

Module 3 Earth Satellites and Weightlessness

This module is designed to help you to:

- analyze the concept of energy of an orbiting satellite;
- explain satellite motion ;
- compare geostationary and polar satellites;
- describe the concept of weightlessness.

Content:

8.3.1 Introduction

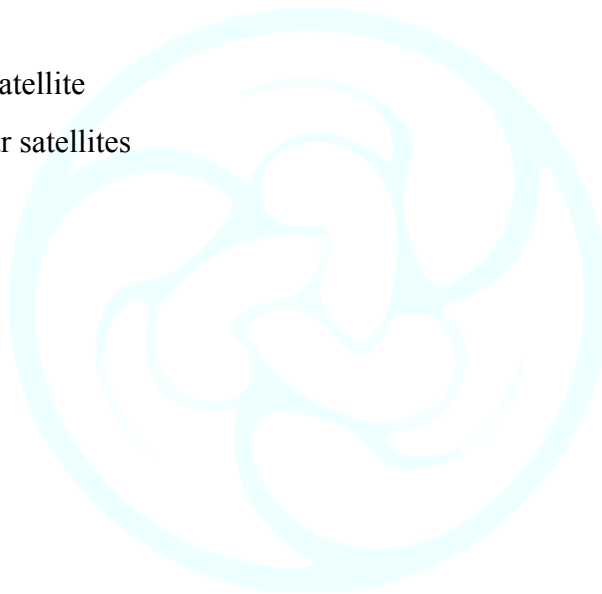
8.3.2 Earth satellites

8.3.3 Energy of an orbiting satellite

8.3.4 Geostationary and polar satellites

8.3.5 Weightlessness

8.3.6 Summary



8.3.1 Introduction

This module discusses the motion of earth's satellites and derives expression for its time period of rotation. The speed of a satellite revolving in the circular orbit around the earth is known as critical speed at that height. We also discuss the conditions required for geostationary satellites which appear stationary with respect to the earth's surface. They are used for communication purposes. Polar satellites are the satellites which orbit at lower altitudes and are used in remote sensing applications. The phenomenon of weightlessness experienced by astronauts in orbiting satellites has also been discussed.

8.3.2 Earth satellites

Earth satellites are objects that revolve around the earth. The orbit of the earth satellite around the earth is circular or elliptical. Moon is the only natural satellite of the earth having a near circular orbit with a time period of approximately 27.3 days.

The artificial satellites are man-made satellites being used extensively for telecommunication. Many countries including India, have launched artificial earth satellites for practical use in fields such as in telecommunication, remote sensing, meteorology, defense, etc. With the help of these artificial earth satellites, communications have become easier and information sharing has increased to a great extent.

Let us consider a satellite in a circular orbit at a distance h above the surface of the earth. If m is the mass of the satellite and v its speed, the centripetal force required for this orbit, is provided by the gravitational force of earth on the satellite (Fig. 8.3.1).

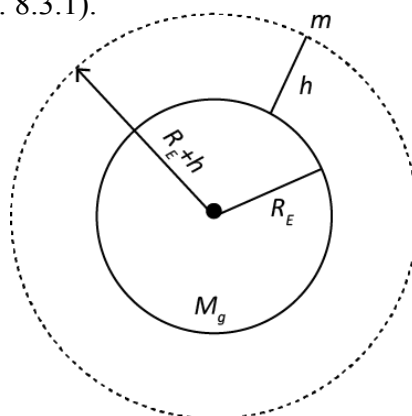


Fig. 8.3.1: *Orbital velocity at a height h .*

$$\frac{mv^2}{(R_E+h)} = \frac{GmM_E}{(R_E+h)^2} \quad (8.3.1)$$

This gives $v = \sqrt{\frac{GM_E}{(R+h)}}$

This speed is known as the critical speed.

$$\text{Critical speed } v_c = \sqrt{\frac{GM_E}{(R+h)}} \quad (8.3.2)$$

v_c is also known as the orbital speed for circular orbit.

As h increases, v_c decreases. If at a given height, orbital speed satellite is greater than critical speed, then it will revolve in an elliptic orbit with the earth at one of the foci of the elliptical orbit.

Using the above equation, we can calculate the time period of a satellite orbiting the earth. Its time period T is given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{v} (R_E + h)$$

$$\therefore T = \frac{2\pi(R_E+h)^{3/2}}{(GM_E)^{1/2}} \quad (8.3.3)$$

We can see that $T^2 \propto (R_E + h)^3$, which is a reflection of Kepler's third law.

Example 8.3.1: A satellite orbits earth at an altitude of 400 km above its surface. What is (a) its orbital speed and (b) its time period of revolution?

$$M_E = 6 \times 10^{24} \text{ kg}$$

$$R_E = 6.4 \times 10^6 \text{ m}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

Solution:

$$r = R_E + h = 6.4 \times 10^6 + 0.4 \times 10^6 \text{ m} = 6.8 \times 10^6 \text{ m}$$

$$\begin{aligned} \text{(a)} \quad v_o &= \sqrt{\frac{GM_E}{(R_E + h)}} \\ v_o &= \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.8 \times 10^6}} \\ &= \sqrt{\frac{66.7 \times 6 \times 10^3}{6.8}} = 7.67 \times 10^3 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad T &= \frac{2\pi r}{v_o} \\ &= \frac{2 \times 3.14 \times 6.8 \times 10^6}{7.67 \times 10^3} \\ \therefore T &= 5.57 \times 10^3 \text{ s} \end{aligned}$$

8.3.3 Energy of an orbiting satellite

An orbiting satellite has kinetic energy as well as potential energy

$$\text{KE} = \frac{1}{2}mv^2 = \frac{GmM_E}{2(R_E + h)} \quad (8.3.4)$$

Potential energy at distance $(R_E + h)$ from the center of earth is

$$\text{PE} = -\frac{GmM_E}{(R_E + h)} \quad (8.3.5)$$

$$\text{Total energy E} = \text{KE} + \text{PE} = -\frac{GmM_E}{2(R_E + h)} \quad (8.3.6)$$

Total energy of a circularly orbiting satellite is negative.

A satellite is revolving around the earth in an orbit of radius $r = (R_E + h)$. The KE, PE and total energy of satellite are

$$KE = + \frac{GM_E m_s}{2r}, \quad PE = - \frac{GM_E m_s}{r},$$

$$\text{Total energy, } E = - \frac{GM_E m_s}{2r}$$

Activity 8.3.1 Sketch the variation of KE, PE and TE of a satellite as a function of its orbit of radius r .

Example 8.3.2: Consider a planet orbiting in an elliptical orbit of eccentricity e and semi-major axis a . Obtain an expression of its total energy.

Solution: Applying law of conservation of angular momentum and law of conservation of energy at A and P (Fig.8.3.2).

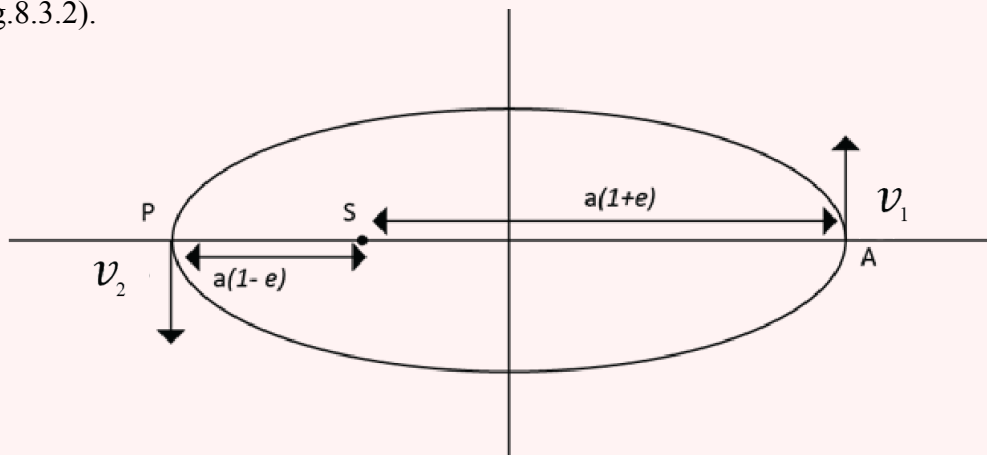


Fig.8.3.2

With symbols having their usual meaning, we get on applying the principles of conservation of energy and momentum, the following:

$$\frac{1}{2}mv_1^2 - \frac{GMm}{a(1+e)} = \frac{1}{2}mv_2^2 - \frac{GMm}{a(1-e)} \quad (8.3.7)$$

$$\text{and} \quad mv_1 a(1+e) = mv_2 a(1-e) \quad (8.3.8)$$

$$\text{From Eq(8.3.8), we get, } \frac{1+e}{1-e} = \frac{v_2}{v_1}$$

$$\text{Or, } \frac{1}{e} = \frac{v_2 + v_1}{v_2 - v_1} \quad (8.3.9)$$

Again from Eq(8.3.7), we have

$$\frac{1}{2}m(v_2^2 - v_1^2) = \frac{GMm}{a} \left(\frac{1}{1-e} - \frac{1}{1+e} \right)b$$

$$\text{or } \frac{m}{2} (v_2 - v_1)(v_2 + v_1) = \frac{GMm}{a} \frac{2e}{1-e^2}$$

Hence, from Eq(8.3.9), we get

$$\frac{me}{2} (v_2 + v_1)^2 = \frac{GMm}{a} \frac{2e}{1-e^2}$$

$$\text{Or, } (v_2 + v_1)^2 = \frac{GM}{a} \frac{4}{1-e^2}$$

$$\text{Or, } v_2^2 \left\{ 1 + \left(\frac{1-e}{1+e} \right) \right\}^2 = \frac{4v_2^2}{(1+e)^2} = \frac{GM}{a} \cdot \frac{4}{1-e^2}$$

$$v_2^2 = \frac{GM}{a} \cdot \frac{1+e}{1-e}$$

$$\text{Total Energy} = \frac{1}{2}mv_2^2 - \frac{GMm}{a(1-e)} = \frac{GMm}{2a} \left(\frac{1-e}{1-e} \right) - \frac{GMm}{a(1-e)}$$

$$E = \frac{GMm}{2a(1-e)} (1 + e - 2) = - \frac{GMm}{2a} \left(\frac{1-e}{1-e} \right)$$

$$\text{So, } E = - \frac{GMm}{2a}.$$

8.3.4 Geostationary and polar satellites

The study of gravitation has enabled us to launch satellites in orbits around the earth. These satellites are used for various applications. For communication, we use geostationary satellites in equatorial orbit and for remote sensing, we use polar orbit satellites which in general have a smaller time period. Another interesting aspect of satellites in circular orbits is that effective gravity in the satellites is zero.

An artificial satellite generally has either equatorial or polar orbit depending on the purpose for which the satellite is launched. Artificial satellites are of following types: geostationary and polar.

Geostationary satellites

If the circular orbit of a satellite lies in the equatorial plane of the earth and has the same period as the period of rotation of the earth about its own axis, the satellite appears stationary in the sky as seen by an observer on the ground. Such satellites are called geostationary satellites.

They are placed at an altitude of approximately 36,000 km directly over the equator and revolve in the same direction as the rotation of the earth (west to east). These satellites are widely used for telecommunication.

Example 8.3.3 A satellite is to be placed in equatorial geostationary orbit around earth for communication.

(a) Calculate the height of such a satellite above the equator.

(b) Find out the minimum number of satellites that are needed to cover the entire earth so that at least one satellite is visible from any point on the equator.

Solution:

$$\begin{aligned} \text{(a)} \quad r &= \left(\frac{GMT^2}{4\pi^2} \right)^{1/3} \\ \therefore h &= \left(\frac{GMT^2}{4\pi^2} \right)^{1/3} - R \\ &= 4.23 \times 10^7 - 6.4 \times 10^6 \\ &= 3.59 \times 10^7 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \theta &= \cos^{-1} \left(\frac{R}{R+h} \right) \\ &= \cos^{-1} \left(\frac{1}{1+h/r} \right) \\ &= \cos^{-1} \left(\frac{1}{1+5.61} \right) \\ &= 81^\circ 18' \end{aligned}$$

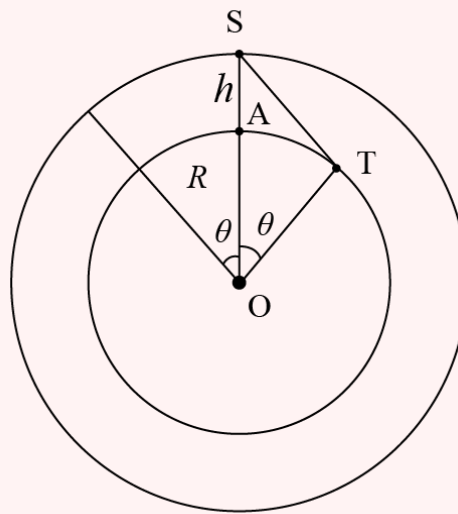


Fig.8.3.3

The satellite S is in an equatorial orbit, say at the point S. O is the center of earth. ST is the tangent from S on the equatorial plane. $\angle SOT = \theta$. So the field of view of the satellite will correspond to a coverage angle 2θ . Since the complete field of view is 360° , the required minimum number of satellites is the integral just above $\frac{360^\circ}{2\theta}$ (Fig.8.3.3).

$$\therefore 2\theta = 162^\circ 36'$$

$$\frac{360^\circ}{2\theta} = 2.21 .$$

Hence, the minimum number is = 3.

Polar satellites

Polar satellites orbit around the earth at a low altitude of 500 to 800 km in a north to south (or vice versa) direction, passing over the poles whereas the earth rotates around its axis in a west to east direction . Polar satellites are extremely useful for remote sensing, meteorology as well as for environmental studies as they

offer a much better resolution due to their closeness to the earth. For the next revolution of the satellite, the earth has rotated a little on an axis so that an adjacent strip becomes visible (Fig. 8.3.4).

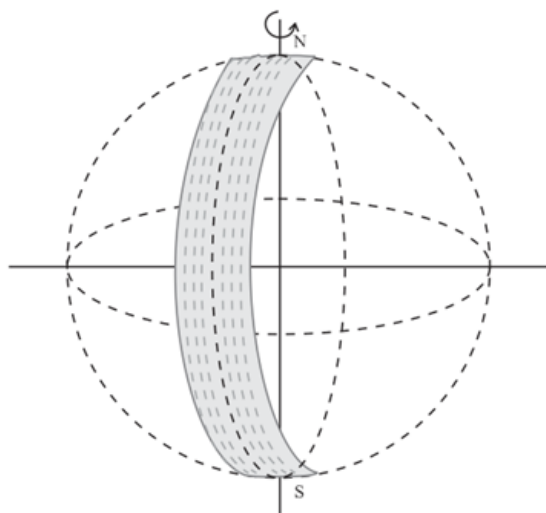


Fig. 8.3.4: A strip on the earth surface (shown shaded) is visible from a polar satellite during one circle



Let Us Reflect

Q.8.3.1 Match the following

A (Application)	B (Types of satellite orbit)
a. Communication	1. Polar orbit
b. Remote sensing	2. Geostationary orbit
c. Astronomical observations	3. Equatorial orbit



Answers

A.8.3.1 a-2, b-1, c-3

8.3.5 Weightlessness

Every object on the surface of the earth experiences a force of gravity. The weight is the gravitational force of attraction that a body experiences. We sense our weight as we stand on the surface of the earth because the surface exerts a force opposite to our weight to keep us at rest. However a freely falling object experiences no upward force and falls downward with acceleration due to gravity. As an object falling freely does not experience upward force, it experiences weightlessness.

A satellite around the earth experiences a constant acceleration towards the center of the earth which is exactly the same as the value of acceleration due to gravity at that position. Thus a satellite or the people inside it experience weightlessness.

This makes astronauts in space a station orbiting earth, experience weightlessness or practically gravity free space.

Weightlessness does not mean absence of weight. It means absence of the perception of weight, which happens due to the reaction experienced by the body. The weightlessness in a space satellite is not because of the fact that the gravitational force is small at that location in space. It is due to the reason that both the astronaut and the satellite are in a free fall situation towards the earth.

8.3.6 Summary

- In order to make a satellite revolve in a fixed circular orbit, it must be put into orbit with a critical speed. Critical speed of a satellite is

$$v_c = \sqrt{\frac{GM_E}{(R+h)}}$$

- A satellite revolving in a circular orbit has a fixed period of revolution depending upon the radius of its orbit. Time period of revolution of a satellite is

$$T = \frac{2\pi(R_E+h)^{\frac{3}{2}}}{(GM_E)^{\frac{1}{2}}}$$

- The KE and PE of an orbiting satellite changes from one position to another keeping its total energy constant. The KE, PE and TE of satellite is

$$KE = + \frac{GM_E m_s}{2r}, \quad PE = - \frac{GM_E m_s}{r}, \quad TE = - \frac{GM_E m_s}{2r}$$

- The height of a geostationary satellite is approximately 36,000 km and its time period is 24 hr.
- An object falling freely does not experience upward force and hence it experiences weightlessness.

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