

Module 2 Work Energy Theorem

This module is designed to help you to:

- explain the meaning of the term energy;
- explain the two forms (kinetic energy and potential energy) in which mechanical energy is available in nature;
- discuss the significance of the principle of conservation of energy;
- describes the work kinetic energy theorem and the reason for calling it so rather than as simply (work-energy theorem);
- explain how the work-energy theorem enables us to calculate the work done even in situations where the details of the relevant force are not available.

Content

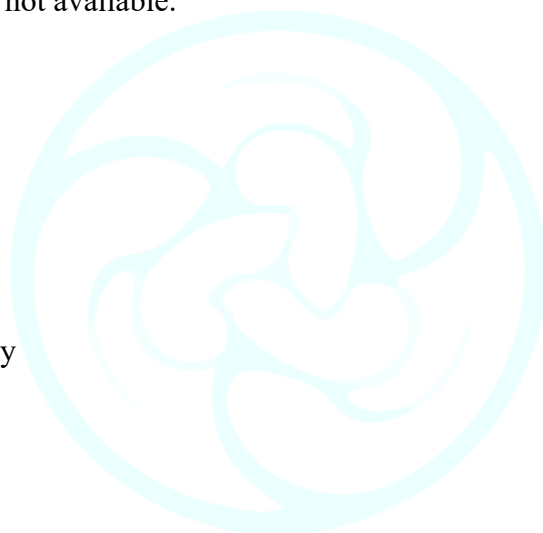
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6.2.1 Introduction

In the previous module you learnt to calculate the work done by a variable force. Here we shall talk about the capacity of doing work which is called energy. It happens to be the lifeline of our existence and so it is important that we study its conservation and related transformations from one form of energy to another. We shall initiate the discussion by restricting ourselves to cases of transformation between forms of energy which are mechanical and thereby develop the concept of work-energy theorem. Further, we shall discuss different examples of the principle of conservation of energy.

6.2.2 Energy

The energy of a body is a measure of its capacity of doing work. It is important to realize that a body, possessing energy, need not necessarily be doing work. It simply needs to have a capacity of doing work.

Kinetic energy and potential energy: The mechanical energy possessed by a body can have two forms: kinetic energy and potential energy.

Kinetic energy: The energy possessed by a body by virtue of its motion, is known as its kinetic energy. The kinetic energy, of a body of mass m , moving with a speed v , is (ordinarily) given by the formula,

$$KE = \frac{1}{2}mv^2.$$

Potential energy: A body can have energy that is a capacity of doing work even when it is at rest. This can be done by virtue of its being in a particular position or condition. We call this energy of the body as its potential energy. Unlike kinetic energy, we do not have any simple general formula for calculation of potential energy. It is also important to realize that we do not calculate the potential energy of a body; we rather calculate the CHANGE in potential energy of a body when it is moved from some initial position to final position or is taken from some initial condition to some final condition. This calculation is done through the general formula for calculation for calculation of work done, i.e.

$$W = \int_{initial}^{final} F \cdot dx$$

In actual practice, we often take some initial position, or condition as the reference position or condition. The potential energy of this (arbitrarily chosen) position or condition is then taken as zero. The change in potential energy, in going from this reference position to some final position can then be taken as the potential energy of the body in this final position.



Let Us Reflect

Q.6.2.1 Two bodies of equal masses have the same kinetic energy. Which one has the larger linear momentum?

Q.6.2.2 Two bodies of equal masses have the same linear momentum. Which one has the larger kinetic energy?



Answers

A.6.2.1 $KE = \frac{1}{2}mv^2 = E$, say

$$\therefore E = \frac{m^2 v^2}{2m} = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

$$\therefore p^2 = 2mE$$

$$\text{Or, } p = \sqrt{2mE}$$

So, the larger mass will have greater linear momentum.

A.6.2.2 $KE = \frac{p^2}{2m}$

$$\text{For equal } p, KE \propto \frac{1}{m}$$

So, kinetic energy is greater for smaller masses.

6.2.3 Work-energy theorem

This theorem may better be called the work-kinetic energy theorem. It is, however, usually referred to as the work-energy theorem.

According to the work-energy theorem, the change in kinetic energy of a particle is equal to the work done on it by the net force.

When ‘net forces’ do work on a body, its kinetic energy increases. However, if work is done by the body, its kinetic energy decreases.

This principle holds good for conservative fields like the gravitational and the electrostatic fields as well as non-conservative force-fields like friction. Note that the work done by a conservative force depends on the initial and final positions only. This theorem helps in calculating the velocities in problems where we do not even know the exact nature of force.

Points to ponder over the work-energy theorem

1. The work-energy theorem holds even when work is being done by a variable force.
2. The work-energy theorem does not, in general, incorporate the complete dynamical information of Newton’s second law. In particular, information with respect to directions is contained in the vector form of Newton’s second law. This concept of directions is not present in the work-energy theorem which has a scalar form.
3. The work-energy theorem does not provide the explicit temporal (time) information provided by Newton’s second law. The second law relates the instantaneous values of acceleration and force. The work-energy theorem, on the other hand, ‘integrates’ this (instantaneous) information over a finite interval of time.
4. The work-energy theorem can be more explicitly labeled as the ‘work-kinetic energy theorem’ and its statement can be made as follows:
‘When a force/ a set of forces acts/act on a system in such a way that the only change in the system is a change in its speed, the work done by the net force equals the change in kinetic energy of the system’.
5. It is interesting to note that the ‘work-kinetic energy’ theorem (as per the statement given above) is valid even for rotational motion. We can give its statement for rotational motion as follows:

‘When a torque/ a set of torques acts/act on a system in such a way that the only change in the system is a change in its angular speed, the work done by the net torque $\left(W = \int_{\theta_1}^{\theta_2} \tau d\theta\right)$ equals the change in rotational kinetic energy $= \left[\frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2\right]$ of the system.

6. It is important to realize that we can have situations where there can be violation of the work-kinetic energy theorem. The condition mentioned in its statement (the only change in the system is a change in its speed) is important. When the forces, acting on a system, cause changes other than changes in its speed (for example, a load lifted vertically upwards), the work done by the forces would not equal the change in kinetic energy of the system. As we know, the forces, in such cases, can (also) cause a change in the potential energy of the system.
7. A simple and straightforward use of the work-kinetic energy theorem, for extended objects, may not always lead us to logically valid results. For extended objects, there can be unforeseen complications. We should therefore, preferably say that the ‘work-kinetic energy theorem’ is valid for a particle or for an object/system that can be thought of/ or can be replaced by an ‘equivalent point particle’.
8. When motion is confined to one dimension we do not need to use vector quantities. However, in real life we experience more motions in two or three dimensions rather than in one dimension. When we consider two or three dimensional cases, mathematical work would be more extensive and involved.

For example, in three dimensions, in terms of its Cartesian components, force would be $F = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$

and $v = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$. The position vector of body under consideration is $r = x \hat{i} + y \hat{j} + z \hat{k}$. Imagine

the mathematics involved in the integration $\int_{K_i}^{K_f} dK = \int_{r_i}^r F \cdot dr = \int_{r_i}^r F \cdot ((dx \hat{i} + dy \hat{j} + dz \hat{k}))$. It is here that the

work-kinetic energy theorem comes handy. Kinetic energy being a scalar quantity, there are no vector calculations.

Example 6.2.1 We consider here an example of using the work-energy theorem to determine the work done by the force during the displacement of the body from $x = 0$ to $x = 2$ m. The velocity of the body has been given to be varying as $v = 5x^{\frac{3}{2}}$.

Solution

By work energy theorem

$$\begin{aligned}\text{work done} &= \text{change in KE} \\ &= \frac{1}{2}m(v_2^2 - v_1^2) \\ &= \frac{1}{2}m\left(5^2(2^{3/2})^2 - 0\right) \\ &= \frac{1}{2} \times 0.5 \times (25 \times 8) \text{ J} = 50 \text{ J}\end{aligned}$$

A direct calculation of the work (by using $W = \int_0^2 F(x)dx$, would be a relatively much more involved calculation, which we will see at the self-Assessment Exercise Q.6.1.8.



Let Us Reflect

Q.6.2.3 A 20 kg gun fires a bullet of mass 20 g. The kinetic energy imparted to the bullet is 800 J. How much kinetic energy gets imparted to the gun due to the recoil?



Answers

A.6.2.3 With symbols having usual meaning.

$$mv = MV$$

$$V = \frac{mv}{M}$$

$$\text{KE of the recoil} = \frac{1}{2}MV^2 = \frac{m^2v^2}{2M} = \frac{m}{M} \left(\frac{1}{2}mv^2 \right)$$

$$\therefore \text{KE of the recoil} = \frac{m}{M}E = \frac{0.02}{20} \times 800 \text{ J} = 0.08 \text{ J}$$

6.2.4 Law of conservation of energy

The law of conservation of energy states that ‘energy can neither be created nor destroyed it can only be changed from one form to another’.

For an idealized mechanical system, viz., the rolling of a steel ball on a perfectly smooth inclined plane, the energy of the ball remains in the form of its kinetic and potential energies. During the course of its motion, a continuous transformation between these energies takes place. The sum of the kinetic and potential energies of the body remains constant provided there is no dissipation of energy due to air resistance, friction, etc.

The following activity can be performed to study the conservation of energy of a ball rolling down an inclined plane (using a double inclined plane).

Activity 6.2.1

Apparatus and materials required

A double inclined plane having smooth hard surfaces as shown in Fig. 6.2.1 (for guided motion of the ball on the double inclined plane, it is suggested that an aluminum channel or rails of two steel wires be used), a steel ball of about 2.5 cm diameter, two wooden blocks, spirit level, tissue paper or cotton, and a half meter scale.

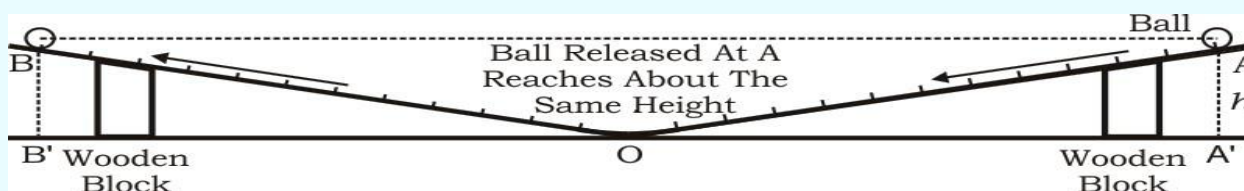


Fig. 6.2.1 A double inclined plane for the demonstration of the principle of conservation of energy

In this experiment, the law of conservation of energy gets illustrated by the motion of a steel ball rolling on a double inclined plane. A steel ball rolling on a hard surface of a smooth inclined plane is an example of

motion with negligible friction. When the ball is released from point A (on inclined plane AO), it will roll down the slope and go up the opposite side on the plane OB to (about) the same height h from which it was released. If the angle of the slope on the left hand plane is decreased, the ball will move over a longer length of the inclined plane till it reaches the same vertical height from which it was released.

At point of release A, on the right-hand inclined plane, the steel ball possesses only potential energy that is proportional to the vertical height, h , (above the base, or reference, level B'OA') of the point of release. At this instant it has zero kinetic energy. This potential energy gets transferred (completely) into kinetic energy when the steel ball rolls down to the lowest point O on the double inclined plane. It then starts rolling up on the second inclined plane during which its kinetic energy changes into potential energy. At point B, where it stops on the left hand inclined plane OB, it again has only potential energy and zero kinetic energy. The law of conservation of mechanical energy gets illustrated by the (near) equality of the two vertical heights AA' and BB'.

Procedure

1. Adjust the experimental table to be horizontal; use a spirit level for checking this.
2. Keep the cleaned double inclined plane on such a horizontal table.
3. Clean the steel ball, and the inclined planes with cotton or tissue paper. Even a minute amount of dust or stain, on the ball or on the plane, can cause some friction.

[Note: In order to reduce friction and thereby reduce loss of energy, one can also design an unbreakable double inclined track apparatus in which the steel ball rolls on stainless steel wire tracks. In a try out, with such an inclined plane, it has been observed that the rolling friction is extremely low and it is very good for this activity. It also does not develop a kink in the center, unlike the apparatus presently in use in many schools].

4. Insert identical wooden blocks (W_1 and W_2) underneath each plane at equal distance from the centre point O. The two planes will be inclined, (nearly) equally, as shown in Fig 6.2.1 The inclined plane should be stable on the horizontal table; otherwise there would be energy loss due to the movement of the inclined plane as well.
5. Release the steel ball from a suitable point A, on the right inclined plane.

6. Find the vertical height AA' (x) of the point A from the table using a scale.
 7. Note the point B up to which the ball reaches the inclined plane on the other side and find its vertical height BB' (y) (Fig. 6.2.1).
- Record the observations. While observing the highest position of the steel ball on the second plane, the observer has to be very alert as the ball stays at the highest position only for a very small instant.
8. Shift one of the two wooden blocks (W_1 and W_2), kept under either of the two planes, towards the centre point O by a small distance. Now the angle of the slope of one of the planes would be larger than that of the other.
 9. Release the ball again from point A on one of the two planes and mark point B on the other plane up to which the steel ball rolls up. Also find the vertical height BB' .
 10. Repeat Steps (8) and (9) for one more angle of the slope of the inclined plane.
 11. Repeat the observations for another point of release on the same inclined plane.

Observations

(1) When the two planes are equally inclined to the horizontal

S. No.	Height on inclined plane from which the ball is rolled down		Height on the inclined plane to which the ball rolls up			Difference ($x - y$) (cm)
	Position of mark A	Vertical height (AA') (x) (cm)	Position of mark up to which the ball rolls up	Vertical height (BB') (y) (cm)	Mean, y (cm)	
1			$B_1 =$			
2			$B_2 =$			
3			$B_3 =$			
1			$B_1 =$			
2			$B_2 =$			
3			$B_3 =$			

(Prepare similar tables for each of the other two adjusted (different) angles of inclination of the two planes)

Result

It is observed that initial vertical height and final vertical height (up to which the ball rolls up) are (nearly) equal. Thus, the rolling steel ball has the same initial and final potential energies, though during the motion, the form of its energy changes. Its total mechanical energy (sum of kinetic and potential energies) remains the same. This provides a verification of the law of conservation of energy.

Precautions

1. Steel balls, as well as the inclined planes, must be cleaned properly with cotton/tissue paper.
2. Both of the inclined planes must lie in the same vertical plane.
3. Both the planes must be stable and should not have any movement, due to rolling of the ball or otherwise.
4. The position of the ball at the highest point while climbing up the plane must be noted quickly and carefully.

Example 6.2.2

Consider a person walking with constant speed v_0 . Estimate the mechanical work he does with the help of the work-energy theorem.

Assume:

- (a) The major work done in walking is due to the acceleration and deceleration of the legs with each stride (Fig. 6.2.2).
- (b) Neglect air resistance.
- (c) Neglect the small work done in lifting the legs against gravity.
- (d) Neglect the swinging of hands etc. as is common in walking.

Solution

As we can see in Fig. 6.2.2, in each stride the leg is brought from rest to a speed, (nearly) equal to the speed of walking, and then brought to rest again.

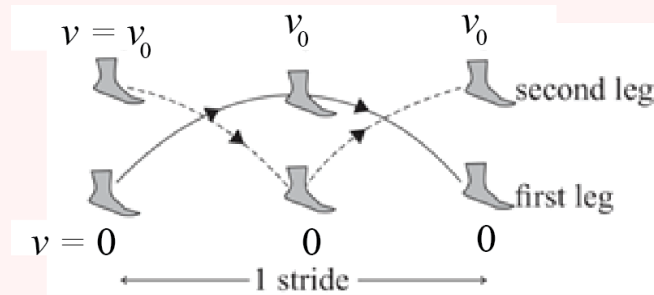


Fig. 6.2.2. *An illustration of a single stride in walking*

While the first leg is maximally off the ground, the second leg is on the ground and vice-versa.

The work done by one leg in each stride is $\frac{1}{2}m_l v_0^2$ (by the work-energy theorem). Here m_l is the mass of the leg.

(Note $\frac{1}{2}m_l v_0^2$ energy is expended by one set of leg muscles to bring the foot from rest to speed v_0 while an additional $\frac{1}{2}m_l v_0^2$ is expended by a complementary set of leg muscles to bring the foot to rest from speed v_0 .)

The total work done, by one set of leg muscles, in one stride, is, therefore, $m_l v_0^2$

Hence work done by both legs in one stride is (observe Fig. 6.2.2 carefully)

$$W_s = 2m_l v_0^2$$

Assuming $m_l = 13$ kg and slow running at (nearly) 7.2 km/h (which translates to about 2 ms^{-1} in SI units), we obtain the

$$W_s = 104 \text{ J / stride}$$

If we take a stride to be 1 m long, person completes 2 strides per second at his speed of 2 ms^{-1} . Thus the power expended

$$P = 104 \text{ J / stride} \times 2 \text{ stride/second} = 208 \text{ W}$$

We must bear in mind that this is a lower estimate since several avenues of power loss (e.g. swinging of hands, air resistance etc.) have been ignored. The interesting point is that we did not worry about the forces

involved.

The forces, mainly friction and those exerted on the leg by the muscles of the rest of the body, are hard to estimate. Static friction does NO work and we have bypassed the very difficult task of estimating the work done by the muscles, by taking recourse to the work-energy theorem.

Here, it is important to understand that the muscles of the leg both (i) in bringing the leg from rest to a speed v and then (ii) in bringing the leg from a speed v to rest. One can ‘feel’ this effort put in the leg muscles, in ‘both parts’ of each stride. By the work (kinetic) energy theorem, work, in each ‘part’ equals $\frac{1}{2}mv_0^2$. Hence the total work done, in one stride, is $\left(\frac{1}{2}mv_0^2 + \frac{1}{2}mv_0^2\right) = mv_0^2$. The total work done, by both legs, in one complete stride, is therefore $2mv_0^2$.

The power expended by the leg muscles would depend not only on this work done but also on the time taken to complete each stride. We therefore, have to have more power to walk faster, i.e. to complete each stride in a shorter time.

The important thing to note, however, is that the work-kinetic energy enables us to calculate the work done by the muscles without knowing any details of how the muscles exert force and how much is that force. There are many such situations where the work-kinetic energy theorem is useful in calculating the work done by a force without knowing much about the force itself.

It is interesting to note that for a heavier person, the force of (limiting) friction would be more than that for a lighter person. We also know that it is the force of friction (between our feet and the earth/ground) that enables us to walk. Since acceleration, for a given force, is inversely proportional to mass, it is clear that the leg muscles of a heavier person would have to spend more power (than those of a lighter person) to attain the same speed in the same time. The heavier person, therefore, feels more exhausted out than a lighter person when their strides bring their legs, from rest to a given speed v_0 , at the same time.



Let Us Reflect

Q.6.2.4 Sand particles are being dropped at the rate of 0.2 kg s^{-1} continually on a conveyor belt moving at a constant speed of 4 ms^{-1} . Find

- (a) the rate at which work is being done by the extra force required to keep the conveyor belt moving.
- (b) the rate of change of kinetic energy per unit time of the sand particles falling on the belt. Is there any difference in the results of (a) and (b)? If yes, then why?



Answers

A.6.2.4 The required force $= \frac{d}{dt}(mv) = v\left(\frac{dm}{dt}\right) = 4 \times 0.2 = 0.8 \text{ (ms}^{-1}\text{)(kgms}^{-2}\text{)} = 0.8 \text{ N}$

(a) The required rate of doing work $= (0.8 \text{ N}) \times (4 \text{ ms}^{-1}) = 3.2 \text{ W}$

(b) The rate of change of KE $= \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}v^2\left(\frac{dm}{dt}\right) = \frac{1}{2} \times 4^2 \times (0.2) \text{ (m}^2\text{s}^{-2}\text{)(kgs}^{-1}\text{)} = 1.6 \text{ W}$

We find the results of (a) and (b) to be different. In fact, the power required at (a) is twice that of (b). The extra power required is due to the fact that the sand particles do not immediately assume the velocity of the conveyor belt. The belt first moves relative to the sand. The extra power is required to overcome the friction between the sand particles and the belt.

6.2.5 Summary

In the module we have learnt

- The energy of a body is a measure of its capacity of doing work.
- The mechanical energy possessed by a body can have two forms: kinetic energy and potential energy.
- The energy possessed by a body by virtue of its motion, is known as its kinetic energy. It is given by,

$$KE = \frac{1}{2}mv^2.$$
- The energy possessed by a body by virtue of its position or condition is called potential energy.
- The change in potential energy of a body when it is moved from some initial position to final position is done through the general formula for calculation of work done, i.e.

$$W = \int_{initial}^{final} F \cdot dx$$

- According to the work-energy theorem, the change in kinetic energy of a particle is equal to the work done on it by the net force.
- This principle holds good for conservative fields like the gravitational and the electrostatic fields. The work done by a conservative force depends on the initial and final positions only.
- The work-energy theorem holds when work is done by a constant force or variable force.
- The law of conservation of energy states that 'energy can neither be created nor destroyed it can only be changed from one form to another'.

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