

Module 1 Work and Power

This module is designed to help you to:

- explain the concept of work and power;
- differentiate between positive and negative work;
- describe the methods/techniques used/needed to calculate the work done by a constant force as well as a variable force;
- explain that, in case of a variable force, the work done is calculated by finding the sum of a very large number of very small amounts of work done for very small displacements over which the (variable) force can still be regarded as constant;
- show the importance and significance of the mathematical technique of integration in problems related to determination of work done by a force;
- explain the difference between work and power and the techniques of calculating power;
- illustrate the need of the alternative formula $(P = \vec{F} \cdot \vec{v})$ for power and the situations where this alternative formula can be used.

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6.1.1 Introduction

We know that the term work has a special meaning in physics. The work done by a force $\vec{F}$, causing a displacement $\vec{s}$ of the object, equals $\vec{F} \cdot \vec{s}$.

We can use the above result only if the force $\vec{F}$ remains constant over the displacement $\vec{s}$. When the force is variable, we have to modify this result to the form

$$W = \int \vec{F} \cdot d\vec{s}$$

In this module, we will discuss the details of the techniques of calculation of the work done by both a constant force and by a variable force. We will also talk about the difference between work and power and the technique of calculating power.

6.1.2 Work done by a constant force

Work done manifests itself in the form of energy. You have learnt that work done by the net force on a body can equal the change in kinetic energy of the body. You have also studied that work done on the body can get stored as potential energy in the system.

Equation for work done by a force $\vec{F}$ on a body is $= \vec{F} \cdot \vec{s} = Fs \cos \theta$. Here, $\vec{s}$ is the displacement and θ is the angle between the force and displacement.

- (a) If a body moves in the direction of the force, both force and displacement are in the same direction ($\theta = 0$). Therefore, $\cos \theta = +1$ and work done by the force on the body is taken as positive.

Example: A body is displaced along the ground by applying a force parallel to the ground. The force and the displacement are here along the same direction. Hence the work done by the force is a positive work.

For a stone falling downward under gravity, the gravitational force and displacement are in the same direction. Hence, the work done by the gravitational force on the falling stone is positive.

- (b) When a body moves opposite to the direction of the applied force, $\theta = 180^\circ$, and $\cos \theta = -1$. In this case, the work done by the force on the body is negative.

For example, when a stone is moving upwards from the surface of the earth, gravitational force and displacement are in opposite directions and work done by the gravitational force on the stone is negative.

(c) If a body moves perpendicular to the force, i.e. $\theta = 90^\circ$; work done is equal to zero.

For a body moving along a horizontal surface the displacement is perpendicular to the gravitational force. Hence, work done by the gravitational force is zero in this case. Work done on such a body by other forces like frictional force, viscous drag, magnetic force etc. may not be equal to zero if they are not perpendicular to its displacement.

(d) If a body moves at any other angle in the range $0 < \theta < 180^\circ$ (except $\theta = 90^\circ$), we need to consider the component of force along, or opposite, to the displacement.

If $\theta = 45^\circ$, the effective component of the force is in the same direction as the displacement of the body. Hence work done, $W = \vec{F} s \cos 45^\circ = + \frac{Fs}{\sqrt{2}}$, (i.e.). Work done on the body is positive.

If $\theta = 135^\circ$, the component of force is opposite in direction to the displacement. Hence, the work done by the force, being opposite to that of the displacement ($W = F s \cos 135^\circ = - \frac{Fs}{\sqrt{2}}$), the body is negative. In such a case, work has to be done by the body to move against the force.

Note that we have defined work done as the DOT PRODUCT of the two vectors. It is, therefore, a scalar quantity. Yet we assign + or – sign to it. The sign, however, does not indicate direction of work. It just indicates as to whether the displacement of the body is in the direction of the force or opposite to that of the force.

For two bodies, the vector sum of the mutual forces exerted by each on the other is zero (from Newton's Third Law),

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

But the sum of the work done by the two forces need not always cancel,

i.e. $W_{12} + W_{21}$ is not necessarily zero.

For example, consider a ball thrown on a wall. The ball rebounds as a result of the reaction force acting on it. However, the work done by the force exerted by the ball on the wall is zero as there is no displacement of the wall. The work done by the reaction force of the wall, on the ball, is, however, not zero. Hence, here

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

$$\text{But, } W_{12} + W_{21} \neq 0.$$



Let Us Reflect

Q.6.1.1. What is work done by the tension force during the oscillatory motion of a simple pendulum?

Q.6.1.2. A man is rowing his boat upstream in such a manner that he remains stationary with respect to the shore. Is he doing any work?

Q.6.1.3. A block of ice is drawn through a distance of 0.5 m along a frictionless horizontal surface. The pull in the rope is 1 N and the angle between the rope and the horizontal is 30° . What is the work done?



Answers

A.6.1.1. In Fig 6.1.1,

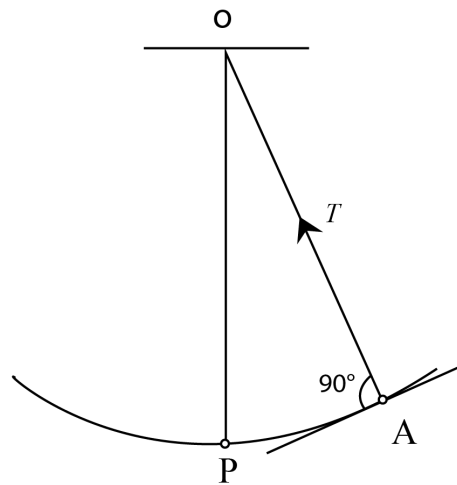


Fig. 6.1.1

We can see that a pendulum bob is suspended from O and the bob is executing oscillatory motion along PA. At every position of its path, the tension T , which acts radially inward is perpendicular to the displacement, which is along the tangent to the path. So, the work done is zero.

A.6.1.2. Since the displacement is zero, no work is being done

A.6.1.3. Refer to Fig 6.1.2

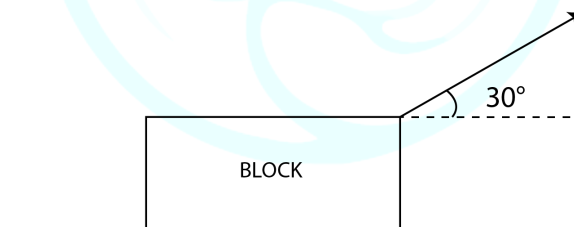


Fig. 6.1.2

With symbols having usual meanings.

$$W = (F \cos \theta) s$$

$$\therefore W = (1 \text{ N}) \cos 30^\circ (0.5 \text{ m}) = 0.5 \times \frac{\sqrt{3}}{2} \text{ J}$$

$$W = 0.433 \text{ J}$$

6.1.3 Work done by a body and work done on a body

A stone, falling from a height, on, say, a nail, in a wooden block, can do work on the nail and press it into the block. Similarly, a stretched spring can do work on a ball kept on it and throw it some distance away. Here it is important to realize that in both these cases (and in all such similar cases), the body does work at the expense of the energy stored in it.

It is more usual to consider situations in which some external force does work on a body. This work done, on the body, can result in an increase in the mechanical energy of the body.

6.1.4 Work done by a variable force

Work done by a force is given by the equation $W = \vec{F} \cdot \vec{s}$ where F and s represent magnitudes of force and resulting displacement respectively.

If magnitude or direction (or both) of force, varies in space, work done (say, δW) for every infinitesimal displacement, in space, has to be determined to know the total work done. Similarly, if force varies with time, work done (say, δW) is to be determined at every instant. The total work W will be the sum of all δW values:

$$i. e. W = \sum \delta W$$

Examples of variable force

We have already learnt about Stokes' formula. There we understand that force on a small spherical body, moving in a fluid experiences a viscous force given by $F \propto v$, where v is the velocity of the body. It becomes evident in experiments for determination of the coefficient of viscosity of a liquid. This is, therefore, an example of a variable force.

A simple example of this type of variable force is provided by a raindrop falling under the influence of the downward gravitational force and opposing (viscous) resistive force. The viscous velocity of raindrops. Velocity of the raindrop changes with time due to gravitational acceleration. Therefore, viscous force also changes with time. In this case, the force on the raindrop is not constant. Hence, we use either the mathematical technique of integration or use a simple technique of calculating the same from graphs.

The spring force is also an example of a variable force. Consider a block attached to a spring and resting on a smooth horizontal surface. Let the other end of the spring be attached to a rigid wall. We assume the spring to be light and hence treat it as massless. In an ideal spring, the spring force F is proportional to x where x is the displacement of the block from the equilibrium position. The displacement could be either positive or negative. Now the restoring force on the spring depends upon its displacement from its equilibrium position. Therefore, as the spring oscillates, its displacement, and corresponding restoring force varies.

Example 6.1.1 For a given object, the plot of the force (acting along the x - axis), with displacement (also along the x - axis), has the form shown. Calculate the work done by the force in displacing the object from $x = a$ and $x = d$

Solution

The net work done, by the (variable) force, equals the (relevant) area enclosed in Fig. 6.1.3. Hence

$$W = \frac{1}{2}f_3(a - 0) + f_4(b - a) + \frac{1}{2}(f_2 + f_1)(c - b) + f_1(d - c)$$

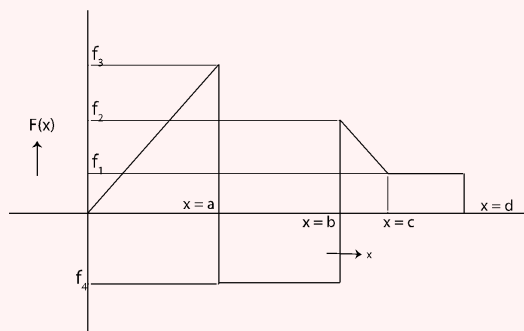


Fig.6.1.3

$$\begin{aligned}
&= f_1 \left[-\frac{1}{2}b - \frac{1}{2}c + d \right] + f_2 \left(\frac{1}{2}(c - b) \right) + f_3 \frac{a}{2} - f_4(b - a) \\
&= f_1 \left[d - \left(\frac{b+c}{2} \right) \right] + \frac{f_2(c-b)}{2} + f_3 \frac{a}{2} - f_4(b - a)
\end{aligned}$$

Let us now verify this result through direct integration also. We have

$$\begin{aligned}
W &= W_1 + W_2 + W_3 + W_4 \\
&= \int_{x=0}^a \left(\frac{f_3}{a} x \right) dx + \int_{x=a}^{x=b} (-f_4) dx + \int_{x=b}^c \left[\left(\frac{f_1 - f_2}{c-b} \right) x + \frac{f_2 c - f_1 b}{(c-b)} \right] dx + \int_{x=c}^{x=d} f_1 dx \\
&= \frac{f_3 a}{2} - f_4(b - a) + \frac{(f_1 + f_2)(c-b)}{2} + f_1(d - c) \\
&= f_1 \left[d - \left(\frac{b+c}{2} \right) \right] + \frac{f_2(c-b)}{2} + f_3 \frac{a}{2} - f_4(b - a)
\end{aligned}$$

This is exactly the same as the result obtained above. This shows that, for a variable force, the work done calculated

- (i) Either by summing up the areas under the graph
- (ii) Integrating the force expression under appropriate limits.

Note: For calculating the work W_3 , we need the equation of the line between $x = b$ and $x = c$. We note that (b, f_2) and (c, f_1) are two points on this line. This easily leads us to the equation of this line which turns out to

$$y = \left(\frac{f_1 - f_2}{c-b} \right) x + \left(\frac{f_2 c - f_1 b}{c-b} \right)$$

Example 6.1.2

An electron is to be moved from the first to the third permitted Bohr orbit in a normal hydrogen atom. Given that the radius of the first orbit equals r_0 and $r_n = n^2 r_0$ ($n = 1, 2, 3, \dots$), find the work done in the process.

Solution

The magnitude of the force, on the electron, due to the nucleus, in a normal hydrogen atom, is given by

where x is the distance of the electron from the nucleus.

$$f(x) = \frac{1}{4\pi\epsilon_0} \frac{e \cdot e}{x^2}$$

Hence, work done in moving the electron to a distance $(x + dx)$, along the direction of the force, is

$$dW = f(x) \cdot dx$$

$$\text{Here } f(x) = \frac{1}{4\pi\epsilon_0} \frac{e^2}{x^2} dx$$

$$\text{Also } r_3 = r_0(3)^2 = 9r_0$$

Hence total work done

$$W = \frac{e^2}{4\pi\epsilon_0} \int_{r_0}^{9r_0} \frac{dx}{x^2} = -\frac{e^2}{4\pi\epsilon_0} \left[\frac{1}{x} \right]_{r_0}^{9r_0} = \frac{e^2}{4\pi\epsilon_0 r_0} \left(\frac{1}{9} - \frac{1}{1} \right) = \left(\frac{8}{9} \frac{e^2}{4\pi\epsilon_0 r_0} \right).$$

In actual practice, it is often important not only to know the work done by a force but also the time in which that work has been done. The concept of power is introduced to take into account the time taken to do a given amount of work.

6.1.5 Power

The term power as we know is used for the rate of doing work. Thus, if a force $\vec{F}$, does a work ΔW , on an object, in a time Δt , the power associated with this force is, $P = \frac{\Delta W}{\Delta t}$

The instantaneous power of a given force $\vec{F}$ at the time t equals the value of $\frac{dW}{dt}$ at that instant. The average power, over a time interval t_1 to t_2 , would equal $\frac{W_{net}}{(t_2 - t_1)}$.

Where W_{net} is the net work done by the force $\vec{F}$, over this time interval.

We often use another useful expression for Power. Since

$$\Delta W = \vec{F} \cdot \Delta \vec{S},$$

We have,

$$P = \frac{\Delta W}{\Delta t} = \vec{F} \cdot \frac{\Delta \vec{S}}{\Delta t}$$

$$P_{inst} = \vec{F} \cdot \vec{v}$$

Here $\vec{F}$ and $\vec{v}$ are the corresponding instantaneous values of the force acting on the body and the velocity $\vec{v}$ of the body.

Example 6.1.3 Let an object, of mass m , initially at rest be acted upon by a constant force $\vec{F}$, for a time t . How does the power, associated with this force, vary with time?

Solution

The velocity $\vec{v}$, acquired by the object in time t , would be given by

$$\vec{v} = 0 + t\vec{a} = 0 + t \cdot \frac{\vec{F}}{m}$$

$\therefore$ The instantaneous power P , at time t , is given by

$$P = \vec{F} \cdot \vec{v} = \vec{F} \cdot \left(\frac{t}{m} \vec{F} \right) = \frac{F^2}{m} t$$

Since $\vec{F}$ is a constant force, we see that the power of this force keeps on increasing linearly with time.

It is interesting to note that if we calculate the power by using

$$P = \frac{W}{t} = \frac{F \cdot s}{t}$$

We get

$$P = \frac{1}{t} \left[\vec{F} \cdot \left(\frac{1}{2} \vec{a} t^2 \right) \right] = \frac{1}{t} \left[\vec{F} \cdot \left(\frac{1}{2} \frac{\vec{F}}{m} t^2 \right) \right] = \frac{1}{2} \frac{F^2}{m} t$$

Why this difference? We need to realize that the power, calculated by using the second method, is the average power over the time interval 0 to t . The first calculation gives the instantaneous power at time t .

The average value, over the interval 0 to t , as per the first calculation is given by

$$P_{AV} = \frac{\int_0^t \frac{F^2}{m} t \, dt}{t} = \frac{F^2}{2m} t$$

This is the same as the average value of power obtained through the second calculation.

Power consumption in walking

Table: 6.1.1 Below lists the approximate power expended by an adult human of mass 60 kg.

Table 6.1.1

Activity	Power (W)
Sleeping	75
Slow Walking	150
Bicycling	500
Heart beat	1.2



Let Us Reflect

Q.6.1.4 A boy whose weight is 60 kg runs up a staircase of height 12 m in 2.5 minutes. How much power gets consumed in the process? (Take $g = 10 \text{ ms}^{-2}$)



Answers

A.6.1.4 Work done = mgh
 $= (60 \text{ kg}) (10 \text{ m/s}^2) (12 \text{ m})$
 $= 7200 \text{ J}$

The work done has been done in 2.5 minutes = $2.5 \times 60 \text{ s} = 150 \text{ s}$

$$\therefore \text{Power} = \frac{7200}{150} \text{ Js}^{-1}$$
$$= 48 \text{ W}$$

6.1.6 Summary

In this module we have learnt

- the work done by a constant force $\vec{F}$, causing a displacement $\vec{s}$ of the object, equals $\vec{F} \cdot \vec{s}$
- for a variable force, the work done is given by the relation

$$W = \int \vec{F} \cdot d\vec{s}$$

- The change in kinetic energy of a body is the work done by the net force on the body.
- Power is defined as the rate of doing work. Thus, if a force $\vec{F}$, does a work ΔW , on an object, in a time Δt , the 'power' associated with this force is, $P = \frac{\Delta W}{\Delta t}$

The instantaneous power of a given force $\vec{F}$ at a given instant of time t equals the value of $\frac{dW}{dt}$ at that instant. The instantaneous power can alternatively be given by

$$P_{inst} = \vec{F} \cdot \vec{v}$$

- the average power, over a time interval t_1 to t_2 , would equal $\frac{W_{net}}{(t_2 - t_1)}$

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