

Oscillations

Answers

Module 1 Simple Harmonic Motion

A.14.1.1 (A)

$$\omega = \text{Angular speed of P along circle of reference} = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\pi \text{ s}^{-1}$$

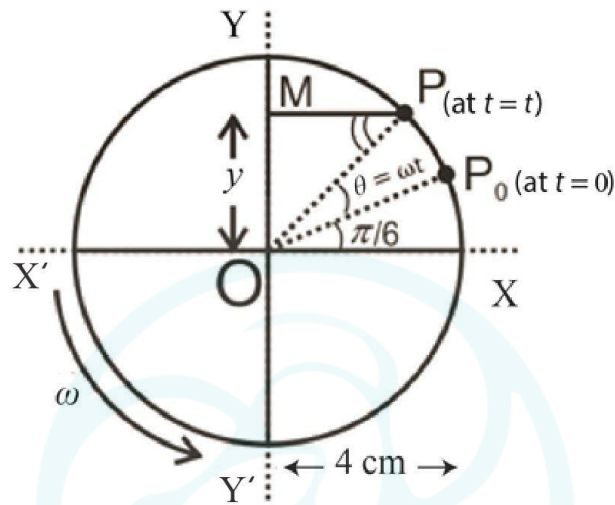


Fig.A.14.1.1

M is the foot of the perpendicular from P on Y'OY.

$$A = \text{Amplitude of motion of M} = 4 \text{ cm}$$

$$\angle \text{OPM} = \angle \text{OPM} = \omega t + \frac{\pi}{6} = 10\pi t + \frac{\pi}{6}$$

From $\angle \text{OPM}$; $y = \text{OM} = \text{instantaneous displacement of M}$

$$= \text{OP} \sin \angle \text{OPM} = 4 \sin \left[10\pi t + \frac{\pi}{6} \right]$$

A.14.1.2 (A.)

(i) Let us $2\sqrt{3} = A \cos \phi$; $2 = A \sin \phi$

$$(2\sqrt{3})^2 + (2)^2 = A^2 \text{ or } A = 4$$

$$\tan \phi = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} \quad \phi = \frac{\pi}{6}$$

Given equation is rewritten as

$$x = 4 \sin(100\pi t) \cos\left(\frac{\pi}{6}\right) + 4 \cos(100\pi t) \sin\left(\frac{\pi}{6}\right)$$

(ii) In the same manner ; we can rewrite

$$x = 4 \sin\left[100\pi t - \frac{\pi}{6}\right]$$

(iii) Put $2 = A \sin \phi$; $2\sqrt{3} = A \cos \phi$

$$\therefore 4 = A ; \phi = \frac{\pi}{6}$$

$$\text{and } x = 4 \cos\left(100\pi t - \frac{\pi}{6}\right)$$

(iv) Can be rewritten as

$$x = 4 \cos\left(100\pi t + \frac{\pi}{6}\right)$$

A.14.1.3 (C)

$$v^2 = 24x - 2x^2 - 70 \tag{14.1.1}$$

$$\therefore \frac{d}{dx} (v^2) = 2v \frac{dv}{dx} = 24 - 4x = -4(x - 6)$$

$$\therefore v \frac{dv}{dx} = -2(x - 6)$$

(14.1.2)

$$v \frac{dv}{dx} = \frac{dx}{dt} \frac{dv}{dx} = \frac{dv}{dt} = \text{acceleration} = \frac{d^2x}{dt^2}$$

$$\text{Let } z = x - 6, \text{ then } \frac{d^2z}{dt^2} = \frac{d^2x}{dt^2}$$

So, from Eq(14.1.2). $\frac{d^2 z}{dt^2} = -2z$, which is the differential equation of SHM, with $\omega^2 = 2$

$$\text{So, } \omega = \sqrt{2} \text{ s}^{-1}$$

$$T = \frac{2\pi}{\sqrt{2}} = \pi\sqrt{2} \text{ s}$$

$$v^2 = -2(x^2 - 12x + 35)$$

$$= -2(x - 5)(x - 7)$$

So, $v = 0$ $x = 5$, $x = 7$. which are the extremities of the motion. Hence,

$$\text{amp} = \frac{1}{2}(7 - 5) = \frac{1}{2} (2) = 1 \text{ cm.}$$

A.14.1.4 (D)

From the given condition, $\frac{d^2 x}{dt^2} = -4\left(x + \frac{1}{4}\right)$

$$\text{Let } z = x + \frac{1}{4}, \quad \frac{dz^2}{dt^2} = \frac{d^2 x}{dt^2}$$

$$\therefore \frac{d^2 z}{dt^2} = -4z$$

$\therefore \frac{d^2 z}{dt^2} + 4z = 0$, which is the differential equation of SHM, with

$$\omega^2 = 4, \quad \omega = 2 \text{ s}^{-1}$$

$$T = \frac{2\pi}{\omega} = \pi \text{ s}$$

The mean position is $z = 0$, i.e. $x + \frac{1}{4} = 0$

$$\text{So, } x_0 = -\frac{1}{4} = -0.25$$

A.14.1.5 (C)

Given equation can be rewritten using trigonometric identity

$\sin 2\theta = 2 \sin \theta \cos \theta$ as,

$$x = 4 \sin 20\pi t \quad (14.1.3)$$

Comparing Eq(14.1.3) with the standard equation of SHM

$x = A \sin(\omega t + \phi_0)$, we have

$$A = 4 \text{ cm}, \omega = 20\pi \text{ rad s}^{-1} \text{ or } \nu = \frac{20\pi}{2\pi} = 10 \text{ Hz}, \phi_0 = 0$$

A.14.1.6 (A)

Comparing $\frac{d^2x}{dt^2} = -9x$ with the standard equation of an SHM.

We get $\omega^2 = 9$, $\omega = 3 \text{ s}^{-1}$

$$T = \frac{2\pi}{\omega} = 2\pi/3 \text{ s}$$

Also, $\omega^2 = k/m$

$$k = 1.8 \text{ N/m}$$

A.14.1.7 (A)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$m' = m - \frac{m}{4} = \frac{3}{4}m, \text{ and } k' = k + \frac{k}{4} = \frac{5}{4}k$$

$$\therefore T' = 2\pi\sqrt{\frac{m'}{k'}} = 2\pi\sqrt{\frac{3m \times 4}{4 \times 5k}} = 2\pi\sqrt{\frac{3m}{5k}} = T\sqrt{\frac{3}{5}}$$

$$\therefore \frac{T'}{T} = \sqrt{\frac{3}{5}}.$$

A.14.1.8 (B)

At $t = 0$; $x = +A$. The equation of SHM is

$$x = A \cos\left(\frac{2\pi}{T}t\right)$$

x_1 = Displacement of particle at P

$$= A \cos\left(\frac{2\pi}{T}x\frac{T}{12}\right) = A \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}A \simeq 0.867A$$

x_2 = Displacement of particle at Q

$$= A \cos\left(\frac{2\pi}{T}x\frac{T}{8}\right) = A \cos\left(\frac{\pi}{4}\right) = \frac{A}{\sqrt{2}} \simeq 0.707A$$

Δx = change in displacement

$$= x_2 - x_1 = (0.707 - 0.867)A = -0.16A$$

% change in displacement

$$= -\left(\frac{0.16A}{0.867A}\right) \times 100 \simeq -18.4\%$$

A.14.1.9 (B)

Let the instantaneous displacement of the two SHM, be X_1 and X_2 , we have

$$X_1 = A \sin 2\pi\nu t$$

$$X_2 = A \sin (2\pi\nu t + \phi_0)$$

here, ϕ_0 is the phase difference between the two motions.

x = instantaneous separation between the two particles = $x_2 - x_1$

$$= A[\sin (2\pi\nu t + \phi_0) - \sin 2\pi\nu t]$$

Using trigonometric identity: $\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$,

x can be rewritten as

$$x = \left[2A \sin\left(\frac{\phi_0}{2}\right) \right] \cos\left(\omega t + \frac{\phi_0}{2}\right) \quad \text{Where } \omega = 2\pi\nu$$

The maximum value of $\cos\left(\omega t + \frac{\phi_0}{2}\right)$ is +1.

$$\text{Therefore } x_m = 2A \sin\left(\frac{\phi_0}{2}\right)$$

Given $x_m = \sqrt{3}A$; therefore

$$\sqrt{3}A = 2A \sin\left(\frac{\phi_0}{2}\right) \text{ or } \frac{\sqrt{3}}{2} = \sin\left(\frac{\phi_0}{2}\right)$$

$$\text{or } \frac{\pi}{3} = \frac{\phi_0}{2}$$

$$\therefore \phi_0 = \frac{2\pi}{3}$$

A.14.1.10 (B)

With symbol having usual meaning, we have the Energy Equation

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{a constant} = E, \text{ say}$$

$$\therefore \frac{p^2}{2mE} + \frac{x^2}{\frac{2E}{k}} = 1, \text{ where } p = mv$$

Since p - x curve is a circle, $2mE = \frac{2E}{k}$, $\therefore m = \frac{1}{k}$

Module 2 Characteristics of Simple Harmonic Motion

A.14.2.1 (D)

For the particle executing SHM; the instantaneous speed v is

$$v = \omega \sqrt{A^2 - x^2} \quad ; \text{ Where } x \text{ is the instantaneous displacement.}$$

Given, $x = 50\%$ of $A = \frac{A}{2}$

Therefore

$$v = \omega \sqrt{A^2 - \left(\frac{A}{2}\right)^2} = \frac{\sqrt{3}}{2} A\omega = \frac{\sqrt{3}}{2} v_m$$

where $v_m = A\omega$ velocity amplitude. Obviously

$$\Delta v = v - v_m = \left(\frac{\sqrt{3}}{2} - 1\right) v_m$$

$$\text{or } \frac{\Delta v}{v_m} = \left(\frac{\sqrt{3}}{2} - 1\right)$$

A.14.2.2 (B)

For a particle executing SHM of amplitude A , angular frequency ω , the instantaneous speed, v , is

$$v = \omega \sqrt{A^2 - x^2} \quad \text{or } \left(\frac{v}{\omega}\right)^2 = A^2 - x^2$$

$$\therefore \frac{v^2}{(A\omega)^2} + \frac{x^2}{A^2} = 1 \quad (14.2.1)$$

Comparing Eq(14.2.1) with the given equation

$$A^2 = 4 \text{ or } A = 2 \text{ cm}$$

$$A\omega = 2\pi \text{ or } \omega = \pi$$

$$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2 \text{ s}$$

A.14.2.3 (D)

Given $T = 2 \text{ s}$; $A = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$; $m = 200 \text{ g} = 0.2 \text{ kg}$; $KE = 1.6 \text{ mJ} = 16 \times 10^{-4} \text{ J}$

Let v and x denote the instantaneous speed and displacement respectively of the bob. Obviously, $K = \frac{1}{2}mv^2$

$$16 \times 10^{-4} = \frac{1}{2} \times 0.2 \times v^2$$

$$\therefore v^2 = 16 \times 10^{-3} (\text{ms}^{-1})^2$$

$$\text{Also } v^2 = \omega^2 [A^2 - x^2] \text{ ; therefore}$$

$$16 \times 10^{-3} = \left(\frac{2\pi}{2}\right)^2 [A^2 - x^2] \quad \left[\because \omega = \frac{2\pi}{T}\right]$$

$$= 10[A^2 - x^2] \quad [\pi^2 \simeq 10]$$

$$16 \times 10^{-4} = (5 \times 10^{-2})^2 - x^2$$

$$\text{or } x^2 = 9 \times 10^{-4}$$

$$\therefore x = \pm 3 \times 10^{-2} \text{ m} = \pm 3 \text{ cm}$$

A.14.2.4 (C)

Using the relation

$$v = \omega \sqrt{A^2 - x^2}, \text{ we have}$$

$$\alpha = \omega \sqrt{A^2 - \beta^2} \quad (14.2.2)$$

$$\text{and} \left(\frac{\alpha}{2}\right) = \omega \sqrt{A^2 - \left(\frac{5\beta}{4}\right)^2} \quad (14.2.3)$$

From Eqs(14.2.2) and (14.2.3).

$$\left[\frac{\alpha}{2}\right]^2 = \frac{A^2 - \beta^2}{\left(A^2 - \frac{25}{16}\beta^2\right)}$$

$$\text{or } 4A^2 = 7\beta^2 \quad \therefore A = \frac{\sqrt{7}}{2}\beta$$

Substituting value of A in Eq(14.2.2), we have

$$\omega = \sqrt{\frac{4}{3}} \left(\frac{\alpha}{\beta}\right)$$

$$\text{and } T = \sqrt{3}\pi \left(\frac{\beta}{\alpha}\right) \quad \left[\because T = \frac{2\pi}{\omega}\right]$$

A.14.2.5 (B)

In SHM.; $a = -\omega^2 x$ i.e. a versus x graph is a straight line having a slope $-\omega^2$. Let ω_1 and ω_2 be the angular frequency of SHM, I and II respectively. From given graph

$$-\omega_1^2 = \tan 120^\circ = -\cot 30^\circ = -\sqrt{3} \quad \text{or } \omega_1^2 = \sqrt{3}$$

And $-\omega_2^2 = \tan 150^\circ = -\cot 60^\circ = -\frac{1}{\sqrt{3}}$ or $\omega_2^2 = \frac{1}{\sqrt{3}}$

$$\frac{T_1}{T_2} = \frac{\omega_2}{\omega_1} = \frac{1}{\sqrt{3}}$$

A.14.2.6 (A)

Given $x = \pm \frac{A}{\sqrt{2}}$. The instantaneous potential energy U is

$$U = \frac{1}{2}kx^2 = \frac{1}{4}kA^2$$

$$E = \text{The total energy of SHM} = U_{\max} = \frac{1}{2}kA^2$$

$$K = \text{The instantaneous kinetic energy} = E - U = \frac{1}{4}kA^2$$

$$\therefore \frac{U}{K} = 1 \text{ and } \frac{E}{U} = 2$$

A.14.2.7 (B)

The average kinetic energy, over one cycle of changes, is half the maximum kinetic energy ($=E$) of SHM. Therefore

$$E = K_{\max} = 2 K_{\text{av}} = 2 \times 16 \text{ mJ} = 32 \times 10^{-3} \text{ J}$$

Given, $k = 10^{-3} \frac{\text{N}}{\text{cm}} = 10^{-1} \text{ Nm}^{-1}$ we have

We know

$$E = U_{\max} = \frac{1}{2}kA^2$$

$$\therefore 32 \times 10^{-3} = \frac{1}{2} \times 10^{-1} A^2 \text{ or } A = 0.8 \text{ m}$$

Also, $E = \frac{1}{2}mA^2\omega^2$, therefore $\omega = 2 \text{ rad s}^{-1}$

A.14.2.8 (A)

In Fig.Q.14.2.2(a) the potential energy in mean position = $U_0 = 0.1$ J. For $x = 1 \text{ cm} = 10^{-2} \text{ m}$; the instantaneous potential energy = $U = 0.2$ J. We know

$$U - U_0 = \frac{1}{2} m A^2 \omega^2$$

$$U - U_0 = \frac{1}{2} k_1 x^2$$

$$\therefore (0.2 - 0.1) = \frac{1}{2} k_1 (1 \times 10^{-2})^2 \text{ or } k_1 = 2000 \text{ Nm}^{-1}$$

In Fig.Q.14.2.2(b); the potential energy (U_0) at the mean position, $x = 0$. Also $U = 0.1$ J for $x = 10^{-2} \text{ m}$

$$\therefore 0.1 = \frac{1}{2} k_2 (1 \times 10^{-2})^2 \text{ or } k_2 = 2000 \text{ Nm}^{-1}$$

$$\therefore \frac{k_1}{k_2} = 1$$

A.14.2.9 (C)

In general, the instantaneous potential energy U and instantaneous displacement x ; in SHM. satisfy the relation

$$U = U_0 + \frac{1}{2} k x^2 \quad (14.2.4)$$

where U_0 is potential energy in the mean position.

but we have $U = \alpha + \beta x^2$

Given relation is the same as Eq(14.2.4); therefore the motion of the particle is SHM. Also $k = 2\beta$. The time period, T ; of motion is $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2\beta}}$

A.14.2.10 (B.)

Refer to Fig.A.14.2.1

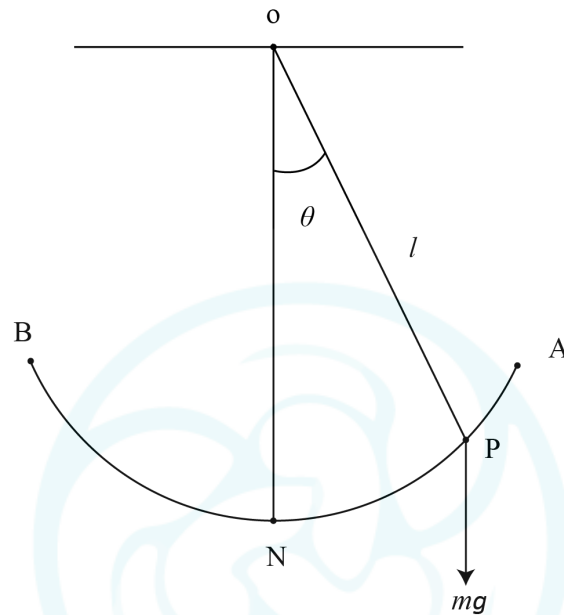


Fig.A.14.2.1

$$ON = OA = l, OM = l \cos \theta$$

$$MN = ON - OM = l(1 - \cos \theta)$$

$$PE = mgl(1 - \cos \theta)$$

$$KE = \frac{1}{2} I \left(\frac{d\theta}{dt} \right)^2 = \frac{1}{2} ml^2 \left(\frac{d\theta}{dt} \right)^2$$

$$\text{So, } \frac{1}{2} ml^2 \left(\frac{d\theta}{dt} \right)^2 + mgl(1 - \cos \theta) = \text{a constant}$$

Dividing both sides by ml , we get

$$\frac{1}{2}l\left(\frac{d\theta}{dt}\right)^2 + g(1 - \cos\theta) = a \text{ constant}$$

$$\therefore \frac{1}{2}l\left(\frac{d\theta}{dt}\right)^2 - g\cos\theta = a \text{ constant}$$

Module 3 Examples of Simple Harmonic Motion

A.14.3.1 (B)

$$T_1 = 2\pi\sqrt{\frac{M}{k_1}} \quad (14.3.1)$$

$$\text{Given, } M' = M - \frac{M}{4} = \frac{3M}{4}; \quad T_2 = T_1 + \frac{T_1}{2} = \frac{3T_1}{2};$$

$$\therefore T_2 = 2\pi\sqrt{\frac{M'}{k_2}} = 2\pi\sqrt{\frac{3M}{4k_2}} \quad (14.3.2)$$

From Eqs(14.3.1) and (14.3.2)

$$\left(\frac{T_1}{T_2}\right)^2 = \frac{\frac{M}{k_1}}{\frac{3M}{4k_2}} \text{ or } \left(\frac{2}{3}\right)^2 = \frac{4k_2}{3k_1}$$

$$\therefore \frac{k_2}{k_1} = \frac{1}{3}$$

A.14.3.2 (C)

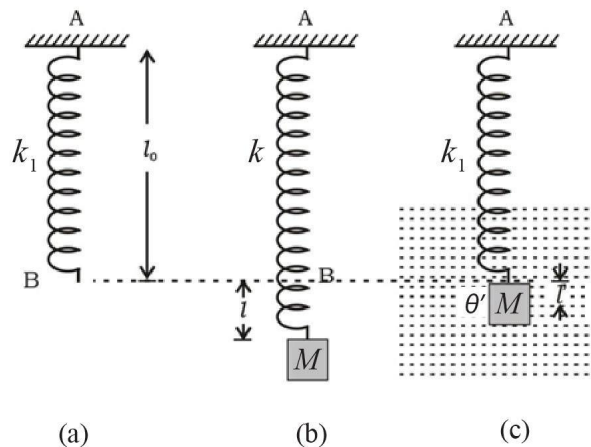


Fig.A.14.3.1

Fig.A.14.3.1(b) equilibrium position O of the system in air. $OB = l$, such that $Mg = kl$. When mass is immersed completely in liquid, due to buoyant force, $g_{eff} = g\left(1 - \frac{\sigma}{\rho}\right)$; σ and ρ denote density of liquid and material of mass M respectively.

Let l' be an extension in equilibrium position O' as shown in Fig.A.14.3.1(c). Obviously, $Mg' = kl'$. Since $g' < g$; $l' < l$. This means equilibrium position moves vertically upwards.

The time period (T) of oscillations is $T = 2\pi\sqrt{\frac{M}{k}}$

When mass is immersed inside liquid, neither M nor k changes; therefore the time-period of oscillations does not change.

A.14.3.3 (B)

At $t = 0$, the time period T_0 is

$$T_0 = 2\pi\sqrt{\frac{M_0}{k}}$$

where $M_0 = m_1 + m_2$

As liquid drips out, mass attached to spring decreases. Let M be the instantaneous mass of the arrangement. Then

$$M = M_0 - \alpha t$$

The instantaneous time-period, T , of oscillations is

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M_0 - \alpha t}{k}}$$

$$\text{or } T^2 = \frac{4\pi^2}{k} [M_0 - \alpha t] \quad (14.3.3)$$

Eq(14.3.3) is the equation of the straight line between T^2 and t . At $t = \tau$; where $\tau = \frac{m_2}{\alpha}$; the entire liquid has dripped out. The mass attached to the spring now has a constant value m_1 . The time period of oscillations is constant for $t > \tau$.

A.14.3.4 (C)

Given

$$T_1 = 2\pi\sqrt{\frac{M}{k_1}} \quad (14.3.4)$$

$$T_2 = 2\pi\sqrt{\frac{M}{k_2}} \quad (14.3.5)$$

For, M is attached to S_1 and S_2 as shown; the springs are joined in parallel. The equivalent spring constant of the arrangement, k , is

$$k = k_1 + k_2$$

$\therefore$

$$T = 2\pi\sqrt{\frac{M}{k_1 + k_2}}$$

(14.3.6)

Squaring and rearranging Eqs(14.3.4), (14.3.5) and (14.3.6) we have

$$\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$$

A.14.3.5 (C)

Springs of constant k and $3k$ are in parallel with one another. The equivalent spring constant $k' = k + 3k = 4k$. k' is joined with a series with spring of spring constant $2k$. The equivalent spring constant, k_{eq} ; of the arrangement is

$$\frac{1}{k_{eq}} = \frac{1}{k} + \frac{1}{2k} = \frac{1}{4k} + \frac{1}{2k}$$

$$\therefore k_{eq} = \frac{4k}{3}$$

The time-period, T , of oscillations of mass M is

$$\begin{aligned} T &= 2\pi \sqrt{\frac{M}{k_{eq}}} \\ &= 2\pi \sqrt{\frac{3M}{4k}} = \pi \sqrt{\frac{M}{k}} \end{aligned}$$

A.14.3.6 (A)

The mass M of the cylinder is $M = \pi R^2 L \rho$

Substituting value of M in given expression for T ; we have

$$T = 2\pi \sqrt{\frac{\pi R^2 L \rho}{\pi R^2 \sigma g}} = 2\pi \sqrt{\frac{L \rho}{\sigma g}}$$

T is independent of R .

A.14.3.7 (C)

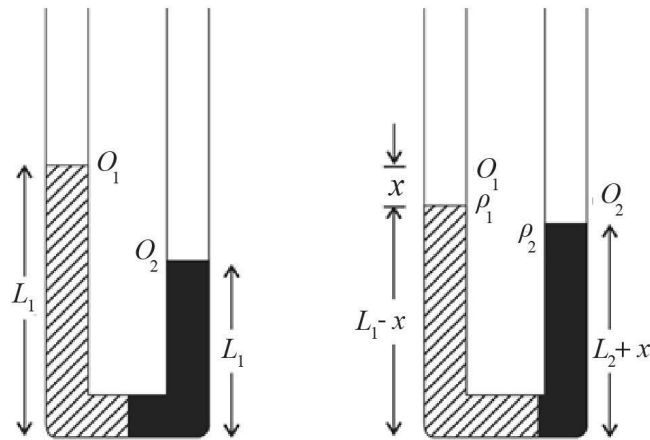


Fig.A.14.3.2

Fig.A.14.3.2(a) shows the equilibrium position of the arrangement. Obviously,

$$L_1 \rho_1 g = L_2 \rho_2 g$$

$$\text{or } L_1 \rho_1 = L_2 \rho_2 \quad (14.3.7)$$

Fig.A.14.3.2(b) shows the level of the liquids in the two limbs. For small, instantaneous displacement x , the net unbalanced restoring force, F , on the liquid column is

$$\begin{aligned} F &= a[(L_2 + x)\rho_2 g - (L_1 - x)\rho_1 g] \\ &= a(\rho_1 + \rho_2)gx \end{aligned} \quad (14.3.8)$$

Let M be the total mass of liquid in the U-tube. Obviously

$$M = a[L_1 \rho_1 + L_2 \rho_2] \quad (14.3.9)$$

The equation of motion of liquid column is

$$\begin{aligned} M \frac{d^2 x}{dt^2} &= - [a(\rho_1 + \rho_2)g]x \\ \text{or } \frac{d^2 x}{dt^2} &= - \omega^2 x \end{aligned}$$

where $\omega^2 = \frac{a(\rho_1 + \rho_2)g}{M} = \frac{a(\rho_1 + \rho_2)g}{a(L_1\rho_1 + L_2\rho_2)}$

$$\frac{4\pi^2}{T^2} = \frac{a(\rho_1 + \rho_2)g}{a(L_1\rho_1 + L_2\rho_2)}$$

T = Time period of oscillations of the liquid column

$$T = 2\pi\sqrt{\frac{L_1\rho_2 + L_2\rho_1}{(\rho_1 + \rho_2)g}}$$

A.14.3.8 (C)

M = mass of liquid in U-tube = $2La\rho$. Substituting value of M in given expression for T ; we have

$$T = 2\pi\sqrt{\frac{2La\rho}{2a\rho g}} = 2\pi\sqrt{\frac{L}{g}}$$

Note: T does not depend on (i) a and (ii) ρ . Therefore

$$T_1 = T \quad \text{or} \quad \frac{T_1}{T} = 1$$

A.14.3.9 (C)

Time period, T , of a seconds pendulum is 2 s Therefore

$$T = 2 \text{ s} = 2\pi\sqrt{\frac{l}{g}}$$

(symbols have their usual meaning)

Inside lift; the time-period, T_1 ; of the pendulum = $T + \frac{T}{2} = 3 \text{ s}$. Let g_{eff} be the effective value of acceleration due to gravity inside the lift. Then

$$T_1 = 3 \text{ s} = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

$$\therefore \frac{4}{9} = \frac{g_{eff}}{g} \quad \text{or} \quad g_{eff} = \frac{4g}{9}$$

Since $g_{eff} < g$; lift accelerates downward; say, with an acceleration a . We know

$$g_{eff} = g - a$$

$$\frac{4g}{9} = g - a$$

$$\therefore a = \frac{5g}{9}$$

A.14.3.10 (B)

The time period, T ; of a second's pendulum having length, l , is 2 s. Therefore

$$T = 2s = 2\pi\sqrt{\frac{l}{g}} \quad (14.3.10)$$

When bob is immersed completely in liquid; because to the buoyant force due to the liquid; the effective value of g is

$$g_{eff} = g\left(1 - \frac{\sigma}{\rho}\right) = g\left(1 - \frac{1}{2}\right) = \frac{g}{2}$$

The time-period; T' , of oscillations is

$$T' = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{2l}{g}} = \sqrt{2}.T = 2\sqrt{2} \text{ s.}$$

A.14.3.11 (D)

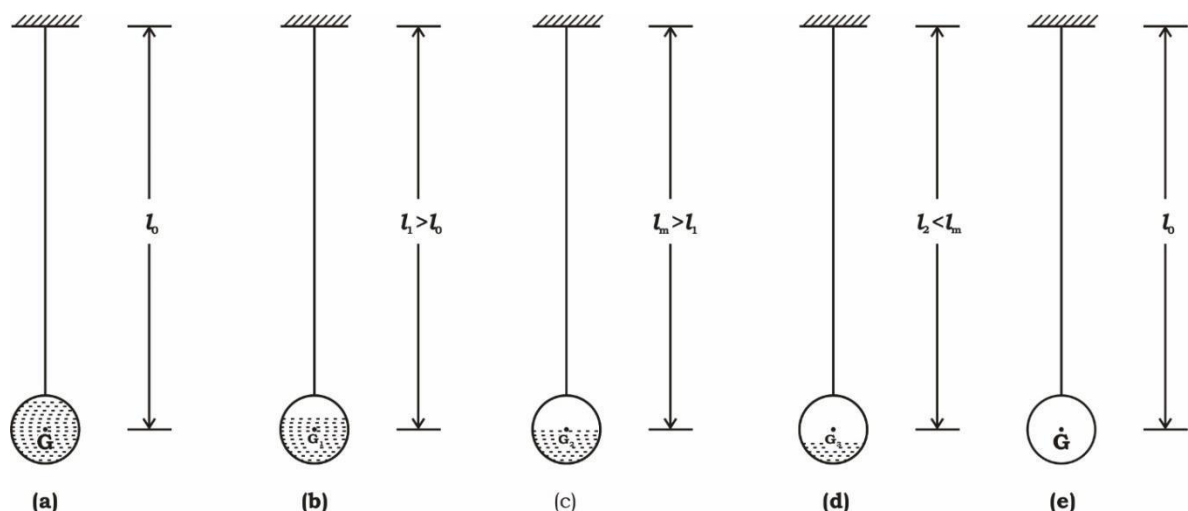


Fig.A.14.3.3

One is tempted to ‘think’ that mass of bob decreases (changes) as liquid drips out and time period should remain same, as time period DOES NOT depend on mass of bob. However, there is a change in length of pendulum (distance between point of suspension and CG of the bob) as liquid drops, as shown in Fig.14.3.3 The CG is at the lowest position when shell is HALF-empty as shown in Fig.A.14.3.3(c). From Fig A.14.3.3; it is clear that the length of the pendulum first increases, acquires a maximum value, l_m ; and then decreases. Finally (when the shell is completely empty) the time period is the same as at the beginning. Therefore the time period first increases, acquires a maximum value, then decreases and finally is same as at $t = 0$.

Module 4 : Free Damped and Forced oscillations

A.14.4.1 (B)

In the said equation $2b \frac{dx}{dt}$ gets added with an acceleration term. So, $2b \frac{dx}{dt}$ must have the unit of acceleration.

So, the unit of b has to be that of (acceleration/velocity), i.e., s^2/s^{-1} , hence, Option B. The distractors have been framed keeping in mind that acceleration may be confused with force, particularly in case of No.(D).

Note: It is crucial to understand that the dimension of b is the same as that of reciprocal of time. The term e^{-bt} appears in the solution of the differential equation of damped harmonic motion. Anything appearing as a power has to be dimensionless. More importantly, e^x is a power series in x . So, it can be understood that if x has a

dimension, a havoc will be created and the series will become untenable. So, bt is dimensionless, and it can only happen when b has the dimension of reciprocal of time.

A.14.4.2 (B)

The highest order of derivative appearing in the equation is two. So, order is two. Again if x_1 and x_2 are two independent solutions, then $(m_1x_1 + m_2x_2)$ is also a solution, where m_1 and m_2 are scalars. So, the equation is linear, and thus, the degree is one. The distractors have been framed by keeping in mind the possible doubts one may have about any possible link between order of the derivatives, and their degrees.

Note: This question has been set despite fear of non-acceptance from the syllabus-oriented persons. Further, it may appear to be more skewed towards mathematics than physics. But the point to be noted is that there are significant links with physics, which are as under –

- In order to solve this equation, we need to have two boundary conditions, one in terms of velocity and the other in terms of displacement; in other words, the two orders of the differential equation are linked directly with the concept of phase, which is the state of motion governed by simultaneous consideration of velocity and displacement.
- The issue of the degree being one is the fountain-head of the ‘principle of superposition’, which finds application in so many phenomena associated with ‘Oscillations and Waves’.

Most importantly, this material is meant for the teachers. So, some aberration has been ventured.

A.14.4.3 (C)

$$\omega = \sqrt{\frac{k}{m}}, \quad \omega_1 = \sqrt{\omega^2 - b^2}$$

$$\text{So, } \omega = \sqrt{\frac{80}{0.28}} = 16.903; \quad \omega_1 = \sqrt{285.7143 - 0.0625} = \sqrt{285.6512} = 16.901$$

$$\text{So, the required error} = \frac{16.901 - 16.903}{16.901} \times 100 \% = \frac{-0.2}{16.901} \% = -0.012 \%$$

One of the possibilities of getting distracted is connected with the sign of the error. Rest is about the calculation.

A.14.4.4 (A)

$$A = A_0 e^{-bt}; \quad A = \frac{A_0}{2}; \quad e^{-bt} = \frac{1}{2}$$

$$\text{So, } \ln\left(\frac{1}{2}\right) = -\ln 2 = -bt$$

$$\text{So, } t = \frac{\ln 2}{b} = \frac{0.693}{0.25} = 2.773 \text{ s}$$

Improper handling of the logarithmic relation and wrong calculation are the possible reasons for distraction.

A.14.4.5 (B)

With symbols having their usual meanings, we have

$$E = \frac{1}{2} kx^2, \text{ where } x = x_0 e^{-bt}; \quad x^2 = x_0^2 e^{-2bt}$$

$$\text{So, } E = E_0 e^{-2bt}$$

So, following the same argument as in case of A.14.4.4, we get that the

$$\text{Required time} = \frac{\ln 2}{2b} = \left(\frac{1}{2}\right)\left(\frac{\ln 2}{b}\right) = \frac{1}{2}(2.773) = 1.387 \text{ s}$$

Incorrect determination of the power of e As a result of variation of energy with time, wrong handling of logarithmic formula and miscalculation are the possible reasons behind distractions.

A.14.4.6 (C)

Option numbers (A), (B), (D) are interchangeably used to mean dangerous.

Option(C) has become synonymous with 'resonant' after several accidents were caused due to resonant vibrations, like the one in the Tacoma Narrows Bridge on November 7, 1940

It is a situation where the affected persons are caught unaware about the sudden damage caused by resonant vibration, and thus the term was coined. Such accidents are testimony to the fact that Forced Vibration generally happens with very small amplitude, but it becomes significantly large in case of Resonance.

A.14.4.7 (A)

During a performance inside an auditorium, multiple reflections are caused primarily due to the walls and surfaces of other objects such as furniture, and even people; and the persistence of such reflected sound with its gradual dying away constitutes the phenomenon of Reverberation. Resonance has hardly any role here.

While in each of the cases of Options (B), (C), (D), we have a system which is subject to vibration, be it a suspension bridge (the only connectivity available or made temporarily available across violently flowing streams in difficult terrains accessed by soldiers under extremely compelling circumstances), or a structure which has its foundation in the soil through which seismic waves (in cases with very small amplitude due to its source being far off) can travel, or an aircraft wing. Each has a natural frequency of vibration based on its mechanical properties, and there is a possibility of each being forced into vibration by some external cause. If the forcing frequency matches the natural frequency of vibration, then resonant vibration with very large amplitude will occur which can cause significant damage to the construction.

A.14.4.8 (B)

Fig 14.4.2 in Module 4 may be seen.

Improper understanding of the nature of the curves is the possible reason behind the distractions.

A.14.4.9 (D)

See Eqn (14.4.3) in Module 4, from where the answer becomes obvious. The distractors have been framed primarily to create confusion.

A.14.4.10 (A)

From Eqn (14.4.4) in Module 4, we have, since $\phi_0 = -45^\circ$, $-1 = \frac{2bp}{\omega^2 - p^2}$

So, $2bp = p^2 - \omega^2$; $p^2 - 2bp - \omega^2 = 0$

$$\therefore p = \frac{2b + \sqrt{4b^2 + 4\omega^2}}{2} = b \pm \sqrt{\omega^2 + b^2}$$

But the negative sign in the numerator is inadmissible. So, $p = b + \sqrt{\omega^2 + b^2}$. The distractors have been framed keeping in mind the possible errors which might be committed by way of not getting the phase angle correctly, confusion between 'lead' and 'lag', and incorrect determination of the value of $\tan(-45^\circ)$.



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