

Oscillations

Self-Assessment

Module 1 Simple Harmonic Motion

Q.14.1.1

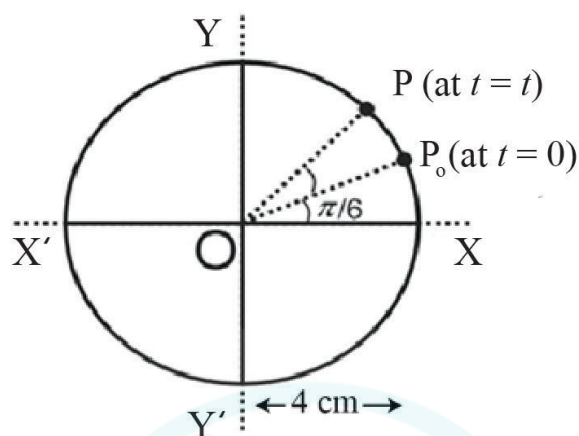


Fig.Q.14.1.1

Fig. Q.14.1.1 shows a particle moving with uniform speed along a circle of radius 4 cm in an anticlockwise direction with a time period 0.2 s. P_0 is the position of the particle at $t = 0$, and P is the position of the particle at $t = t$ as shown, consider the foot of the perpendicular from P on the diameter $Y'OY$. If it is given by y (in cm).

Then -

(A) $y = 4 \sin \left[10\pi t + \frac{\pi}{6} \right]$

(B) $y = 4 \sin \left[10\pi t - \frac{\pi}{6} \right]$

(C) $y = 4 \cos \left[10\pi t + \frac{\pi}{6} \right]$

(D) $y = 4 \cos \left[10\pi t + \frac{\pi}{6} \right]$

Q. 14.1.2. The equation describing SHM of a particle along X-axis with symbols having their usual meaning,

$$\text{is } x = 4 \sin \left[100\pi t + \frac{\pi}{6} \right]$$

The equation can be rewritten as

(A) $x = 2\sqrt{3} \sin(100\pi t) + 2 \cos(100\pi t)$

(B) $x = 2\sqrt{3} \sin(100\pi t) - 2 \cos(100\pi t)$

(C) $x = 2 \sin(100\pi t) + 2\sqrt{3} \cos(100\pi t)$

(D) $x = 2\sqrt{3} \cos(100\pi t) - 2 \sin(100\pi t)$

Q. 14.1.3. The magnitude (v) of the velocity of a particle moving along the x-axis is expressed as a function of x , as under

$$v^2 = 24x - 2x^2 - 70 \quad (\text{all quantities are cgs units})$$

Then, which among the following statements is/are correct about the motion?

- I. it is anharmonic with maximum velocity at $x = 12$
- II. it is anharmonic with minimum velocity at $x = 12$
- III. it is simple harmonic with amplitude equal to 1 cm
- IV. it is a simple harmonic with a time period equal to π sec.

(A) only I

(B) only II

(C) only III

(D) III and IV

Q. 14.1.4 A particle of mass m , moving along X-axis has instantaneous acceleration; is given by

$$a = -[4x+1]$$

x is the instantaneous position coordinate of the particle. The motion of a particle is an SHM with time period T and mean position at x_0 , where

- (A) $T = \pi, \quad x_0 = 0$
- (B) $T = 2\pi, \quad x_0 = 0$
- (C) $T = 2\pi, \quad x_0 = 0.25$
- (D) $T = \pi, \quad x_0 = -0.25$

Q. 14.1.5 The instantaneous displacement, x , (in cm) of a particle moving along X-axis is

$$x = 8 \sin 10\pi t \cos 10\pi t$$

Which of the following statements is correct?

- (A) Motion is not simple harmonic
- (B) Motion is simple harmonic, with $A = 8 \text{ cm}, \nu = 5 \text{ Hz}$
- (C) Motion is simple harmonic, with $A = 4 \text{ cm}, \nu = 10 \text{ Hz}$
- (D) Motion is simple harmonic, with $A = 8 \text{ cm}, \nu = 10 \text{ Hz}$

Q. 14.1.6 The differential equation of motion of a particle of mass 200 g, executing SHM, along X-axis is

$$\frac{d^2x}{dt^2} = -9x, \quad x \text{ is in meters; } t \text{ is in seconds.}$$

The time period T ; and the spring constant k are

- (A) $T = \frac{2\pi}{3} \text{ s}; \quad k = 1.8 \text{ Nm}^{-1}$
- (B) $T = \frac{2\pi}{9} \text{ s}; \quad k = 0.18 \text{ Nm}^{-1}$

(C) $T = \frac{2\pi}{3} s$; $k = 1.8 \text{ dyne cm}^{-1}$

(D) $T = \frac{2\pi}{3} s$; $k = 1.8 \times 10^3 \text{ Nm}^{-1}$

Q. 14.1.7 A particle of mass m executes SHM of time period T , under a restoring force of spring constant k . The mass is decreased by 25% and k is increased by 25%. The time period of oscillation is T' , $\frac{T'}{T}$ is

(A) $\sqrt{\frac{3}{5}}$

(B) 1

(C) $\sqrt{\frac{5}{3}}$

(D) $\frac{1}{\sqrt{2}}$

Q. 14.1.8 A particle executes SHM of amplitude A and time period T along the X-axis. At $t = 0$; the particle is in its extreme position along the positive direction of displacement. The particle is in position P and Q at $t = \frac{T}{12}$ and $t = \frac{T}{8}$ respectively. The percentage change in displacement, as a particle moves from P to Q, is nearly

(A) 16%

(B) – 18.4%

(C) + 18.4%

(D) – 24%

Q. 14.1.9 Two particles execute SHM of same amplitude A and same frequency ν along the X-axis. The maximum separation between the two particles during their motion is $\sqrt{3} A$. The initial phase difference, ϕ_0 , between the two SHM's is

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$

Q. 14.1.10 The momentum vs. position curve for a particle of mass m executing SHM, of force constant, k , is a circle. Then the numerical value of m is equal to

(A) k

(B) $\frac{1}{k}$

(C) k^2

(D) $\frac{1}{k^2}$



Module 2 Characteristics of Simple Harmonic Motion

Q.14.2.1 A particle executes SHM of amplitude A , angular frequency ω . v is the instantaneous speed of the particle when the instantaneous displacement is 50% of A . v_m is the velocity amplitude of the motion. Δv , the difference of v and v_m , expressed as a fraction of v_m is

(A) $\left(1 - \frac{\sqrt{3}}{2}\right)$

(B) 0.5

(C) $\left(1 - \frac{1}{\sqrt{2}}\right)$

(D) $\left(\frac{\sqrt{3}}{2} - 1\right)$

Q.14.2.2 For a particle moving along the X-axis, x and v denote the instantaneous displacement and the instantaneous speed respectively. Given

$$\frac{x^2}{4} + \frac{v^2}{4\pi^2} = 1$$

x is in cm and time in seconds. If A is the amplitude in cm, and T , the time period in sec, then which among the following statements is correct?

(A) The motion is not simple harmonic.

(B) The motion is simple harmonic, with $A=2$, $T=2$

(C) The motion is simple harmonic with $A=4$, $T=\frac{1}{2\pi}$

(D) The motion is simple harmonic with $A=2$, $T=1$

Q.14.2.3 The amplitude and the time period of a simple harmonic oscillator of mass 200 g, are 5 cm and 2 sec respectively. When the oscillator is at a distance x cm from the mean position its kinetic energy is equal to 1.6 mJ; the value(s) of x is/are (given $\pi^2 = 10$).

- (A) ± 2
- (B) Only 2
- (C) Only -3
- (D) ± 3

Q.14.2.4 A particle executing SHM has a speed α when its displacement from mean position is β . If the magnitude of displacement increases by 25% , the magnitude of the speed decreases by 50%. The amplitude (A) and time-period (T) of SHM is

- (A) $A = \frac{\beta}{2\alpha}$; $T = \frac{\alpha}{\beta}$
- (B) $A = \frac{\sqrt{5}}{2}\beta$; $T = \pi\left(\frac{\beta}{\alpha}\right)$
- (C) $A = \frac{\sqrt{7}}{2}\beta$; $T = \sqrt{3}\pi\left(\frac{\beta}{\alpha}\right)$
- (D) $A = \frac{2}{\sqrt{7}}\beta$; $T = \sqrt{3}\pi\left(\frac{\alpha}{\beta}\right)$

Q.14.2.5 The instantaneous acceleration (a) versus the instantaneous displacement (x) graph, for two SHM's I and II is shown in fig.Q.14.2.1.

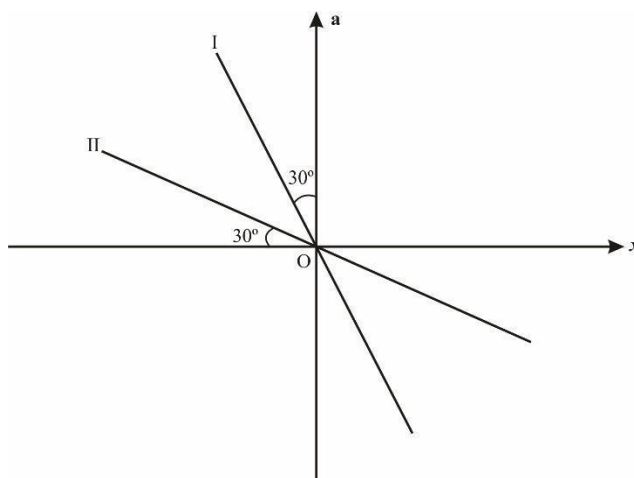


Fig.Q.14.2.1

The ratio of the time periods T_1 and T_2 in the two cases is

- (A) 1:1
- (B) $1:\sqrt{3}$
- (C) $\sqrt{3}:1$
- (D) 2:1

Q.14.2.6 A particle executes SHM of amplitude A and total energy E . When the instantaneous displacement of the particle is $\pm \frac{A}{\sqrt{2}}$, K and U denote the kinetic and potential energy of the particle. What is (i) $\frac{U}{K}$ and (ii) $\frac{E}{U}$?

- (A) $\frac{U}{K} = 1$; $\frac{E}{U} = 2$
- (B) $\frac{U}{K} = 1$; $\frac{E}{U} = 1$

(C) $\frac{U}{K} = 2; \frac{E}{U} = \frac{3}{2}$

(D) $\frac{U}{K} = \frac{1}{2}; \frac{E}{U} = \frac{2}{3}$

Q.14.2.7 A particle of mass 25 g executes SHM under a force of spring constant 10^{-3} Ncm^{-1} . The average kinetic energy over one cycle of changes is 16 mJ. The amplitude(A); and angular frequency (ω) of SHM are

(A) $A = 0.4\sqrt{2} \text{ m} ; \omega = 1 \text{ rad s}^{-1}$

(B) $A = 0.8 \text{ m} ; \omega = 2 \text{ rad s}^{-1}$

(C) $A = 0.8 \text{ m} ; \omega = 1 \text{ rad s}^{-1}$

(D) $A = 0.4 \text{ m} ; \omega = 1 \text{ rad s}^{-1}$

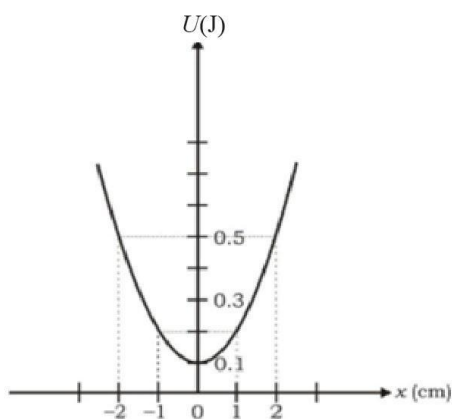
Q.14.2.8 The potential energy(U) versus the displacement(x) graph for two SHMs is as shown in Fig.Q.14.2.2(a) and Fig.Q.14.2.2(b). The ratio of the spring constant in Fig.Q.14.2.2(a) and Fig.Q.14.2.2(b) is

(A) 1 : 1

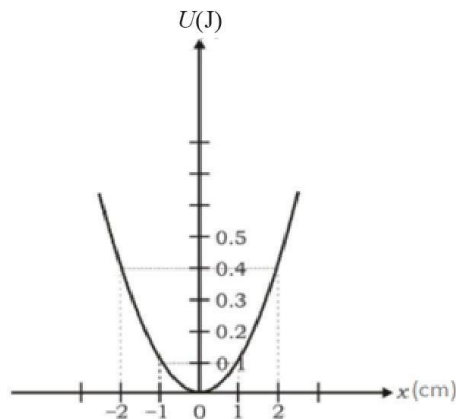
(B) 1 : 2

(C) 2 : 1

(D) 1 : 4



(a)



(b)

Fig.Q.14.2.2

Q.14.2.9 $U = \alpha + \beta x^2$ is the relation between the instantaneous potential energy(U) and instantaneous displacement(x) of a particle of mass m . α and β are constants. Which of the following statements is correct?

- (A) Motion is periodic but not simple harmonic
- (B) Motion is a simple harmonic of time-period $2\pi\sqrt{\frac{m}{\beta}}$
- (C) Motion is a simple harmonic of time-period $2\pi\sqrt{\frac{m}{2\beta}}$
- (D) Motion is neither periodic nor simple harmonic

Q.14.2.10 The diagram of a simple pendulum executing SHM is shown in fig.Q.14.2.3. The symbols have their usual meaning. The energy conservation equation corresponding to angular displacement θ reduces to-

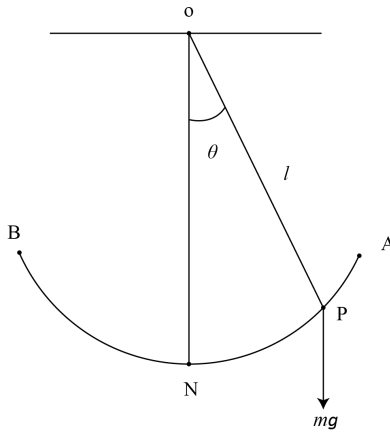


Fig.Q.14.2.3

- (A) $\frac{1}{2}l\left(\frac{d\theta}{dt}\right)^2 + g \cos \theta = \text{a constant}$
- (B) $\frac{1}{2}l\left(\frac{d\theta}{dt}\right)^2 - g \cos \theta = \text{a constant}$
- (C) $l\left(\frac{d\theta}{dt}\right)^2 + g \sin \theta = \text{a constant}$
- (D) $l\left(\frac{d\theta}{dt}\right)^2 - g \sin \theta = \text{a constant}$

Module 3 Examples of Simple Harmonic Motion

Q.14.3.1 A mass M attached to a spring of spring constant k_1 executes vertical oscillations of time period T_1 . The mass is reduced by 25% and attached to another spring of spring constant k_2 . The time period of vertical oscillations is 50% more than T_1 . $\frac{k_2}{k_1}$ is.

- (A) $\frac{2}{3}$
- (B) $\frac{1}{3}$
- (C) $\frac{9}{4}$
- (D) $\frac{4}{9}$

Q.14.3.2 A mass M attached to the free end of a vertical spring executes SHM of time-period T . The mass is immersed completely in a non-viscous liquid as shown in Fig.Q.14.3.1(b). What change, if any, is observed in the oscillations of mass M ? (The mass remains inside liquid throughout the motion)

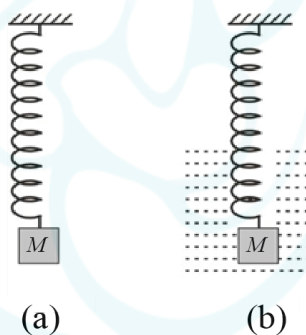


Fig.Q.14.3.1

- (A) There is no change
- (B) Time period increases.
- (C) The mean (or equilibrium) position moves upwards. Time period remains the same.
- (D) The mean (or equilibrium) position moves downwards. Time period remains the same.

Q.14.3.3 A hollow container of mass m_1 is filled completely with a liquid of mass m_2 . The arrangement attached to the free end of a vertical spring of spring constant k , executes SHM (Fig.Q.14.3.2). There is a small hole in the container through which liquid drips out at a constant rate α . The graph showing the variation of (T^2) with time(t) is

[T = instantaneous time period of the system]

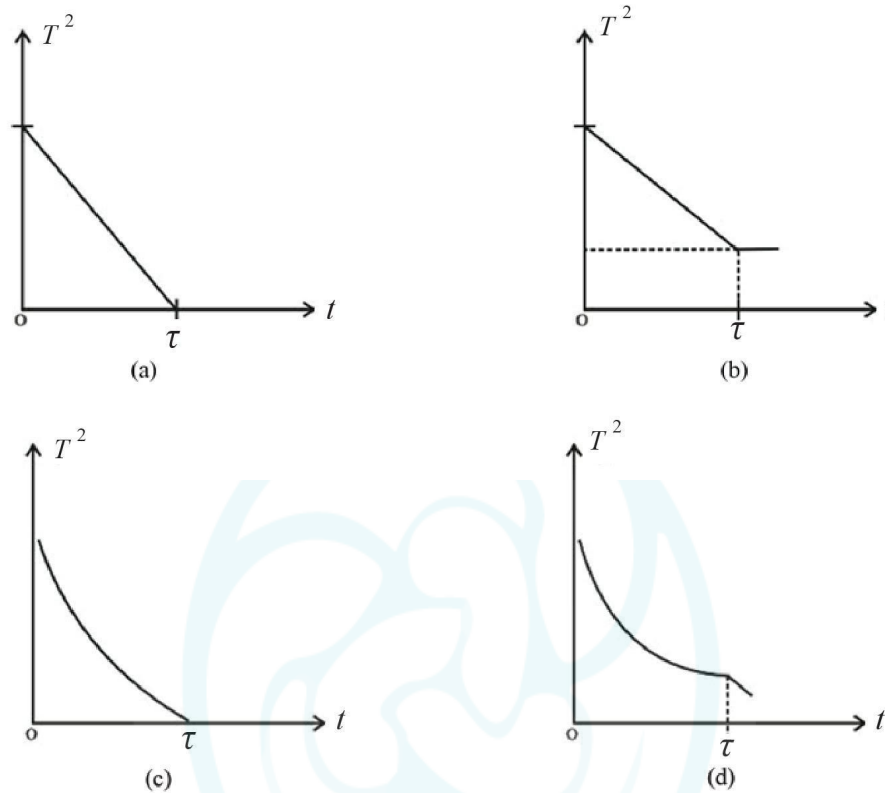


Fig.Q.14.3.2

Q.14.3.4 A mass M attached to a spring S_1 , (spring constant = k_1) ; and S_2 (spring constant = k_2) one by one executes horizontal oscillations of time period T_1 and T_2 respectively. The same mass attached to both S_1 and S_2 simultaneously as shown in Fig.Q.14.3.3; executes oscillations of time-period T Which of the following is correct?

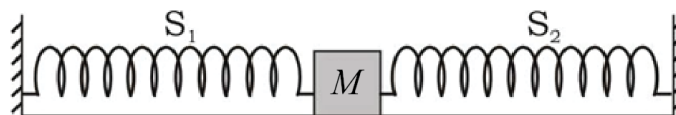


Fig.Q.14.3.3

- (A) $T = T_1 + T_2$
- (B) $T^2 = T_1^2 + T_2^2$
- (C) $\frac{1}{T_2} = \frac{1}{T_1} + \frac{1}{T_2}$
- (D) $\frac{2}{T} = \frac{1}{T_1} + \frac{1}{T_2}$

Q.14.3.5

For the arrangement shown in Fig.Q.14.3.4

; the time-period of oscillations of mass M is

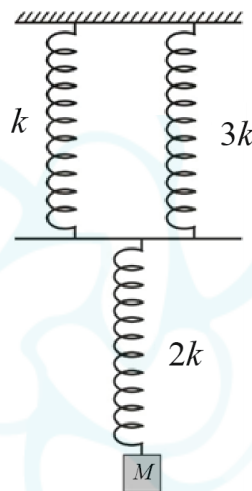


Fig.Q.14.3.4

- (A) $2\pi\sqrt{\frac{M}{6k}}$
- (B) $2\pi\sqrt{\frac{3M}{k}}$
- (C) $\pi\sqrt{\frac{3M}{k}}$
- (D) $2\pi\sqrt{\frac{2M}{3k}}$

Q.14.3.6 The time period, T ; of the vertical oscillations of a cylinder of radius R ; length L ; mass M and density ρ ; immersed in a liquid of density σ is

$$T = 2\pi\sqrt{\frac{M}{\pi R^2 \sigma g}}$$

The radius of the cylinder is doubled without any other change. The time-period of oscillations of cylinder now is

- (A) T
- (B) $2T$
- (C) $\frac{T}{2}$
- (D) $\frac{T}{\sqrt{2}}$

Q.14.3.7 A U-tube contains two immiscible liquids. In equilibrium the length of the two liquid columns in the two limbs of the tube is L_1 and L_2 as shown in Fig.Q.14.3.5 ρ_1 and ρ_2 are the densities of the two liquids. For small displacements of the liquid columns, the time period, T , of oscillations is

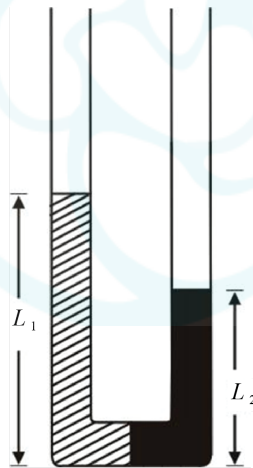


Fig.Q.14.3.5

- (A) $2\pi\sqrt{\frac{L_1 + L_2}{2(\rho_1 + \rho_2)g}}$

(B) $2\pi\sqrt{\frac{L_1 + L_2}{(\rho_1 + \rho_2)g}}$

(C) $2\pi\sqrt{\frac{L_1\rho_2 + L_2\rho_1}{(\rho_1 + \rho_2)g}}$

(D) $2\pi\sqrt{\frac{2L_1\rho_1}{(\rho_1 + \rho_2)g}}$

Q.14.3.8 A vertical U-tube has limbs of cross-sectional area (a). A liquid of density (ρ) fills the tube up to a length (L), in equilibrium. The time period of oscillations of liquid is (T). The area of cross-section of each limb of the U-tube is doubled. It is filled up to the same length (L), by a liquid of density (2ρ) in equilibrium. The time-period of oscillation of liquid now is (T_1). What is ($\frac{T_1}{T}$)?

[Given T = Time period of oscillations]

$$= 2\pi\sqrt{\frac{M}{2A\rho g}}$$

M = mass of liquid in U-tube]

(A) 0.5

(B) 0.707

(C) 1

(D) 1.414

Q.14.3.9 A second pendulum is suspended from the ceiling of an accelerating lift. The time-period is observed to increase by 50% of its earlier value. The acceleration of lift is

- (A) $\frac{2g}{3}$; upwards
- (B) $\frac{4g}{9}$; downwards
- (C) $\frac{5g}{9}$; downwards
- (D) $\frac{5g}{9}$; upwards

Q.14.3.10 A second pendulum has a bob of a material of density ρ . The bob is immersed completely in a non-viscous liquid of density $\sigma = \frac{\rho}{2}$. The time period of oscillations is

- (A) 2 s
- (B) $2\sqrt{2}$ s
- (C) $\sqrt{2}$ s
- (D) 1 s

Q.14.3.11 A spherical shell filled completely with a liquid is used as the bob of a simple pendulum. As the pendulum starts oscillating at $t = 0$; the liquid drips out at a constant rate; till the shell is empty. The correct statement, describing time period of pendulum is:

- (A) Remains the same throughout.
- (B) Increases
- (C) Decreases
- (D) First increases acquire a maximum value, then decreases and finally is same as at $t = 0$.

Module 4 : Free, Damped and Forced oscillations

Q.14.4.1 What is the unit of b used in the differential equation of motion given by

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega^2x = 0$$

- (A) kg s^{-1}
- (B) s^{-1}
- (C) kg m^{-1}
- (D) kg ms^{-1}

Q.14.4.2 What are the Order (O) and Degree (D) of the differential equation of damped harmonic motion?

- (A) O-2, D-2
- (B) O-2, D-1
- (C) O-1, D-2
- (D) O-1, D-1

Common information for Q.14.4.3, Q.14.4.4, Q.14.4.5.

For a damped harmonic oscillator of mass 0.28 kg, the force constant is 80 Nm^{-1} and the damping constant 0.25 s^{-1} .

Q.14.4.3 If instead of the angular frequency of damped harmonic motion, the same for undamped vibration is determined, what will be the percentage error (correct up to three decimal places)?

- (A) + 0.023 %
- (B) + 1.128 %
- (C) – 0.012 %
- (D) – 1.114 %

Q.14.4.4 In what time (correct up to three decimal places in seconds) does the amplitude reach to half its value?

- (A) 2.773

- (B) 1.387
- (C) 0.694
- (D) 0.347

Q.14.4.5 In what time (correct up to three decimal places in seconds) does the mechanical energy get reduced to half its original value?

- (A) 5.546
- (B) 2.773
- (C) 1.387
- (D) 0.694

Q.14.4.6 Resonant vibration is also called _____ vibration.

- (A) menacing
- (B) threatening
- (C) sympathetic
- (D) savage

Q.14.4.7 For which among the following, the primary reason IS NOT 'Resonance'?

- (A) Reverberation in an auditorium.
- (B) Soldiers are being asked to break steps while marching on a suspension bridge.
- (C) Collapsing of some buildings far away from the epicenter of an earthquake, while not much damage is caused to many structures located close to it.
- (D) Violent flapping of aircraft wings corresponds to some engine speed.

Q.14.4.8 Which among the following statements is/are INCORRECT in respect of (the displacement of the oscillator) vs. (driving frequency) curve in the case of forced vibration accompanied by damping?

- I. For weak damping the curve shows a higher peak at resonance than in the case of high damping.
- II. The curve is more skewed towards high frequency.
- III. The curves for high as well as low damping are bell shaped.

Options

- (A) Only I
- (B) Only II
- (C) Only III
- (D) I and III

Common information for Q.14.4.9 & Q.14.4.10

A system having a natural angular frequency of oscillation, ω is subject to forced oscillation by the application of an external periodic force proportional to $\sin pt$, in presence of damping, characterized by the constant b which is equal to half the damping constant per unit mass and unit velocity of the oscillator.

Q.14.4.9 On which among the following does the amplitude of forced oscillation depend?

- (A) ω , but not on p and b
- (B) ω and p , but not on b
- (C) b , but not on ω and p
- (D) b, ω and p

Q.14.4.10 If the forced oscillation lags behind the free oscillation by 45° , then the value of p is given by:

- (A) $b + \sqrt{(\omega^2 + b^2)}$
- (B) $\sqrt{(\omega^2 + b^2)} - b$
- (C) $\sqrt{(\omega^2 + b^2)} - \omega$
- (D) $\omega + \sqrt{(\omega^2 + b^2)}$



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