

Work , Energy and Power

Answers

Module 1 Work and Power

A.6.1.1 (C) The force 500N acts in a direction opposite to the motion of the bicycle .So work done by the road = (-500N) (10m) = - 5000 J. The road does not move .So the work done on the road is zero.

Option (A), if selected, would be incorrect as here, the student is assuming the validity of Newton's third law for the two cases of work also. The work done by the cycle, on the road, is zero as the cycle does not cause any displacement of the road.

In option (B), we have the additional error of attaching a +ve sign to the work in the first case. The work, done by the road on the cycle, is a negative work, as the force exerted by the road on the cycle is directed opposite to its displacement.

In option (D), Work in the first case has been incorrectly taken as zero and in the second case work (which is zero) has been given a finite value.

A.6.1.2 (A)

Force of friction = $\mu N = \mu mg$

$$= 0.1 \times 2 \times 10 \text{ N} = 2 \text{ N}$$

$$\therefore \text{Net force on the body} = 18 \text{ N} - 2 \text{ N} = 16 \text{ N}$$

$$\text{Net acceleration} = 16 \text{ N} / 2 \text{ kg} = 8 \text{ m/s}^2$$

$$\therefore \text{Distance moved in } 10 \text{ s} = 0 + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 8 \times 10^2 = 400 \text{ m}$$

$\therefore$ Work done by the applied force

$$= 18 \text{ N} \times 400 \text{ m} = 7200 \text{ J}$$

Work done by the force of friction

$$= -2 \text{ N} \times 400 \text{ m} = -800 \text{ J}.$$

A.6.1.3 (B)

We have,

$$\text{Instantaneous power} = \vec{F} \cdot \vec{v}$$

$$= \vec{F} \cdot \left[\frac{\vec{F}}{m} t \right] = \frac{t}{m} F^2$$

$$\therefore \frac{t}{m} F^2 = k t^{\frac{3}{2}}$$

Where k = a constant

$$\therefore F^2 = (km) t^{\frac{1}{2}} = c t^{\frac{1}{2}}$$

Where $c = km$ = a constant

$$\text{Hence, } |\vec{F}(t)| = (\text{a constant}) t^{\frac{1}{2}}$$

The other options, if selected, would be due to an incorrect use of the formula for instantaneous power.

A.6.1.4 (D)

We have, for a constant force, $W = \vec{F} \cdot \vec{S}$ Here $\vec{F} = (-2\hat{i} + 3\hat{j} + 4\hat{k})$

Also,

$$\vec{s} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} = (3 - 1)\hat{i} + (4 - 2)\hat{j} + (5 - 3)\hat{k} = 2\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\therefore W = \vec{F} \cdot \vec{s} = (-4 + 6 + 8) = 10 \text{ units}$$

A.6.1.5 (C)

$$\text{Here, } W = \int_0^H \frac{GMm}{(R+x)^2} dx = \left| -\frac{GMm}{(R+x)} \right|_0^H = GMm \left[\frac{1}{R} - \frac{1}{R+H} \right] = \frac{GMmH}{R(R+H)} = m \cdot \frac{GMH}{R^2} \left(1 + \frac{H}{R} \right)^{-1}$$

A.6.1.6 (C) It is given that, $R = 6400$ km. So, 3200 km $= \frac{R}{2}$. So, the object is being raised from $x = R$ to

$$x = R + \frac{R}{2} = \frac{3R}{2}$$

$$W = - \int_{x=R}^{x=\frac{3}{2}R} \frac{GMm}{x^2} dx = GMm \left| -\frac{1}{x} \right|_{\frac{3}{2}R}^R = GMm \left| \frac{2}{3R} - \frac{1}{R} \right| = -GMm \left(\frac{1}{3R} \right) = -\frac{GM}{R^2} m \frac{R^2}{3R} = -mg \frac{R}{3}$$

$$W = -100 \times 10 \times \frac{6400}{3} \times 10^3 \text{ J} = -2.13 \times 10^9 \text{ J}$$

$\therefore$ Work by the gravitational force

$$\cong -2.13 \times 10^9 \text{ J}$$

[Note: By using $W = m g H$, we get

$$W = 100 \times 10 \times 3200 \times 10^3 \text{ J} = 3.2 \times 10^9 \text{ J}$$

This is incorrect as it does not take into account the variation of g with H . Here H is not negligible compared to R]

A.6.1.7 (B)

$$P = \vec{F} \cdot \vec{v} = \text{a constant} = k, \text{ say}$$

$$\therefore m \frac{d\vec{v}}{dt} \cdot \vec{v} = k$$

$$\therefore \frac{1}{2} \frac{d}{dt} (v^2) = k$$

$$\text{On } (v^2) = 2kt + A, A = \text{a constant}$$

It is given that, $v = 0$ at $t = 0$.

$$\text{So, } A = 0$$

$$\text{Or } v^2 = 2kt$$

$$\therefore v = \sqrt{2k} t^{\frac{1}{2}}$$

$$\therefore \frac{ds}{dt} = \sqrt{2k} t^{\frac{1}{2}}$$

$$\therefore s = \sqrt{2k} t^{\frac{3}{2}} + B, \text{ where } B = \text{a constant}$$

$$\text{As } s = 0 \text{ for } t = 0, B = 0$$

$$\therefore s = \sqrt{2k} t^{\frac{3}{2}}$$

As per Option (A), the displacement varies linearly with time, whereas displacement varies as $t^{\frac{3}{2}}$. Hence Option (A) is incorrect.

For (B), the displacement varies with time as $t^{\frac{3}{2}}$. The graph should be a non-linear one. Also, its slope, given by $(\text{slope} = \frac{ds}{dt} = v)$ should increase with time. Hence the Option (B) is correct.

For(C), the displacement varies with time as $t^{\frac{3}{2}}$. The graph should be a non-linear one. But as per Option (C) displacement remains constant with time. Hence Option (C) is incorrect.

For(D), the displacement varies with time as $t^{\frac{2}{3}}$. The graph should be a non-linear one. Also, its slope is given by $(\text{slope} = \frac{ds}{dt} = v)$ should increase with time. We therefore have to rule out Option (D) as its slope is

decreasing with time.

For option (B) the instantaneous slope is increasing with time as required.

A.6.1.8 (B)

$$\text{Here, } v = \left(\frac{dx}{dt} \right) = ax^{3/2}$$

$$\therefore \text{Acceleration} = \frac{dv}{dt} = \frac{3}{2}ax^{1/2} \frac{dx}{dt}$$

$$= \frac{3}{2}ax^{1/2}(ax^{3/2})$$

$$= \frac{3}{2}a^2x^2$$

$$\therefore \text{Force} = F(x) = \frac{3}{2}ma^2x^2$$

$$\text{Work done} = \int_{x=0}^{x=2} F(x)dx$$

$$= \frac{3}{2}ma^2 \left| \frac{x^3}{3} \right|_0^2$$

$$= \frac{1}{2}ma^2 \left| x^3 \right|_0^2$$

$$= 4ma^2$$

$$= 4 \times 0.5 \times (5)^2 \text{ J} = 50 \text{ J}$$

A.6.1.9 (A)

$$F_{net} = mg - kv = mg - k(\sqrt{2gx})$$

$$dW = F(-dx) = [mg - k\sqrt{2gx}](-dx)$$

$$W = \int_H^0 F(-dx) = \int_H^0 [mg - k\sqrt{2gx}]dx = mgH - k\sqrt{2g} \left| \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right|_0^H = mgH - \frac{2}{3}\sqrt{2gk^2H^3}$$

A.6.1.10 (C)

The instantaneous acceleration $\vec{a}(t)$ of the object is given by

$$\vec{a}(t) = \frac{\vec{F}(t)}{m} = \left(\frac{k}{m}t\right)\hat{i}$$

$$\therefore \frac{d\vec{v}}{dt} = \left(\frac{k}{m}t\right)\hat{i}, \quad \therefore \vec{v}(t) = \left(\frac{k}{m}\frac{t^2}{2}\right)\hat{i}$$

$$(\text{as } \vec{v}(0) = 0)$$

$$\text{Hence } \frac{d\vec{x}}{dt} = \left(\frac{k}{m}\frac{t^2}{2}\right)\hat{i}$$

$$\therefore \vec{x}(t) = \left(\frac{k}{m}\frac{t^3}{6}\right)\hat{i} \quad (\text{as } \vec{x}(0) = 0)$$

The instantaneous power, $P(t)$, is given by

$$P(t) = \vec{F}(t) \cdot \vec{v}(t) = (kt\hat{i}) \cdot \left(\frac{k}{m}\frac{t^2}{2}\right)\hat{i} = \frac{k^2}{2m}t^3$$



Module 2 Work Energy Theorem

A.6.2.1 (B)

Total energy of the ball, at the point $P = \frac{1}{2}mv^2 + mgh$. At the point Q, the ball has only PE, its KE, there, is zero as it (momentarily) comes to rest there.

$\therefore$ Total energy at $Q = mgR$

By the energy conservation principle,

$$\frac{1}{2}mv^2 + mgh = mgR$$

This gives, $v = \sqrt{2g(R - h)}$

A.6.2.2 (B)

Let the required speed = u .

We know from work energy theorem that the change in $KE = \text{Work done}$

$$\text{Initial } KE = \frac{1}{2}mu^2$$

$$\text{Final } KE = 0$$

$$\therefore \frac{1}{2}mu^2 - 0 = (\text{Average force}) (\text{length}) = Fl$$

$$\therefore \frac{1}{2}mu^2 = Fl$$

$$\text{Or } u^2 = \frac{2Fl}{m},$$

$$u = \sqrt{\frac{2Fl}{m}} = \left(\frac{2Fl}{m}\right)^{\frac{1}{2}}$$

A.6.2.3 (C) If the mass of the particle is m , and if the slope of the inclined plane is θ , and the uniform force of dynamic friction F is then the KE of the particle is given by, $KE = mgh\sin\theta - Fx$. So, the graph of KE vs. x is a straight line with a negative slope and the intercept with $x = 0$ is $mgh\sin\theta$. So, the KEY is option (C)

A.6.2.4 (A)

$$\text{Force of friction} = 0.1 \times 2 \times 10 \text{ N} = 2 \text{ N}$$

$$\text{Net Force} = (7 - 2) \text{ N} = 5 \text{ N}$$

$$\text{Net acceleration} = \frac{5}{2} \text{ ms}^{-2} = 2.5 \text{ ms}^{-2}$$

$$\therefore s_{10} = \left(\frac{1}{2} \times 2.5 \times 10^2 \right) \text{ m} = 125 \text{ m}$$

$\therefore$ Works done are

- a) $(7 \times 125) \text{ J} = 875 \text{ J}$
- b) $(-2 \times 125) \text{ J} = -250 \text{ J}$
- c) $(875 - 250) \text{ J} = 625 \text{ J}$
- d) change in kinetic energy = 625 J

A.6.2.5 (D) $10 \text{ cm} = 0.1 \text{ m}$, $1 \text{ g cm}^{-3} = \left(\frac{10^{-3}}{10^{-6}} = 10^3 \right) \text{ kg m}^{-3}$, $5 \text{ km} = 5000 \text{ m}$

So, the reqd. loss in PE = $10^5 \times 0.1 \times 10^3 \times 10 \times 5000 \text{ J} = 5 \times 10^{11} \text{ J}$. So, $n = 11$

So, correct option is (D)

A.6.2.6 (B)

$$\text{Mass of the drop} = \frac{4\pi}{3} r^3 \rho$$

$$= \frac{4\pi}{3} (2 \times 10^{-3})^3 \times 10^3 \text{ kg}$$

$$= \frac{32\pi}{3} \times 10^{-6} \text{ kg}$$

∴ Work done by the gravitational force

$$= mgh = \left(\frac{32\pi}{3} \times 10^{-6} \times 9.8 \times 500 \right) \text{ J} = \left(\frac{160\pi}{3} \times 9.8 \times 10^{-4} \right) \text{ J} = 0.1642 \text{ J}$$

The gain in K.E. of the drop

$$= \left[\frac{1}{2} m (10)^2 - 0 \right] \text{ J} = \frac{1}{2} \times \frac{32\pi}{3} \times 10^{-6} \times 10^2 \text{ J} = \left(\frac{16\pi}{3} \right) \times 10^{-4} \text{ J} = 0.001675 \text{ J}$$

Hence, work done by the resistive force

$$= - [(0.1642 - 0.001675)] \text{ J}$$

$$= - 0.1625 \text{ J}$$

A.6.2.7 (C) The kinetic energy is given by (with symbols having their usual meanings)

$$KE = \frac{1}{2} m (u + ft)^2 = \frac{m}{2} (u^2 + 2uft + f^2 t^2);$$

$$KE = E = a + bt + ct^2; E - a = c \left(t^2 + \frac{bt}{c} \right) = c \left\{ \left(t + \frac{b}{2c} \right)^2 - \frac{b^2}{4c^2} \right\},$$

$$\text{Where } a = \frac{1}{2} mu^2, b = muf, c = \frac{1}{2} mf^2.$$

$$\therefore E - a + \frac{b^2}{4c} = c \left(t + \frac{b}{2c} \right)^2; E' = E - \left(a - \frac{b^2}{4c} \right) = c \left(t + \frac{b}{2c} \right)^2$$

$$\text{So, } E' = ct_1^2, \text{ where } E' = E - E_0, \text{ where } E_0 = a - \frac{b^2}{4c} \text{ and } t_1 = t + \frac{b}{2c}.$$

So, E' vs. t_1 curve will be a parabola with pole at $E = E_0$ and $t = -\frac{b}{2c}$ (Fig.A.6.2.1).

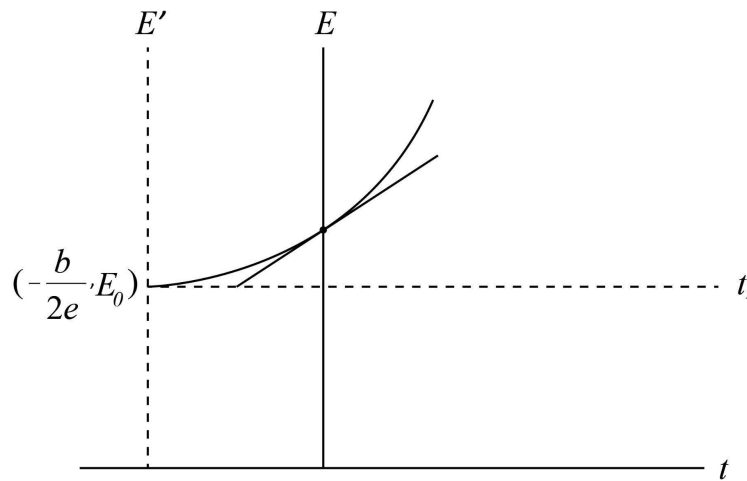


Fig.A.6.2.1

Since the parabola has a pole at $(-\frac{b}{2c}, E_0)$ The parabola will have zero slope at the pole, but it will intersect the E axis in the manner shown in Fig. 6.2.1, and at the point of intersection the tangent to the parabola will be making an acute angle with the positive direction with the time axis. Hence, the correct option is (C).

A.6.2.8 (B)

Initial PE of the bob = mgh

$$= m \times 9.8 \times 1.5 \text{ J} = (m \times 14.7) \text{ J}$$

$$\text{Energy dissipated} = \frac{5}{100} \times (m \times 14.7) \text{ J} = (m \times 0.735) \text{ J}$$

$\therefore$ If v is the speed (when it (first) comes back) to its lowest position, we have

$$\frac{1}{2}mv^2 = m(14.7 - 0.735)$$

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$$\therefore v^2 = 2 \times 13.965 = 27.930$$

$$\therefore v = \sqrt{27.930} = 5.28 \text{ ms}^{-1}$$

A.6.2.9 (B)

We have

$$\frac{mv_1^2}{r} = k \cdot \frac{e^2}{r^2}$$

$$\therefore mv_1^2 = \frac{k e^2}{r}$$

$$\therefore \text{initial } KE = \frac{1}{2} mv_1^2 = \frac{1}{2} \frac{k e^2}{r}$$

For the orbits of radius $r/2$, we would similarly have

$$\frac{mv_2^2}{(\frac{r}{2})} = k \frac{e^2}{(\frac{r}{2})^2}$$

$$\therefore mv_2^2 = \frac{k e^2}{\frac{r}{2}} = \frac{2 k e^2}{r}$$

$$\therefore \text{Final } KE = \frac{1}{2} mv_2^2 = \frac{k e^2}{r}$$

$$\therefore \text{Change in } KE = \frac{k e^2}{r} - \frac{k e^2}{2r} = \frac{k e^2}{2r}$$

The KE , therefore, increases by $\left(\frac{k e^2}{2r}\right)$

A.6.2.10 (B)

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The change in kinetic energy of the drop is

$$\begin{aligned} K &= \left(\frac{1}{2}mv^2 - 0 \right) \\ &= \frac{1}{2} \times 10^{-3} \times 50 \times 50 \\ &= 1.25 \text{ J} \end{aligned}$$

Assuming that g is (nearly) a constant (with a value $\approx 10 \text{ m/s}^2$), the work done by the gravitational force is,

$$W_g = mgh = 10^{-3} \times 10 \times 10^3 \text{ J} = 10.0 \text{ J}$$

From the work-energy theorem

$$\Delta K = W_g + W_r$$

where W_r is the work done by the resistive forces on the raindrop. Thus

$$W_r = \Delta K - W_g = (1.25 - 10) \text{ J} = -8.75 \text{ J}$$

The negative sign, for this work, implies that the resistive forces act in a direction opposite to the direction in which the ball is falling.



Module 3 Potential Energy and Energy of Spring

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A.6.3.1 (B)

We have, for the first case, $2^2 = u^2 - 2 \times 10 \times 3$; $u^2 = 64$

And for second case, $0 = u^2 - 2 \times 10 \times 3$; $u_0^2 = 60$

The work done in case is the imparted KE . So, their ratio is equal to the ratio of the squares of the initial speed.

So, if the work done in the first case and second case are W and W' respectively, then, we have,

$$\frac{W}{W'} = \frac{60}{64} = \frac{15}{16}. \text{ So, fraction saved} = 1 - \frac{15}{16} = \frac{1}{16} \equiv \frac{100}{16} \% = \frac{25}{4} \% = 6\frac{1}{4} \% = 6.25\%$$

So, correct option is (B)

A.6.3.2 (D)

We know that, for a spring, stretched by an amount x ,

$$W = \frac{1}{2} kx^2$$

(i) in case (a), we would have

$$x_{1a} = \frac{F}{k_1} \text{ and } x_{2a} = \frac{F}{k_2}$$

$$\therefore W_{1a} = \frac{1}{2} \frac{F^2}{k_1} \text{ and } W_{2a} = \frac{1}{2} \frac{F^2}{k_2}$$

Since $k_1 < k_2$, we have $W_{1a} > W_{2a}$

(ii) In case (b), we would have

$$F_{1b} = k_1 x \quad \text{and} \quad F_{2b} = k_2 x$$

$$\therefore W_{1b} = \frac{1}{2} k_1 x^2 \quad \text{and} \quad W_{2b} = \frac{1}{2} k_2 x^2$$

Since $k_1 < k_2$, we would have $W_{1b} < W_{2b}$

A.6.3.3 (B)

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When the mass m is at a depth x below the surface of the earth, the gravitational force on it is due only to the inner sphere of radius $(R-x)$. Hence

$$F(x) = \frac{G \cdot \frac{4\pi}{3}(R-x)^3 \rho m}{(R-x)^2} = \frac{4\pi}{3} G \rho (R-x)m$$

The infinitesimal work done, in moving it a distance dx away from the center, towards the surface of the earth, is

$$dW = F(x)dx = \frac{4\pi}{3} G \rho m (R-x)dx$$

Hence total work done in moving the mass from the centre, to the surface of the earth, is

$$W = \frac{4\pi}{3} G \rho m \int_{x=0}^{x=R} (R-x)dx = \frac{4\pi}{3} G \rho m \left[Rx - \frac{x^2}{2} \right]_0^R = \frac{4\pi}{3} G \rho m \left[R^2 - \frac{R^2}{2} \right] = \frac{4\pi}{3} G \rho m \frac{R^2}{2}$$

But $\rho = \frac{M}{\frac{4}{3}\pi R^3} \therefore W = \frac{GMm}{2R} = \frac{m}{2} \frac{GM}{R^2} R = \frac{mgR}{2}$

A.6.3.4 (B)

When a conservative force does work on a particle the P.E of the particle depends only on the distance (r) between the point source of the force and the particle.

We are given here that the given conservative force is doing a positive work when the particle is moving towards the origin of the source of the force. Positive work implies that the displacement of the particle is along the direction of the force on it due to the source. The particle, in this case, must, therefore, be experiencing an attractive force due to the 'source'.

By convention, the PE of the particle, under an attractive force, is always taken with a negative sign. It is also proportional to $1/r$ for a conservative inverse square force.

It follows that (because of its negative sign), the P.E., in such a case, will decrease in r (It would become more

negative). Its maximum value is zero which (because of the inverse proportionality with r) will be attained as $r \rightarrow \infty$.

A.6.3.5 (D)

The P.E., stored in the compressed spring $= \frac{1}{2}kx_0^2$. It is this PE that raises the given object through height H .

Hence

$$\frac{1}{2}kx_0^2 = mgh$$

or
$$k = \frac{2mgh}{x_0^2}$$

A.6.3.6 (C)

The spring constant k , of a spring of a given material, is inversely proportional to the length of the spring and directly proportional to its area of cross section, i.e. d^2 .

In case of springs (1) and (2), we would, therefore, have $k_1 < k_2$

and In case of springs (3) and (4), we would, therefore, have $k_3 > k_4$

The PE, stored in a spring, when it is stretched by an amount x , equals $U = \frac{1}{2}kx^2$. Here x being the same in all cases, we have $U \propto k$. Hence we would have

$$U_1 < U_2$$

$$U_3 > U_4$$

[Note: These (approximate) results are obtained by applying Hooke's law to the stretching of a spring. We have

$$Y = \frac{4f/\pi d^2}{(\Delta l/L)}$$

Now force per unit change in length $= k$

$$\therefore k = Y \cdot \frac{1}{L} \frac{\pi d^2}{4} \text{ (Here } Y \text{ is a constant for a given material.)}$$

$\therefore$ For same d , $k \propto \frac{1}{L}$

and for same L , $k \propto d^2$

A.6.3.7 (D)

$$\int_1^2 F(x)dx = \text{Work done is taking the particle from 1 to 2,}$$

= change in KE

$$= K_2 - K_1$$

$$\text{If } F(x) = -\frac{dU}{dx}, \text{ then } \int_1^2 F(x)dx = -\int_1^2 dU = U_1 - U_2$$

$$\text{Which will give } K_2 - K_1 = U_1 - U_2$$

Or $K_2 + U_2 = K_1 + U_1$, which is consistent with the principle of conservation of energy

So, case I is a possibility.

case II will lead to

$$K_2 - U_2 = K_1 - U_1$$

Which does not conform to the principle of conservation of energy.

A.6.3.8 (D)

For path 1

$$W_1 = [q(c_1 \hat{i} + c_2 \hat{j})] \cdot [\{(1 - 0) \hat{i} + (2 - 0) \hat{j}\} + \{(4 - 1) \hat{i} + (5 - 2) \hat{j}\}] = q[4C_1 + 5C_2]$$

For path 2

$$W_2 = [q(C_1 \hat{i} + C_2 \hat{j})] \cdot [(4 - 0) \hat{i} + (5 - 0) \hat{j}] = q(4C_1 + 5C_2)$$

For path 3

$$W_3 = [q(C_1 \hat{i} + C_2 \hat{j})] \cdot [\{(2 - 0) \hat{i} + (1 - 0) \hat{j}\} + \{(4 - 2) \hat{i} + (5 - 1) \hat{j}\}] = q[4C_1 + 5C_2]$$

Thus $W_1 = W_2 = W_3$. Hence the electric force is a conservative force.

A.6.3.9 (D)

Over a complete orbit, the final position (and final speed) of the comet is the same as its initial position. Hence, there is no change in the PE, or the KE, of the comet. This implies that the net work done on the comet, by the gravitational force of the sun, (over a complete orbit) is zero.

A.6.3.10 (C)

We have

$$\text{Centripetal force} = \left(\frac{mv^2}{r} \right) = \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$\therefore mv^2 r = \frac{1}{4\pi\epsilon_0} e^2$$

$$\text{or } \left(\frac{1}{2} mv^2 \right) r = \frac{1}{2} \cdot \frac{e^2}{4\pi\epsilon_0}$$

This equation appears to indicate that as the electron radiates energy, its kinetic energy $\left(\frac{1}{2} mv^2 \right)$ should decrease and, therefore, r would increase (Fig. A.6.3.1)

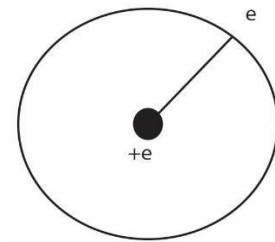


Fig.A.6.3.1

However, we need to remember that the electron, moving around the nucleus, also has a PE

$\left(= -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = -mv^2 = -2KE \right)$ associated with it. As the electron radiates energy, its total energy (and not its KE) decreases. The PE (because of its negative sign) decreases twice as fast as the KE when r decreases.

Hence, the total energy can decrease even when the KE increases.

The electron, therefore continuously moves towards the nucleus with a continuous increase in its KE {to keep $\left(\frac{1}{2}mv^2 \right)r = \text{constant}$ }.

There is, however, a continuous decrease in its total energy.



Module 4 : Elastic and Inelastic Collision

A.6.4.1(A) The component of linear momentum of the gas molecule before collision along AB and CD are respectively $mv \sin \theta$ and $mv \cos \theta$ (Fig.6.4.1)

And the component along BA and CD are respectively $mv \sin \theta$ and $mv \cos \theta$

For calculating the force, we need to determine the change in momentum.

For determining the change, we find that equal components along CD neutralize each other.

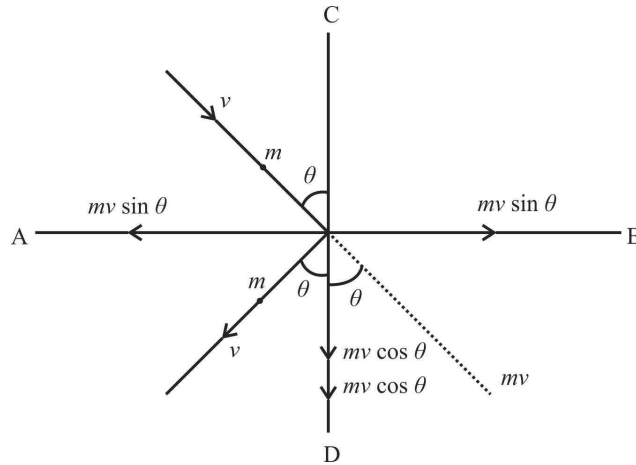


Fig.A.6.4.1

We are left with the components along AB and BA.

So the change along BA, i.e. OA is equal to $mv \sin \theta - (-mv \sin \theta) = 2mv \sin \theta$

Since, the time of impact of the collision is τ , the force = $\frac{2mv \sin \theta}{\tau}$ along OA.

A.6.4.2 (C)

The velocity, of object (2), at $t = 0$, equals v_0 , (the velocity with which object (1), of same mass, collides elastically with it).

At time $t = t_0$, objects (2) and (3) have equal velocities, say, v each. Also, at this instant, the spring has been compressed by an amount x_0 .

Hence, by principle of energy conservation,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2} \cdot 2mv^2 + \frac{1}{2}kx_0^2 \quad (6.4.1)$$

Also, by momentum conservation, we get

$$mv_0 = mv + 2mv \quad (6.4.2)$$

This gives $v = v_0/3$

Substituting the value of v in Eq(6.4.1), we get

$$\text{or } kx_0^2 = mv_0^2 - \frac{3mv_0^2}{9} = \frac{2}{3} mv_0^2$$

$$\therefore k = \frac{2}{3} \frac{mv_0^2}{x_0^2}$$

A.6.4.3 (B)

At the start, the bob is raised to a height L .

Let the coefficient of restitution corresponding to bob wall collision be e .

When the angular amplitude of the oscillation becomes 60° , the height of the bob, would be

$$h = L(1 - \cos 60^\circ) = \frac{L}{2}$$

The velocities of the bob, before and after the first collision, are $\sqrt{2gl}$ and $(e\sqrt{2gl})$, respectively. Hence, the height h_1 , to which the bob would rise after the first collision, equals $e^2 L$. So, the corresponding height, after two collisions would be $e^4 L$.

The height h_n , to which the bob would rise after n collisions, would, in a similar way, be given by

$$h_n = e^{2n} L$$

We want the value of n for which $h_n = \frac{L}{2}$

$$\therefore (e)^{2n} \cdot L = \frac{L}{2}, \quad \therefore (e)^{2n} = \frac{1}{2}$$

$$\therefore \left(\frac{2}{\sqrt{5}}\right)^{2n} = \frac{1}{2}$$

Taking logarithm to the base 10 on both sides, we get

$$2n(\log 2 - \log \sqrt{5}) = (-\log 2)$$

$$2n[0.3 - 0.35] = (-0.3)$$

$$\therefore 2n = \frac{0.3}{0.05} = 6$$

$$\text{or } n = 3$$

A.6.4.4 (A)

The ball would take a time t_1 , to make its first impact with the floor, where

$$H = 0 + \frac{1}{2}gt_1^2$$

$$\therefore t_1 = \sqrt{\frac{2H}{g}}$$

The ball would rebound with a velocity $e\sqrt{2gH}$. Hence the time, t_2 taken by it to rise to its new (reduced) height, say, h_1 , is given by

$$0 = e\sqrt{2gH} - gt_2$$

$$\text{or } t_2 = \frac{e\sqrt{2gH}}{g} = e\sqrt{\frac{2H}{g}}$$

The ball would again take a further time t_2 , before it makes its second impact with the floor. Hence the time instant, at which it would make its second impact with the floor, is

$$\begin{aligned} t_1 + t_2 + t_2 &= t_1 + 2t_2 \\ &= \sqrt{\frac{2H}{g}} + 2e\sqrt{\frac{2H}{g}} = \sqrt{\frac{2H}{g}}(1 + 2e) \end{aligned}$$

A.6.4.5 (C)

The collision being elastic, we have

$$m\vec{v} = m\vec{v}_1 + m\vec{v}_2$$

$$\therefore \vec{v} = \vec{v}_1 + \vec{v}_2 \quad (6.4.3)$$

$$\text{and } \frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$\therefore v^2 = v_1^2 + v_2^2 \quad (6.4.4)$$

From the Eq(6.4.3), we have

$$\vec{v} \cdot \vec{v} = (\vec{v}_1 + \vec{v}_2) \cdot (\vec{v}_1 + \vec{v}_2)$$

$$v^2 = v_1^2 + 2\vec{v}_1 \cdot \vec{v}_2 + v_2^2$$

Using the Eq(6.4.4), we get

$$v_1^2 + v_2^2 = v_1^2 + v_2^2 + 2\vec{v}_1 \cdot \vec{v}_2$$

$$\therefore \vec{v}_1 \cdot \vec{v}_2 = 0$$

$$\text{Or } v_1 v_2 \cos(\theta_1 + \theta_2) = 0$$

$$\therefore \theta_1 + \theta_2 = \frac{\pi}{2}$$

$$\therefore \theta_2 = \frac{\pi}{3}$$

A.6.4.6 (A)

The height of rise, after the first impact is $e^2 H$. Hence

$$162 = e^2 H = e^2 \cdot 200$$

$$\therefore e^2 = \frac{162}{200} = 0.81$$

$$\therefore e = 0.9$$

The height of rise after the second impact would be $e^2 \times 162$ cm.

Hence this height is

$$0.81 \times 162 = 131.22 \text{ cm or } 131 \text{ cm (nearly)}$$

A.6.4.7 (B)

The velocity v_1 , with which the ball hits the floor, just before the first impact, is

$$v_1 = \sqrt{2gH}$$

Hence, its velocity of rebound, is

$$v_2 = ev_1 = e(\sqrt{2gH})$$

The height, h_1 , to which it would rise, would be given by

$$h_1 = \frac{v_2^2}{2g} = e^2 H$$

At the second impact, the ball would hit the floor with a velocity v_2 and would rebound with a velocity ev_2

Hence the height, h_2 , to which it would rise, after to second impact, is

$$h_2 = \frac{(ev_2)^2}{2g} = e^4 H$$

Continuing, in the same way, the height h_3 , to which it would rise, after the third impact, is

$$h_3 = e^6 H$$

A.6.4.8 (A)

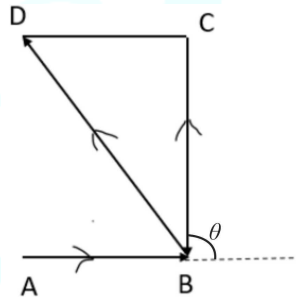


Fig A.6.4.1

Let $\vec{AB}$ represent the original momentum vector in magnitude and direction. $\vec{CD}$ represents the same after impact (Fig A.6.4.1).

$$\vec{\Delta p} = \vec{BC} - \vec{AB} = \vec{BC} + \vec{CD} = \vec{BD}$$

$$|\vec{\Delta p}| = \sqrt{BC^2 + CD^2} = \sqrt{BC^2 + AB^2}$$

Fig.A.6.4.2

The angular displacements of P and Q at the instant of collision is given by,

$$\frac{3}{3+2} \times 360 = 216, \quad \frac{2}{3+2} \times 360 = 144$$

So, P and Q meet at a point B such that the greater among the angles AOB is 216° and the smaller is 144° . BN is the perpendicular from B on the radial direction perpendicular to OA. So angle BON is equal to 54° .

$\therefore \angle NBD = 54^\circ$ (Fig.A.6.4.2).

The momenta of P and Q at the time of collision are given by $3mu$ and $2mu$ along the direction of BD and BC respectively. So, from the principle of conservation of momentum

$$3mu - 2mu = mv - 0$$

$$\therefore u = v$$

Hence the final momentum of particle Q will be mu along a direction perpendicular to the radius at B.

The component of final momentum of particle Q will therefore be given by

$$mu \cos 54^\circ = mu \sin 36^\circ .$$

A.6.4.10 (C)

$$\text{Initial K.E} = \frac{1}{2} m u^2$$

From the law of conservation of angular momentum

$$mu = (m + M)v$$

$$v = \frac{mu}{m+M}$$

So, decrease in KE

$$= \frac{1}{2} mu^2 - \frac{1}{2} (m + M) \left(\frac{mu}{m+M} \right)^2 = \frac{1}{2} mu^2 \left(1 - \frac{m}{m+M} \right) = \frac{1}{2} \left(\frac{Mm}{M+m} \right) u^2 = \frac{1}{2} m_0 w^2$$

So, $m_0 = \left(\frac{Mm}{M+m} \right)$ and $w = u$

$$m_0 < m$$

So answer is (C)

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