

Module 4 Elastic and Inelastic Collision

This module is designed to help you to:

- differentiate between elastic and inelastic collisions;
- explain that it is only in the case of one-dimensional (or head-on) collisions that one can work out the complete details of the velocities of the two colliding objects after the collision;
- explain the techniques of solving a wide variety of one-dimensional elastic/inelastic collision problems;
- compare the perfectly elastic and perfectly inelastic collisions.

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6.4.1 Introduction

A collision is an event where two or more bodies exert forces on each other for a relatively short interval of time. The study of collisions finds its applications in a number of fields like safety studies in automobiles, collision of molecules in kinetic theory of gasses, collision of charged particles in motion in a conductor etc.

In this module, we shall differentiate between elastic and inelastic collisions and solve a wide variety of one dimensional elastic and inelastic collisions. We will also understand the concept of coefficient of restitution and find its limiting values for perfectly elastic and inelastic collisions.

6.4.2 Elastic and inelastic collision

For any collision, the momentum before collision and after collision is equal. Thus total momentum is always conserved. However, total kinetic energy may not always be conserved in a collision. This may be because a part of the kinetic energy may get converted to heat, sound, or some other form of energy.

Inelastic collisions are those collisions in which kinetic energy is not conserved. For example, when a rubber ball hits the ground, some of its kinetic energy can get lost in the deformation of the ball.

When two objects collide, and stick together, the collision is called completely inelastic. If two pieces of (say) molding wax collide, they (usually) get stuck together and move with some common velocity after collision. Such an inelastic collision (in practice), does not always lead to a total loss of kinetic energy.

An elastic collision is one in which both momentum and kinetic energies are conserved. Billiard ball collisions are highly elastic. But even in the case of billiard balls collision, some kinetic energy is lost in deformation of the surface or as sound energy. A pure elastic collision is, therefore, an idealized concept which, generally, does not get realized in practical situations.

Perfectly elastic collisions do occur between atomic and subatomic particles.

[Note: We, sometimes, speak of what may be called super-elastic collisions. These are collisions in which the total KE of the colliding particles, after the collision, is more than their total KE before the collision. This can

happen if some other 'source' contributes to the kinetic energies of the colliding particles. For example, if two moving billiard balls collide near the edge of a table and slide over smoothly to a (slightly) lower surface, their total kinetic energy after the collision, can be more than that before the collision. This increase is due to the decrease in their potential energy. However, since such collisions would involve some external source they are strictly speaking, not restricted to the two colliding objects. It is, therefore, debatable as to whether such interactions, between two objects, should be regarded only as a collision between those two objects.]

6.4.3 Coefficient of restitution (e)

We know that if a ball (tennis) were dropped vertically from a height (H) on a floor, it bounces back. We also know that it does not bounce back to the same height; this height (h), to which it bounces back, depends on the nature of the floor on which it has been dropped. We introduce a quantity, e , called the coefficient of restitution, that is characteristic of 'each floor' and that helps us to determine the height to which the ball would bounce back.

The velocity of the ball, at the instant, it just hits the floor, equals $\sqrt{2gH}$. Since the ball bounce back to a height h , its velocity of rebound, from the floor, is $\sqrt{2gh}$. The floor, of course, remains at rest, both before and after the ball hits it/rebounds from it. Hence, the magnitudes of the relative velocity, between the ball and the floor, before and after impact, are $\sqrt{2gH}$ and $\sqrt{2gh}$, respectively.

We define the coefficient of restitution, e , through the relation,

$$e = \frac{\text{relative velocity just after impact/collision}}{\text{relative velocity just before impact/collision}}$$

In the above example, we then have

$$e = \sqrt{\frac{h}{H}}$$

The definition of e , given above, is generalized to any collision between any two given objects. Thus, in general, for a collision between any two objects,

Coefficient of restitution $(e) = \frac{\text{relative velocity of the colliding objects just after collision}}{\text{relative velocity of the colliding objects just before collision}}$

There is no loss of kinetic energy during a perfectly elastic collision. Also, there is a complete loss of kinetic energy during a perfectly inelastic collision. We, therefore, say

(i) for a perfectly elastic collision,

Coefficient of restitution, $(e) = 1$

(ii) for a perfectly inelastic collision,

Coefficient of restitution, $(e) = 0$

For all practical situations, we would have

$$0 < |e| < 1$$



Let Us Reflect

Q.6.4.1 A ball of mass m_1 is moving along a straight line with a velocity u_1 collides with another ball of mass m_2 moving with a velocity u_2 along the same straight line ($u_1 > u_2$). After the collision the balls of masses m_1 and m_2 move along the same straight line with velocities v_1 and v_2 respectively ($v_2 > v_1$). Prove that if the collision is elastic, the coefficient of restitution is equal to 1.



Answers

A.6.4.1



Fig. 6.4.1

Refer to Fig 6.4.1. It represents the collision between m_1 and m_2 . The * symbol schematically represents collision.

Since the collision is elastic, both linear momentum and kinetic energy are conserved. So we have,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad (6.4.1)$$

$$\text{And, } \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad (6.4.2)$$

And, the coefficient of restitution is given by

$$e = \frac{\text{relative velocity of approach}}{\text{relative velocity of separation}}$$

$$\text{So, } e = \frac{u_1 - u_2}{v_2 - v_1} \quad (6.4.3)$$

From Eq(6.4.1), we get

$$m_1(u_1 - v_1) = m_2(v_1 - u_2) \quad (6.4.4)$$

From Eq (6.4.2), we get

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad (6.4.5)$$

Dividing Eq(6.4.5) by Eq(6.4.4), we get

$$u_1 + v_1 = v_2 + u_2$$

So,

$$u_1 - u_2 = v_2 - v_1$$

∴ From Eq.(6.4.3), we get

$$e = \frac{u_1 - u_2}{v_2 - v_1} = 1$$

Example 6.4.1

Three balls of masses m_1 , m_2 , and m_3 are all positioned on a smooth straight track as shown in fig 6.4.2. The first ball, of mass m_1 , is given a velocity v_1 , along the straight track and hit ball 2. The ball 2 in turn, hits ball 3.

Assuming all collisions to be (ideal) elastic collisions, show that the velocity acquired by ball 3, would be maximum if the mass m_2 , of the second ball, is the geometric mean of the masses m_1 and m_3 .

Solution

Since, collisions are elastic in nature. Both KE and momentum are conserved during these collisions. The velocity v_2 , acquired by the second ball, due to the impact of the first ball is then given by

$$v_2 = \frac{2m_1}{m_1 + m_2} v_1$$



Fig.6.4.2

The velocity v_3 , acquired by the third ball, due to the impact of the second ball, is similarly given by

$$v_3 = \frac{2m_2}{m_2 + m_3} v_2 = \left[\frac{4m_1 m_2}{(m_1 + m_2)(m_2 + m_3)} \right] v_1$$

The value of m_2 , for which v_3 becomes maximum (for given values of m_1 , m_3 and v_1), is given by

$$\frac{dv_3}{dm_2} = 0$$

Now,

$$v_3 = \left[\frac{4m_1 m_2}{(m_1 + m_2)(m_2 + m_3)} \right] v_1$$

$$\therefore \frac{dv_3}{dm_2} = \left[\frac{(m_1 + m_2)(m_2 + m_3)(4m_1) - 4m_1 m_2[(m_1 + m_2) \times 1 + (m_2 + m_3) \times 1]}{(m_1 + m_2)^2 (m_2 + m_3)^2} \right]$$

The numerator would be zero if

$$(m_1 + m_2)(m_2 + m_3)m_1 - m_1 m_2(m_1 + 2m_2 + m_3) = 0$$

$$\text{Or if } m_1[m_1 m_2 + m_2^2 + m_1 m_3 + m_2 m_3] - m_1 m_2(m_1 + 2m_2 + m_3) = 0$$

Or if $m_1 m_2^2 + m_3 m_1^2 - 2m_1 m_2^2 = 0$

Or if $m_1 m_3 - m_2^2 = 0$

Or if $m_2^2 = m_1 m_3$

i.e. $m_2 = \sqrt{m_1 m_3}$

Thus v_3 becomes maximum if m_2 is the geometric mean of m_1 and m_3 .

6.4.4 Head on collision

A collision is said to be a one-dimensional, or head-on collision if the two colliding particles, (or (effectively) point objects), after their collision, keep on moving along the line joining them before the collision.

Activity 6.4.1

An experiment on head on collision

In performing an experiment on collision on a horizontal surface, we face three difficulties.

- (i) There will be friction and bodies will not move with uniform velocities.
- (ii) If two bodies of different sizes collide on a table, it would be difficult to arrange them for a head-on collision unless their centers of mass are at the same height above the surface.
- (iii) It will be fairly difficult to measure velocities of the two bodies, just before, and just after, the collision.

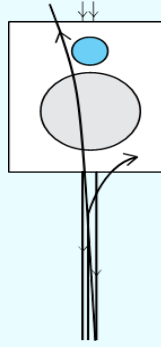


Fig. 6.4.3 *Head on collision*

By performing this experiment in a vertical direction, all these three difficulties can be overcome. Take two balls, one of which is heavier (basketball/football/volleyball) and the other lighter (tennis ball/rubber ball). First take only the heavier ball and drop it vertically from some height, say 1 m. Note the height to which it rises. This gives the velocities near the floor or ground, just before and just after the bounce (by using $v^2 = 2gh$). Hence, you will get the coefficient of restitution. Now take a big ball and a small ball and hold them in your hands one over the other, with the heavier ball below the lighter one, as shown here. Drop them together, taking care that they remain together while falling, and see what happens. You will find that the heavier ball rises less than when it was dropped alone, while the lighter one shoots up to about 3 m. With practice, you will be able to hold the balls properly so that the lighter ball rises vertically up and does not fly sideways. This is then a head-on collision.

You can try to find the best combination of balls which gives you the best effect. You can measure their masses on a standard balance.

We leave it to you to think how you can determine the initial and final velocities of the balls.

6.4.5 Collisions in two dimensions

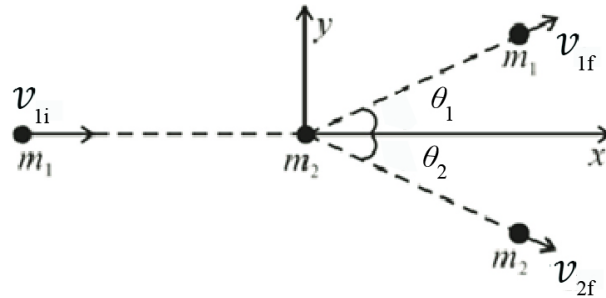


Fig.6.4.4: The collision of a moving mass, m_1 , with a stationary mass, m_2

Fig. 6.4.4 shows the collision of a moving mass, m_1 , with a stationary mass, m_2 . Linear momentum is conserved in such a collision. Since momentum is a vector, this implies three equations for the three directions $\{x, y, z\}$. Consider the plane determined by the final velocity directions of m_1 and m_2 and choose it to be the $x - y$ plane. The conservation of the z - component of the linear momentum implies that the entire collision is in the x - y plane. The x - and y - component equations are

$$m_1 v_{1i} = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2 \quad (6.4.6)$$

$$0 = m_1 v_{1f} \sin \theta_1 - m_2 v_{2f} \sin \theta_2 \quad (6.4.7)$$

One knows $\{m_1, m_2, v_{1i}\}$ in most situations. There are thus four unknowns $\{v_{1f}, v_{2f}, \theta_1 \text{ and } \theta_2\}$, If $\theta_1 = \theta_2 = 0$, we will regain the equation for a one dimensional collision.

Further, for an elastic collision;

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (6.4.8)$$

We thus obtain an additional equation. That however, still leaves us one equation short. At least one of the four unknowns, say θ_1 , must be known for the problem to be solvable. For example, θ_1 can be determined by rotating a detector from the x to the y axis. Given $\{m_1, m_2, v_{1i}, \theta_1\}$ we can then determine $\{v_{1f}, v_{2f}, \theta_2\}$ from Eqs. (6.4.6)-(6.4.8).

Example 6.4.2

Two balls, of masses m_1 and m_2 , moving with velocities $\vec{v}_1 = v_1 \hat{i}$ and $\vec{v}_2 = v_2 \hat{j}$ collide at the origin where they coalesce to form a single ball. Find the velocity of this single ball and also calculate the kinetic energy lost in this collision.

Solution:

Here, the two balls undergo an inelastic collision at the origin. Hence, as per the law of conservation of momentum, we have

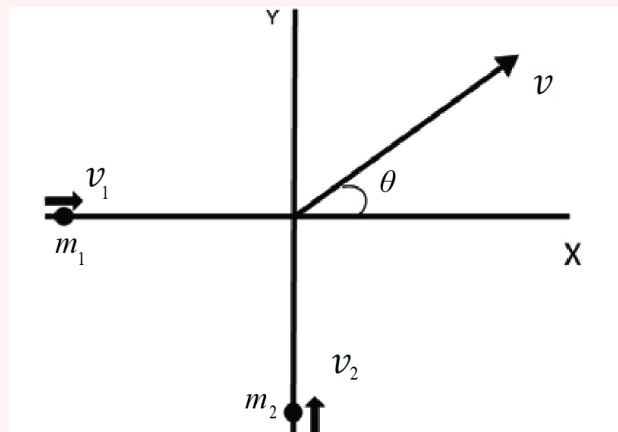


Fig 6.4.5

$$(m_1 + m_2)v \cos \theta = m_1 v_1 \quad (6.4.9)$$

$$\text{And } (m_1 + m_2)v \sin \theta = m_2 v_2 \quad (6.4.10)$$

Dividing Eq(6.4.10) by Eq(6.4.9) we get

$$\therefore \tan \theta = \left(\frac{m_2 v_2}{m_1 v_1} \right) \quad (6.4.11)$$

Also, by squaring and adding Eq(6.4.9) and Eq(6.4.10), we get

$$\begin{aligned} (m_1 + m_2)^2 v^2 &= m_1^2 v_1^2 + m_2^2 v_2^2 \\ \therefore v &= \left(\frac{m_1^2 v_1^2 + m_2^2 v_2^2}{(m_1 + m_2)^2} \right)^{1/2} \end{aligned} \quad (6.4.12)$$

The coalesced single ball thus moves with a velocity v (given by Eq(6.4.12)) along a direction inclined at an angle θ (given by Eq(6.4.11)) to the x-axis.

Loss in KE

$$= \frac{1}{2} [m_1 v_1^2 + m_2 v_2^2] - \frac{1}{2} \left(\frac{m_1^2 v_1^2 + m_2^2 v_2^2}{(m_1 + m_2)} \right)$$

It is interesting to note here that in this inelastic collision, we are able to get complete information about the velocity after the collision. This is because, in this inelastic collision, we need to calculate v and θ for only one object. Hence, we need only two equations which are provided by the law of conservation of momentum.



Let Us Reflect

Q.6.4.2 If in case of Q. 6.4.1 under let us reflect, $m_1 = 3$ kg, $m_2 = 1$ kg and with respect to an initial frame of reference, if u_1, u_2 and v_2 are 4, 2 and 5 in ms^{-1} , then what is the value of v_2 in ms^{-1} ?

If the velocities get transformed to u'_1, u'_2, v'_1, v'_2 with respect to another inertial frame of reference moving with a uniform velocity of 1 ms^{-1} with respect to the earlier frame of reference, will the linear momentum remain conserved in the new inertial frame of reference?



Answers

A.6.4.2

$$m_1 u_1 + m_2 u_2 = (3)(4) + (1)(2) = 14$$

$$m_1 v_1 + m_2 v_2 = 3 v_1 + (1)(5) = 3v_1 + 5$$

$$\therefore 3v_1 + 5 = 14$$

$$\text{Or, } 3v_1 = 9,$$

$$v_1 = 3 \text{ ms}^{-1}$$

So, we have the values of the velocities in ms^{-1} as

$$u_1 = 4, \quad u_2 = 2, \quad v_1 = 3, \quad v_2 = 5$$

So, with respect to the new inertial frame of reference, the values (in ms^{-1}) of the velocities would become,

$$u'_1 = u_1 + 1 = 4 + 1 = 5$$

$$u'_2 = u_2 + 1 = 2 + 1 = 3$$

$$v'_1 = v_1 + 1 = 3 + 1 = 4$$

$$v'_2 = v_2 + 1 = 5 + 1 = 6$$

Now,

$$m_1 u'_1 + m_2 u'_2 = (3)(5) + (1)(3) = 15 + 3 = 18 \text{ kgms}^{-1}$$

$$\text{And, } m_1 v'_1 + m_2 v'_2 = (3)(4) + (1)(6) = 12 + 6 = 18 \text{ kgms}^{-1}$$

$$\text{So, } m_1 u'_1 + m_2 u'_2 = m_1 v'_1 + m_2 v'_2$$

Hence, the linear momentum remains conserved.

6.4.6 Ballistic pendulum

A ballistic pendulum is an arrangement which demonstrates the practical use of inelastic collision.

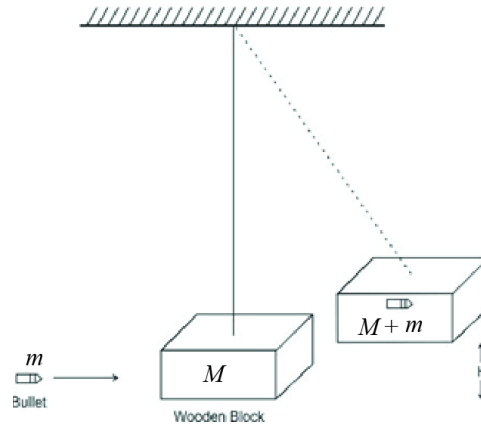


Fig 6.4.6: ballistic pendulum

Fig 6.4.6 shows an arrangement for finding the speed of a bullet. The system is called a ballistic pendulum. The velocity of the bullet can be found by using conservation of energy and conservation of momentum.

m = Mass of bullet

v = velocity of bullet

M = mass of wooden block

From the law of conservation of momentum

$$(M + m)V = mv + 0$$

$$\text{Or, } V = \left(\frac{mv}{M+m}\right)$$

Here V is the velocity of the system (Block + Bullet) after the bullet gets embedded in the block.

Because of this velocity the combined system would rise through a height H .

$$\begin{aligned} (M + m)gH &= \frac{1}{2} (M + m)V^2 \\ &= \frac{1}{2} (M + m)\left(\frac{mv}{M+m}\right)^2 \end{aligned}$$

$$\therefore H = \frac{1}{2} \left[\frac{m^2 v^2}{g(M+m)} \right]$$

$$\therefore v = \frac{\sqrt{2g(M+m)H}}{m}$$

Thus, knowing the masses, the height to which the combined system rises, we can find the velocity of the bullet.

Here it is interesting to note that during this collision, kinetic energy is not conserved. We see that kinetic

$$\begin{aligned}
 \text{energy of the system after collision is} &= \frac{1}{2} (M + m)V^2 \\
 &= \frac{1}{2} (M + m)\left(\frac{mv}{M+m}\right)^2 \\
 &= \left(\frac{m}{M+m}\right)\frac{1}{2}(mv)^2
 \end{aligned}$$

Thus, the kinetic energy after the collision is less than the kinetic energy before collision. This collision is, therefore, inelastic.

Note: The kinetic energy decreases due to the work done by the bullet in overcoming the resistance of the block during penetration and the energy lost is evolved as heat.



Let Us Reflect

Q.6.4.3 A bullet of mass 0.012 kg and horizontal speed 70 ms^{-1} strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.



Answers

A.6.4.3 We know that, with symbols having their usual meanings,

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad (6.4.13)$$

Here, m_1 = mass of the bullet = 0.012 kg

$$m_2 = 0.012 + 0.4 = 0.412 \text{ kg}$$

$$v_{1f} = 70 \text{ ms}^{-1}$$

From Eq(6.4.13)

$$0.012 \times 70 = 0 + 0.412 \times v_{2f}$$

$$v_{2f} = 2.04 \text{ ms}^{-1}$$

We know,

$$v_{2f}^2 = 2gh$$

$$h = \frac{v_{2f}^2}{2g} = \frac{(2.04)^2}{2 \times 9.8} = 21.2 \text{ cm}$$

$$\text{Kinetic energy before collision} = \frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} \times 0.012 \times (70)^2 \text{ J} = 29.4 \text{ J}$$

$$\begin{aligned} &= \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \\ &= [0 + \frac{1}{2} (0.012 + 0.4) \times (2.04)^2] \text{ J} \\ &= 0.857 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{Thus, the amount of heat produced in the block} &= [29.4 - 0.857] \text{ J} \\ &= 28.5 \text{ J} \end{aligned}$$

6.4.7 Summary

- A collision is an event where two or more bodies exert forces on each other for a relatively short interval of time.
- For any collision, total momentum is always conserved.
- Inelastic collisions are those collisions in which kinetic energy is not conserved.
- An elastic collision is one in which both momentum and kinetic energies are conserved.
- Coefficient of restitution (e) = $\frac{\text{relative velocity of the colliding objects just after collision}}{\text{relative velocity of the colliding objects just before collision}}$
- There is no loss of kinetic energy during a perfectly elastic collision. Also, there is a complete loss of kinetic energy during a perfectly inelastic collision. We, therefore, say
 - (i) for a perfectly elastic collision,

Coefficient of restitution, $(e) = 1$

(ii) for a perfectly inelastic collision,

Coefficient of restitution, $(e) = 0$

- For all practical situations, we would have

$$0 < |e| < 1$$

- A collision is said to be a one-dimensional, or head-on collision if the two colliding particles, (or (effectively) point objects), after their collision, keep on moving along the line joining them before the collision.
- A ballistic pendulum is an arrangement which demonstrates the practical use of inelastic collision.



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