

**This module is designed to help you to:**

- explain the concept of potential energy;
- distinguish between conservative and non-conservative force;
- calculate the potential energy of a stretched/compressed spring;
- solve problems based on the potential energy of a spring;
- calculate potential energy, associated with systems with changed configuration, like a compressed spring;
- examine the value of potential energy in cases where the particle gets displaced from one position to another under a variable force;
- describe the fact that potential energy of a particle/system can be both positive and negative, as against kinetic energy, which is always positive;
- identify the situations where the total mechanical energy of a system may decrease despite losing kinetic energy. .

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### 6.3.1 Introduction

In the previous module we have understood kinetic energy is associated with the moving body. Then we introduce another form of energy which is associated with the position or configuration of the body, known as potential energy.

In this module we shall discuss the concept of potential energy, conservation of potential energy, potential energy of spring and their conversion in other forms of energy.

### 6.3.2. Potential energy

The term potential energy relates to energy 'stored' in a system. A stretched bow-string possesses potential energy. When it is released, the arrow flies off with a good speed. The (stored) potential energy is getting converted into kinetic energy.

Every force, encountered in mechanics, does not have a (unique) potential energy associated with it. For example, work done by the force of friction, over a closed path, is not zero; no unique value of potential energy can be associated with the force of friction. In a later Article, where we will describe conservative and non-conservative forces, we shall see that force of friction comes under the second category.

The potential energy of a body, subjected to a conservative force is unique, but undeterminable to the extent of an additive constant. The point where the potential energy is taken as zero is a matter of choice. For gravitational potential energy near the earth surface, the zero of the potential energy is taken to be at the ground itself whereas for distances large compared to the radius of the earth, we take zero at far away (infinity) distance.

Conventionally, we regard the potential energy of a particle, moving in an attractive force field as negative, and in a repulsive force field as positive. That for an attractive force, PE is negative means that work has to be done on the system to break the bounds of attraction, whereas a positive PE in case of a repulsive force means that work can be done by the system towards generation of KE. For example, gravitational force is attractive and the corresponding PE is negative, which means that positive work is done when a mass approaches the source of attraction.

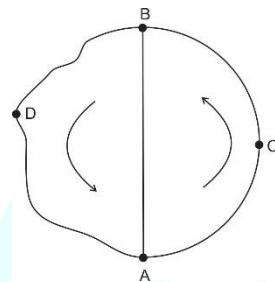
For the spring-mass system, ( $PE = kx^2/2$ ), potential energy is zero at the equilibrium position of the oscillating mass.

[**Note:** It is a common misconception that once an arrow is released from a bow, it will not have potential energy; it will then have only kinetic energy. One should realise that the arrow will be at some height from the surface of the earth. The arrow will therefore have both kinetic as well as potential energy when the earth's surface is considered as the reference level (zero of gravitational potential energy)].

### 6.3.3. Conservation of mechanical energy

#### 6.3.3.1 Conservative and non- conservative forces

A force is said to be conservative if the work it does on an object, in moving it between two given points, is independent of the path the object takes to go between the points (Fig. 6.3.1).



**Fig. 6.3.1:** *Conservative force*

Thus the work done by a conservative force depends only on the initial and final positions of the object.

In contrast, a force is non-conservative if it involves a dissipation of mechanical energy. The work done by such a force, depends on the path followed in going from one point to another. Frictional force is an example of a non-conservative force.

Imagine a particle moving along a closed path; made up of two parts: ACB and BDA. There is a possibility that the magnitude of the  $W_{ACB}$  and  $W_{BDA}$  may happen to be equal (non-equal) if the work done by the relevant force, in moving a particle between two points, is independent of (dependent on) the path followed between these two points.

Consider a particle describing a closed path under a conservative/(non-conservative) force. The work done in moving the particle over the two parts of the path ACB and BDA would then be equal in magnitude, but opposite in sign. The net work done is, therefore, zero for a conservative force. However, for the same motion

under a non-conservative force, (e.g. frictional force), the net work done for the motion, along ACB and BDA, will be non - zero.

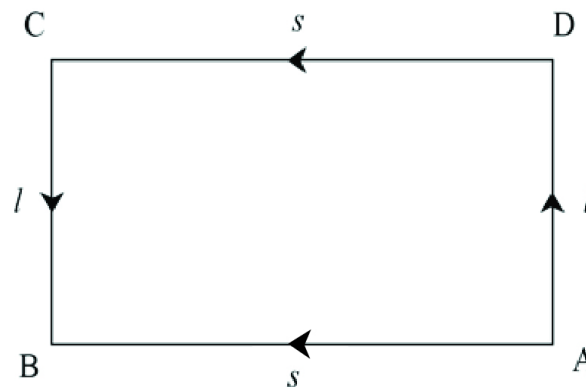
How do we understand this zero or non-zero value of the work done in terms of the simple definition:  
work = force  $\times$  displacement?

Here for the closed path, the net displacement is zero.

From this, it may appear that the work done should be zero irrespective of whether the force is conservative or non-conservative in nature. However, we must remember that, in general, for a closed path,

$$W = \int_s F \cdot dS$$

It is, therefore, the net value of this integral that would determine the net work done. For a conservative force this integral over a closed path is necessarily zero. However, for non-conservative forces, this integral is in general, not zero. Hence, the net work done (over a closed path) can be non-zero for non-conservative forces.



**Fig.6.3.2:** *work done is dependent on path taken in non-conservative force*

In Fig.6.3.2, if a block is moved through distance  $s$ , between two points A and B, then the change in its mechanical energy, due to the force of friction, will be  $-F \cdot s$ . When the block is moved through the path  $A \rightarrow D \rightarrow C \rightarrow B$ , then the change in its mechanical energy will be  $F \cdot (s + 2l)$ .

Thus, for a non-conservative force, the work done in going from one point (A) to another point (B), is dependent on the path taken to do that work.

### 6.3.3.2 Conservation of mechanical energy involves only conservative forces

The principle of conservation of mechanical energy states that the total mechanical energy of a system, or objects, remains constant if the objects interact through conservative forces only.

$$(\text{Initial kinetic energy}) + (\text{Initial potential energy}) = (\text{Final kinetic energy}) + (\text{Final potential energy})$$

i.e.

$$K_f - K_i = W = -\Delta V$$

$$\Delta(K + V) = 0$$

Thus, if the kinetic energy of a conservative system increases by some amount, the potential energy of this system must decrease by the same amount.

When a non-conservative force acts on a system, then its work contribution has to be added to (or subtracted from) the initial energy of the system in order to express the conservation of energy correctly.

Consider work done by conservative forces, (such as gravity), and also by non-conservative forces, (such as friction). The total work,  $W$  is the sum of the contributions of the conservative and non-conservative forces:

$$W = W_{\text{conservative}} + W_{\text{non-conservative}}$$

For conservative part, the work done  $W_{\text{conservative}}$  can be expressed in terms of change in potential energy,  $-\Delta V$ . From the work- kinetic energy theorem, we can write  $\Delta K = W$ .

$$\therefore \Delta(K + V) = W_{\text{non-conservative}}$$

The change, in mechanical energy, is thus seen to be equal to the work done by non-conservative forces.

### Example 6.3.1

Let a force  $\vec{F}$ , given by

$$\vec{F} = xy(\hat{i} + \hat{j})$$

act on a particle moving in the xy plane. Let the work done by this force, in moving the particle from point O to point P, via three paths as shown in Fig.6.3.3.

(i) OAP (ii) OBP and (iii) OP be  $W_1$ ,  $W_2$  and  $W_3$ , respectively.

Choose the correct option:

We can say that

- (A)  $W_1=W_2=W_3$ ; the force  $\mathbf{F}$  is a conservative force.
- (B)  $W_3 \neq W_1=W_2$ ; the force  $\mathbf{F}$  is a non-conservative force.
- (C)  $W_1 \neq W_2=W_3$ ; the force  $\mathbf{F}$  is a non-conservative force.
- (D)  $W_2 \neq W_3=W_1$ ; the force  $\mathbf{F}$  is a non-conservative force.

**Solution :**

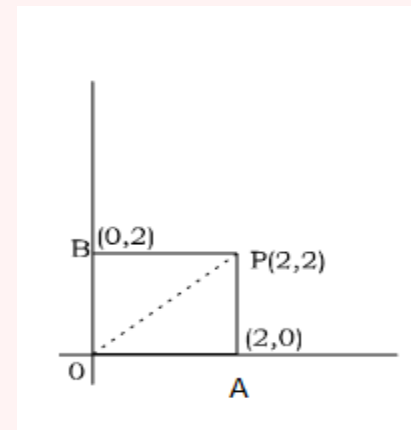
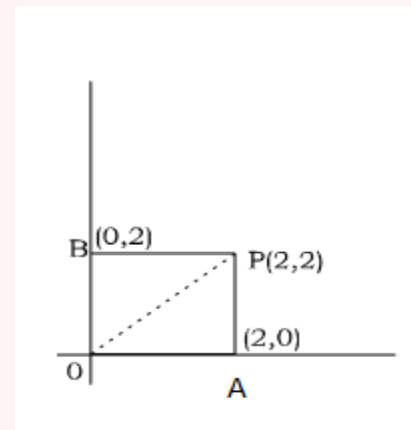
Correct option is option (B)

We have 
$$W = \int \vec{F} \cdot \vec{s} = \int [xy(\hat{i} + \hat{j}) \cdot (dx\hat{i} + dy\hat{j})]$$

$$= \int (xy)(dx + dy)$$

We, therefore, have

$$W_1 = \int_0^2 (x \times 0)(dx + 0) + \int_0^2 2 \cdot y(0 + dy) = 4 \text{ units}$$



**Fig. 6.3.3**

$$W_2 = \int_0^2 (0 \times y)(0 + dy) + \int_0^2 x \cdot 2(dx + 0) = 4 \text{ units}$$

$$W_3 = \int_0^2 x^2 (2dx) [\because \text{Along path OP, } x = y \text{ and } dx = dy, \text{ throughout}] = \frac{16}{3} \text{ units}$$

We thus find that

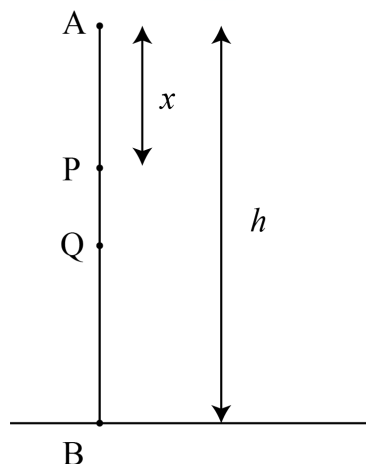
$$W_3 \neq W_1 = W_2$$

Hence, the force  $\vec{F}$  is a non-conservative force.



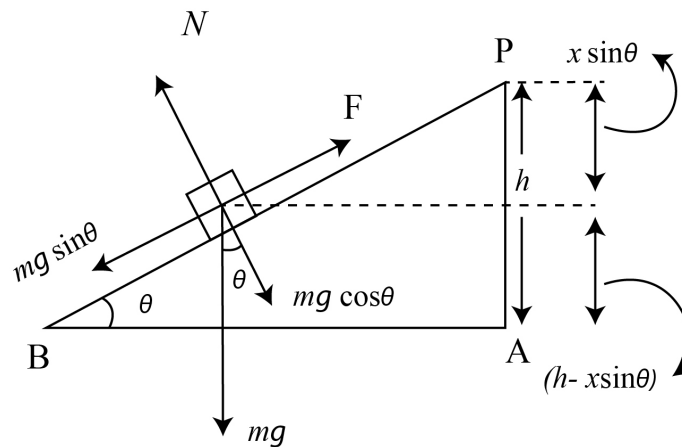
## Let Us Reflect

**Q 6.3.1** A body of mass  $m$  falls freely under gravity from a point A. It hits the ground at point B, which is directly below A, such that  $AB = h$ . Assuming that there is no air resistance, show that for any two positions P and Q in the path, the loss in potential energy (PE) is equal to the gain in kinetic energy (KE).



**Fig. 6.3.4**

**Q.6.3.2** A body of mass  $m$  is let slide down an inclined plane (Fig 6.3.5).



**Fig. 6.3.5**

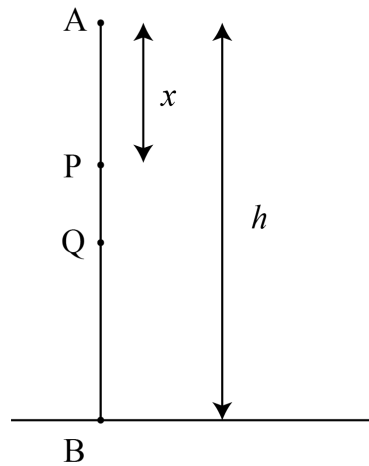
from the highest point P, which is at a height  $h$  from the bottom. The plane offers a uniform force of friction,  $F$  is the sliding motion of the body. If  $PQ = x$ , then what is the value of the total mechanical energy ( $KE + PE$ ) of the body at Q. Is it a constant? If not, then how do you attribute the loss in energy and what happens to this loss?

**Q 6.3.3** Show that for a projectile of mass  $m$ , projected with a velocity  $u$  in free space, the total mechanical energy ( $KE + PE$ ) remains constant. What is the value of the constant?



## Answers

**A.6.3.1** Refer to Fig. 6.3.4 (Redrawn for convenience)



**Fig. 6.3.4 (Redrawn)**

Let the velocity of the body at P be  $v$ .

Then,  $v^2 = 2gx$

$$\therefore (KE)_p = \frac{1}{2}mv^2 = \frac{1}{2}m(2gx) = mgx$$

$$(PE)_p = mg(PB) = mg(h - x) = mgh - mgx$$

$\therefore (KE)_p + (PE)_p = mgh$  = a constant independent of  $x$ , and it is equal to the initial  $PE$ . It may be noted that the total mechanical energy ( $KE + PE$ ) at A is fully potential and is equal to  $mgh$ .

So,  $KE + PE$  = a constant throughout the path

$$(KE)_p + (PE)_p = (KE)_Q + (PE)_Q$$

$$(PE)_p - (PE)_Q = (KE)_Q - (KE)_p$$

So, the loss in  $PE$  = the gain in  $KE$

**A.6.3.2** Total energy of the body at P is fully potential, and is equal to  $mgh$ .

The  $PE$  at Q =  $mg(h - x \sin \theta)$

$$= mgh - mgx \sin \theta \quad (6.3.1)$$

At Q, we have shown the two perpendicular components of the weight,  $mg$ , which are  $mg \cos \theta$  normal to the plane and  $mg \sin \theta$  along the plane. The normal reaction,  $N$  adjusts itself to be equal to  $mg \cos \theta$ .

The body

slides down due to  $mg \sin \theta$ , which is opposed by  $F$ , the force of friction. So, the resultant force, down the plane is equal to  $mg \sin \theta - F$ .

$$\begin{aligned}\text{So, the acceleration down the plane} &= \frac{mg \sin \theta - F}{m} \\ &= g \sin \theta - \frac{F}{m}\end{aligned}$$

So, the velocity of the body,  $v$ , at Q is given by

$$v^2 = 2(g \sin \theta - \frac{F}{m})x$$

$$\begin{aligned}\therefore KE \text{ at Q} &= \frac{1}{2}mv^2 \\ &= m(g \sin \theta - \frac{F}{m})x = mgx \sin \theta - Fx\end{aligned}\quad (6.3.2)$$

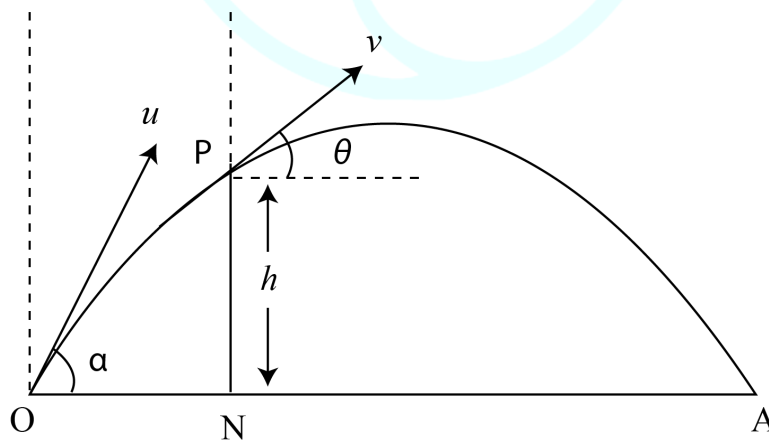
Adding Eq(6.3.1) and Eq(6.3.2), we get

$$(KE + PE)_Q = mgh - Fx \quad (6.3.3)$$

Eq(6.3.3) is not independent of  $x$ . So, the total energy is not constant. The loss =  $Fx$ , which is the work done against the force of friction.

This loss is called dissipation of energy and it gets transferred primarily into heat energy.

**A.6.3.3** Refer to Fig.6.3.6



**Fig.6.3.6**

The initial velocity of projection =  $u$

The velocity vector makes an angle  $\alpha$  with the horizon. The horizontal and the vertical components are  $u \cos \alpha$  and  $u \sin \alpha$  respectively.

At a point P, at a height  $h$  above the horizontal level, it attains a velocity  $v$  whose direction makes an angle  $\theta$  with the horizontal.

Since, the horizontal components remains unchanged, we have

$$v \cos \theta = u \cos \alpha \quad (6.3.4)$$

For the vertical component, we have

$$\begin{aligned} (v \sin \theta)^2 &= (u \sin \alpha)^2 - 2gh \\ \text{Or } v^2 \sin^2 \theta &= u^2 \sin^2 \alpha - 2gh \end{aligned} \quad (6.3.5)$$

Squaring both sides of Eq(6.3.4) and adding with Eq(6.3.5), we get

$$v^2 = u^2 - 2gh \quad (6.3.6)$$

So, the KE of the projectile at P is equal to  $\frac{1}{2}mv^2$

$$= \frac{1}{2}m(u^2 - 2gh) = \frac{1}{2}mu^2 - mgh$$

The PE =  $mgh$

So,  $KE + PE = \frac{1}{2}mu^2 = \text{a constant}$

= the initial value of energy, which is solely kinetic.

### 6.3.4 Potential energy of a spring

Elastic potential energy is the energy stored in elastic materials as a result of their stretching or compressing. Springs, rubber bands, etc., are the type of materials that are usually regarded as 'elastic'.

The amount of elastic potential energy stored in such an object depends on the fractional amount by which the device is stretched or extended.

Springs can store elastic potential energy due to either compression or extension. During compression, or extension, the restoring force developed in the spring, is proportional to the amount by which the spring is extended or compressed.

$$F = -kx \quad (6.3.7)$$

Here,  $k$  is called the spring constant of the spring.

The above expression implies that the force exerted, to extend or compress the spring (within its elastic limits), results in a restoring force that is equal in magnitude but opposite direction, to the applied force. Hence the negative sign.

Consider a spring-mass system. As the mass moves from point  $x_i$  to  $x_f$ , the spring force ( $F = -kx$ ) does work on the mass. The corresponding change in the elastic potential energy, of the spring-mass system, is given by,

$$\begin{aligned} \Delta U &= - \int_{x_i}^{x_f} F \cdot dx = - \int_{x_i}^{x_f} (-kx) dx = k \int_{x_i}^{x_f} x dx = \frac{1}{2} k [x^2]_{x_i}^{x_f} \\ &= \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2 \end{aligned} \quad (6.3.8)$$

To associate a potential energy value  $U$  with the block at position  $x$ , we choose the reference configuration to be the one for which the spring is at its relaxed length and the block is at  $x_i = 0$ . For this position, the elastic potential energy  $U_i$  is taken as 0. Hence Eq(6.3.8) becomes

$$U - 0 = \frac{1}{2} k x^2 - 0$$

which gives us,

$$U(x) = \frac{1}{2} k x^2 \quad (6.3.9)$$

Thus, the potential energy is the energy that is stored in an object due to its position relative to some standard reference, or 'zero' position.

From Eqs (6.3.7) and (6.3.9) we can say that if the potential energy,  $U(x)$  of a spring, increases with increasing  $x$ , the force (developed in the spring) will tend to move it toward smaller  $x$  to decrease the potential energy. Hence, this force is called a restoring force.

### 6.3.5 Spring constant and its dependence on material and geometry

The spring constant is a measure of the stiffness of the spring. The higher the value of  $k$ , the stiffer the spring. The value of this constant depends on the nature of the material and a number of other parameters of the spring. The spring constant is related to the elastic modulus of the spring material and to the spring parameters, (such as wire diameter, outer diameter of the spring and the number of turns in its windings).

#### Dependence of spring constant on length of the spring

The spring constant  $k$ , for a given spring, is inversely proportional to the length of the spring. Thus

$$k \propto \frac{1}{l}$$

This means that if a given spring is cut into two equal pieces, the spring constant of each piece will get doubled i.e. becomes  $2k$ .

#### Activity 6.3.1

Try to get different springs having different spring constants. Determine the value of their spring constants by the static and the dynamic methods. Also study the dependence of the spring constant, of a given spring, on its length.

##### (a) Static method:

$$\text{Extension of Spring} = \frac{g}{k} \left( M + \frac{M_s}{2} \right)$$

where  $M_s$  is the mass of the spring.

##### (b) Dynamic method:

$$\text{Time Period of oscillation} = 2\pi \sqrt{\frac{M + \frac{M_s}{3}}{k}}$$

Take a spring. Find its spring constant by any of the two methods mentioned above. Now cut the spring into two equal parts. Find the spring constant of one of the cut parts and compare it with that of the complete spring.

[It is known that for a helical spring, consisting of  $N$  windings, with wire diameter  $d$ , and outer spring diameter  $D$ , the spring constant  $k$  is given by,  $k = \frac{Gd^4}{8(D-d)^3N}$  where  $G$  is the shear modulus of the material of the spring.]

### 6.3.6 Changes in mechanical energy

When the (mechanical) energy of an object changes, we usually tend to think of it only in terms of the change in its kinetic energy. Thus, when a moving object loses energy, we tend to say that it would 'slow down', i.e. its kinetic energy would decrease.

This, however, is not true in general. We should view the change in (mechanical) energy of an object as the change in the sum total of its kinetic and potential energies. Thus, in general, change in (mechanical) energy

$$= (KE + PE)_{\text{Final}} - (KE + PE)_{\text{Initial}}$$

This general result can help us to understand situations where the kinetic energy of an object may increase even when its total energy is decreasing.

A well-known example of this type of situation is provided by the well-known Rutherford model of the hydrogen atom. Here the orbiting electron was thought of as 'radiating energy' so that its total energy was decreasing. It was also said to be 'spiralling inwards' towards the nucleus which was leading to an increase in its kinetic energy. However, this (inward spiralling) led to a decrease in its potential energy and this decrease in its potential energy of the electron was more than the increase in its kinetic energy. The total energy of the electron was, therefore, decreasing. Here, in this case, the kinetic energy of the electron was increasing even when its total energy was decreasing as quoted above, i.e., We, therefore, need to remember the result;

$$\text{Change in (Mechanical) energy} = (KE + PE)_{\text{Final}} - (KE + PE)_{\text{Initial}}$$



### Let Us Reflect

**Q.6.3.4** A spring whose spring constant is equal to  $k$ , is divided into three equal parts. What will be the spring constant of each part?

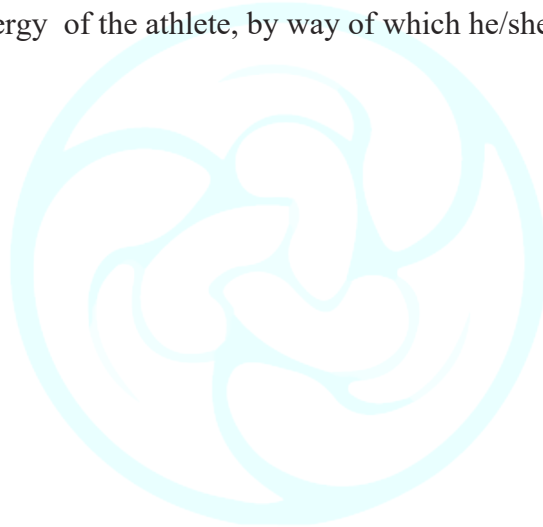
**Q.6.3.5** Watch a video of any pole vaulter executing the jump. Describe the nature of transformation of mechanical energy that takes place.



## Answers

**A.6.3.4** We know that the spring constant is inversely proportional to the length of the spring . So, the spring constant for each part will be  $3k$ .

**A.6.3.5** Sum of kinetic energy of running of the athlete and elastic potential energy of the pole is converted into the gravitational/potential energy of the athlete, by way of which he/she attains height.



### 6.3.7 Summary

In this module we have learnt that

- The term potential energy relates to energy stored in a system.
- changes in potential energy can be uniquely defined and determined.
- The value of potential energy, assigned (or associated) to an object, depends on the choice of the zero of potential energy; the value, therefore, changes with the change in the reference level chosen.
- The potential energy of an object moving in an attractive/(a repulsive) force field is (conventionally) taken as negative/(positive).
- A force is said to be conservative if the work it does on an object, in moving it between two given points, is independent of the path the object takes to go between the points. Thus the work done by a conservative force depends only on the initial and final positions of the object.
- a force is non-conservative if it involves a dissipation of mechanical energy. The work done by such a force, depends on the path followed in going from one point to another.
- under the action of a conservative force, the total mechanical energy of a system remains conserved.
- there is a potential energy associated with any spring that has been compressed / extended from its normal length.
- The spring constant is a measure of the stiffness of the spring. The higher the value of  $k$ , the stiffer the spring. The value of this constant depends on the nature of the material and a number of other parameters of the spring.

## Development Team

N.K.Sehgal, Former Associate Professor, Hans Raj College, Delhi University

## Review Team

C.K Ghosh, Former Director, NCIDE, IGNOU, New Delhi

Surender Kumar, Vice Principal, KVS, Kannaur, Kerala

Anuradha Rajesh, Physics Teacher, Shaheed Rajpal DAV Public School, Delhi

Vikrant Bhardwaj, PGT, KVS, Gole Market, New Delhi

Parvinder Singh Bhatia, Physics Teacher, Lajpat Nagar, New Delhi

## Member- Coordinator

Shashi Prabha, Professor, CIET, NCERT, New Delhi

## Technical support

Riya Rani, JPF, CIET, NCERT, New Delhi

Bhawana Bhatt, Technical Content Developer, CIET, NCERT, New Delhi

Rajat Kumar, Graphic Designer, CIET, NCERT, New Delhi