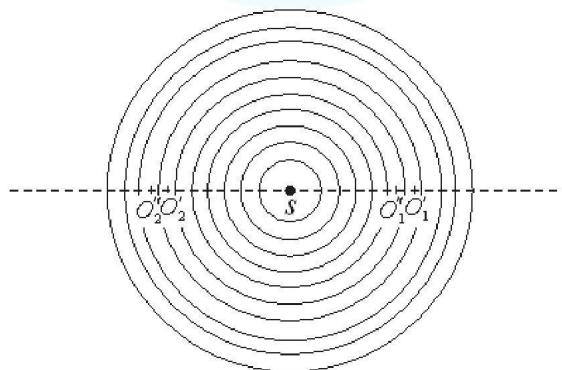


Fig. 15.3.1: (a) *the positions of waves emitted by a stationary source S at equal intervals of time.*

Consider two different observers O_1 and O_2 at the same distance from the source but on diametrically opposite sides. Both receive the same number of waves emitted by the source every second. Suppose both O_1 and O_2 start moving to the right with the same speed, say v_o . After one second O_1 reaches O_1' and O_2 reaches O_2' . As can be seen in the figure, during this second, O_1 receives the number of waves less than n_o , whereas O_2 receives number of waves higher than n_o , given by formula $n_2 = n_0 \left(\frac{v+n_o}{v} \right)$. (15.3.7)

Here v_o to be taken positive for O_2 (observer moving towards the source) and negative for O_1 (observer moving away from the source).

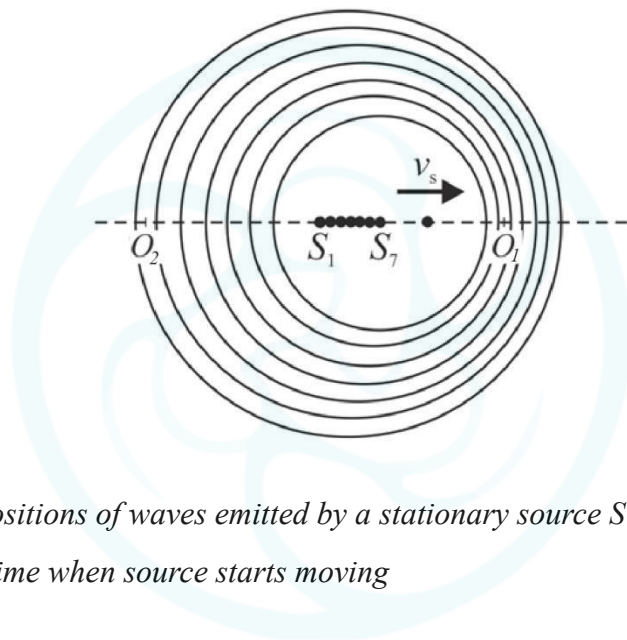


Fig. 15.3.1(b) the positions of waves emitted by a stationary source S at equal intervals of Time when source starts moving

Now let the source be moving towards the right with a speed v_s and both the observers be at rest as shown in Fig. 15.3.1(b).

Let S move from position S_0 to positions $S_1, S_2, \dots$ every second. The corresponding wavefronts emitted (with speed v_s) are circular (spherical in 3-dimensions) but with their centres shifting to the right. Consider two observers O_1 and O_2 diametrically opposite but at the same distance from S_0 . As the source is approaching O_1 , but moving away from O_2 , O_1 receives a higher number of waves, whereas O_2 receives a smaller number of waves than n_o every second, given by

$$n_2 = n_0 \left(\frac{v}{v - v_0} \right) \quad (15.3.8)$$

Here v_s to be taken positive for O_1 (source approaching) and negative for O_2 (source receding away).

This effect implies that the frequency of a source of waves, as observed by an observer, is dependent on the nature, and extent, of the relative motion, between the source and the observer.

It follows that the true frequency n_0 of a source would correspond to its value, as observed (or detected) by an observer (or detector) at rest relative to the source. It follows that the frequency n' detected by an observer when both the source as well as the observer are in relative motion,

$$n' = n_0 \left(\frac{v + v_o}{v - v_s} \right) \quad (15.3.9)$$

The effect was originally discovered by Doppler for the sound waves. However, it was soon realised that it also holds for other types of waves, including electromagnetic waves. ‘Doppler red shift’, in the light received from distant stars is a very significant evidence in favour of the ‘big-bang’ theory of the origin of the universe.

15.3.4.1 Mathematical treatment of doppler effect

It is possible to mathematically calculate the change in frequency due to the Doppler effect, by assuming that the motion of the source, observer are along the line of propagation of the waves themselves.

The overall effect of the relative motion between the source and the observer can be obtained by considering that these effects are the compounded effects of the motion of the (i) source only and (ii) observer only. We, therefore, first discuss these two effects, one by one, and then obtain a general formula for the general case where both the source and the observer are in relative motion.

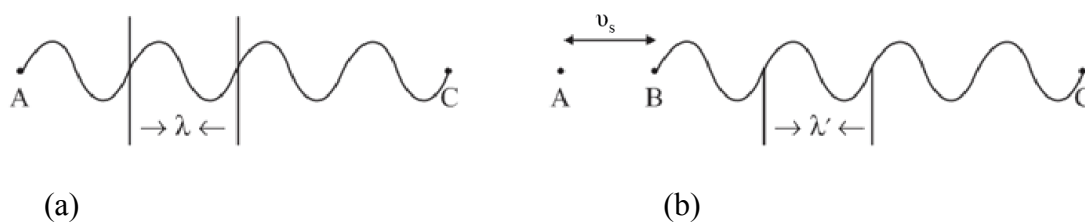


Fig. 15.3.2 Effect of the motion of the source only

Let the source emitting sound waves of velocity v , initially at position A, start moving towards the observer with a speed v_s . After 1 second, the source would emit its n_0^{th} wave ($n_0 = \text{true frequency}$) when it is at position B, where $AB = v_s$. At this instant, the first wave (the wave emitted by the source when at position A) would have reached point C, where $AC = v$. The n_0 waves, emitted by the source, thus effectively get contained in a distance $BC (= v - v_s)$. The effective wavelength λ' of the waves is, therefore,

$$\lambda' = \left(\frac{v - v_s}{n_0} \right) \quad (15.3.10)$$

The motion of the source towards the observer can, therefore, be visualised as resulting in a compression of waves so that their effective wavelength decreases. The observer O, therefore, perceives these waves having a frequency n' where

$$n' = \left(\frac{v}{\lambda'} \right) \quad (15.3.11)$$

$$n' = \left(\frac{v}{v - v_s} \right) n_0 \quad (15.3.12)$$

If the source were moving away from the observer, the 'effective wavelength' can be visualised as increasing. The frequency, perceived by the observer, would then be

$$n' = \left(\frac{v}{v + v_s} \right) n_0$$

This is a result, obtained from Eq.(15.3.12), by attaching a negative sign to the value of v_s .

(2) Effect of the motion of the observer only

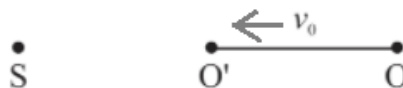


Fig. 15.3.3

Let the observer, initially at position O, start moving towards the stationary source with a speed v_0 refer Fig. 15.3.3. The waves emitted by the source are approaching the observer with a speed v . Thus the number of waves an observer will receive in 1 s will be

$$n' = \left(\frac{v+v_0}{v} \right) n_0 \quad (15.3.13)$$

If the observer were moving away from the source, the apparent velocity of approach of the waves towards the observer would become $(v - v_0)$. The apparent frequency, perceived by the observer, would then be

$$n' = \left(\frac{v-v_0}{v} \right) n_0$$

This is a result obtained from Eq. (15.3.13), by attaching a negative sign to the value of v_0 .

(3) Combined effect of the motion of the source and the observer

Let us consider the case where the source and the observer are both approaching each other, with speeds v_s and v_0 , respectively. We can then say

- (i) The motion of the source causes the (true) frequency n_0 to have a value, say n'' , where

$$n'' = \left(\frac{v}{v-v_s} \right) n_0 \quad (15.3.14)$$

- (ii) The motion of the observer causes this (apparent) frequency n'' to have a value n' , where

$$\therefore n' = \left(\frac{v+v_0}{v} \right) n'' \quad (15.3.15)$$

$$\therefore n' = \left(\frac{v+v_0}{v} \right) \cdot \left(\frac{v}{v-v_s} \right) n_0$$

$$\therefore n' = \left(\frac{v+v_0}{v-v_s} \right) n_0 \quad (15.3.16)$$

Eq (15.3.16) can be regarded as the general formula for the ‘Doppler change in frequency’ of a source, as perceived by an observer, when there is a relative motion between the two. It is easy to see that Eqs (15.3.14) and (15.3.15), are special cases of this general formula, corresponding to (i) $v_0 = 0$ and (ii) $v_s = 0$, respectively.

An important point in relation to Eq (15.3.16) has to be kept in mind. We have to attach a negative sign to the value of

- (i) v_0 , when the observer is moving away from the source but the source is moving towards the observer,
- (ii) v_s , when the source is moving away from the observer but the observer is moving towards the source, and
- (iii) v_0 and v_s , when both the source and the observer are moving away from each other.

15.3.4.2 Change in frequency at the instant of crossing

When the source and the observer are initially moving towards each other, they would cross each other at some instant and after that they would both be moving away from each other. This would result in a ‘sharp dip’ in frequency at the instant of crossing. It is this feature of the Doppler effect that is most often observed in day-to-day life.

The apparent frequency, before crossing, is n_1 , where,

$$n_1 = \left(\frac{v+v_0}{v-v_s} \right) n_0 \quad (15.3.17)$$

The apparent frequency, after crossing, is n_2 , where

$$n_2 = \left(\frac{v-v_0}{v+v_s} \right) n_0 \quad (15.3.18)$$

The change, in frequency, at the instant of crossing, would, therefore, be

$$\begin{aligned} \Delta n &= n_1 - n_2 \\ &= \left\{ \frac{v+v_0}{v-v_s} - \frac{v-v_0}{v+v_s} \right\} n_0 \\ &= \left\{ \frac{2v(v_0+v_s)}{v^2-v_s^2} \right\} n_0 \end{aligned} \quad (15.3.19)$$

An important point:

A careful look at Eqs (15.3.17) and (15.3.18) shows that even for identical values of v_s and v_o , the values of the apparent frequency, n_1 and n_2 , are not the same. This implies that the effects of identical (towards/away) velocities of the source and the observer do not cause the same change in frequency due to the Doppler effect. We can understand this by noting that the basic cause of this change, in the two cases, is not the same. Whereas the motion of the source causes an apparent change in the wavelength of the waves, the motion of the observer towards the stationary source results in receiving the higher number of waves during the same interval of time. So the observer perceives a higher frequency of the source. The frequency of the sound appears to be lower when the observer moves away from the stationary source.

15.3.5 Doppler effect for light

The above discussion of the Doppler effect, strictly speaking, cannot be applied, as such, to light waves where relativistic concepts and effects become quite relevant. However, we can use the above formulae to an approximate extent.

When the source moves with a speed v_s , the apparent wavelength is

$$\begin{aligned}\lambda' &= \left(\frac{c - v_s}{c} \right) \lambda \\ &= \left(1 - \frac{v_s}{c} \right) \lambda\end{aligned}$$

(c = speed of light)

Hence the change in wavelength due to the motion of the source towards the observer would be

$$\Delta\lambda = \lambda' - \lambda = \left(\frac{-v_s}{c} \right) \lambda$$

The motion of the source towards the observer, thus, causes an apparent decrease in wavelength which can be referred to as a 'blue shift'. For a source moving away from the observer, the wavelength would show an apparent increase in wavelength. This, as we know, is referred to as a Red shift.

The velocity of light, as per Einstein's theory of relativity, does not change, due to a motion of the observer or the source. Hence, we can take the formula

$$\frac{\Delta\lambda}{\lambda} = \frac{-v_s}{c}$$

as the formula for finding the Doppler shift in the wavelength of a source of light. Here it is important to remember that we need to attach a negative sign, to the value of v_s , when the source is moving away from the observer. We can also say

- (i) an ‘away’ motion of the source results in a Doppler red shift, that is, an apparent increase in wavelength, and
- (ii) a ‘towards’ motion of the source results in a Doppler blue shift, that is, an apparent decrease in wavelength.

Example:15.3.1

Two cars A and B, each moving with a speed of 25 ms^{-1} , are approaching each other on long straight two-way roads. Find the apparent frequency of the horn of car A, as observed by the occupant of car B, (a) before and (b) after the cars cross each other. The frequency of the sound of the horn of the car is 700 Hz and the speed of sound in air is 325 ms^{-1} .

Solution

Here the source and the observer are

- (i) both approaching each other before the cars cross each other.
- (ii) both moving away from each other after the cars cross each other.

Hence

- (a) Apparent frequency, before the cars cross each other, is

$$\begin{aligned} n' &= \left(\frac{v+v_A}{v-v_B} \right) n \\ &= \frac{325+25}{325-25} \times 700 \text{ Hz} \\ &= \frac{350}{300} \times 700 \text{ Hz} \\ &= 816.67 \text{ Hz} \end{aligned}$$

- (b) Apparent frequency, after the cars cross each other, is

$$n'' = \left(\frac{v-v_A}{v+v_B} \right) n$$

$$\frac{325-25}{325+25} \times 700 \text{ Hz}$$

$$\begin{aligned}\frac{300}{350} \times 700 \text{ Hz} \\ = 600 \text{ Hz}\end{aligned}$$

Example:15.3.2

A stationary detector, positioned as shown behind a source of sound, can detect direct sound of the source as well as that reflected by a wall (Fig. 15.3.4).

When the source starts moving towards the reflecting wall, the detector observes beats of frequency 2 Hz between the direct and the reflected sounds. Find the speed of the source.

(The speed of sound is 340 ms^{-1} and the true frequency n of the source is 256 Hz)

**Fig.15.3.4****Solution**

Let v_s be the speed of the source.

For the direct sound, the source is moving away from the detector, with a speed v_s .

$$\therefore n_D = \frac{v}{v+v_s} n$$

For the reflected sound, the (virtual image) source of the reflected sound is moving towards the detector with a speed v_s . Hence

$$n_R = \frac{v}{v-v_s} n$$

$\therefore$ Frequency of beats is $n_b = n_D - n_R$

$$\begin{aligned}\therefore n_b &= vn \left[\frac{1}{v-v_s} - \frac{1}{v+v_s} \right] \\ &= vn \left[\frac{2v_s}{v^2 - v_s^2} \right] = \frac{2vv_s}{v^2 + v_s^2} n\end{aligned}$$

As $v_s \ll v$

We can put this as

$$\begin{aligned}n_b &= \frac{2vv_s}{v^2} n \\ &= \frac{2v_s}{v} n \\ \therefore v_s &= \frac{vn_b}{2n} = \frac{340 \times 2}{2 \times 256} \text{ ms}^{-1} \\ &= 1.33 \text{ ms}^{-1}\end{aligned}$$



Let Us Reflect

Q.15.3.3 An observer, standing on the platform, hears the ‘note’ of the whistle, sounded by the engine of a train, approaching the platform at a speed of 72 km per hour. Find the apparent frequency of the ‘note’ (frequency = 512 Hz) as heard by the observer.

Q.15.3.4 A stationary source blows a whistle of frequency 500 Hz. The sound of the whistle is heard by a person sitting on a vehicle moving towards the source at 54 km per hour. Find the apparent frequency of the note, as heard by the observer in the vehicle, before and after, crossing the source.

Q.15.3.5 The whistle of the engine of a moving train (frequency = 512 Hz) is heard by a person sitting in another train, on a parallel track. If both the trains are moving at 108 km per hour, find the ‘dip’ in the frequency of the whistle at the instant the two trains just cross each other.

Q.15.3.6 Two tuning forks, each of frequency 256 Hz, are mounted on two blocks that move with equal speeds, v , in the same direction. An observer (at rest) positioned in between the moving blocks, hears beats of frequency 2 Hz. Find the (approximate) value of v .

Q.15.3.7 The whistle of an engine, moving at 36 kmh^{-1} , is observed by a motorist, moving at 54 kmh^{-1} . If the apparent frequency of the whistle appears to be 500Hz, find its actual frequency if the motorist and the engine were

- moving in opposite directions and approaching each other.
- moving in opposite directions and going away from each other.

Note: Take the velocity of sound in air = 340 ms^{-1}



Answers

A.15.3.3 $[340/(340-20)] 512 = 544 \text{ Hz}$

A.15.3.4 Before: $[(340+15)/340]500 = 522 \text{ Hz}$. After: $[(340-15)/340]500 = 478 \text{ Hz approx.}$

A.15.3.5 $[\{2 \times 340 \times (30+30)\} / \{(340+30) \times (340-30)\}] \times 512 = 182 \text{ Hz}$

A.15.3.6 Let v_s be the speed of the source.

For the direct sound, the source is moving away from the detector, with a speed v_s .

$$\therefore n_D = \frac{v}{v+v_s} n$$

For the reflected sound, the (virtual image) source of the reflected sound is moving towards the detector with a speed v_s . Hence

$$n_R = \frac{v}{v-v_s} n$$

$\therefore$ Frequency of beats is $n_b = n_D - n_R$

$$\begin{aligned}\therefore n_b &= vn \left[\frac{1}{v-v_s} - \frac{1}{v+v_s} \right] \\ &= vn \left[\frac{2v_s}{v^2-v_s^2} \right] = \frac{2vv_s}{v^2+v_s^2} n\end{aligned}$$

As $v_s \ll v$

We can put this as

$$\begin{aligned}n_b &= \frac{2vv_s}{v^2} n \\ &= \frac{2v_s}{v} n \\ \therefore v_s &= \frac{vn_b}{2n} = \frac{340 \times 2}{2 \times 256} \text{ ms}^{-1} \\ &= 1.33 \text{ ms}^{-1}\end{aligned}$$

A.15.3.7 (a) $[(340-15)/(340+10)]500 = 464.28 \text{ Hz}$ (b) $[(340+15)/(340-10)]500 = 537.8 \text{ Hz}$

15.3.6 Summary

- When two vibrations of very nearby equal frequencies are made to superimpose, then it creates an effect of waxing and waning of intensity with a frequency equal to the difference of the component frequencies. This is called the phenomenon of beats.
- When there is a relative motion between a source of sound and an observer, there occurs an apparent change in frequency from what is originally emitted by the source. This phenomenon is known as the Doppler effect.



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