

Gravitation

ANSWERS

Module 1 Kepler's Laws and Universal Law of Gravitation

A.8.1.1 (B)

The force of attraction experienced by m due to $2M$ at C will be $F_1 = G \frac{2Mm}{(BC)^2}$ (along BC).

The force experienced by m due to M at A will be $F_2 = G \frac{Mm}{(AB)^2} = G \frac{4Mm}{(BC)^2}$ (along AB).

Thus $F_2 = 2F_1$. Hence, m will move towards A, i.e. M .

A.8.1.2 (B) Gravitational force between two masses is independent of the medium separating masses.

A.8.1.3 (C)

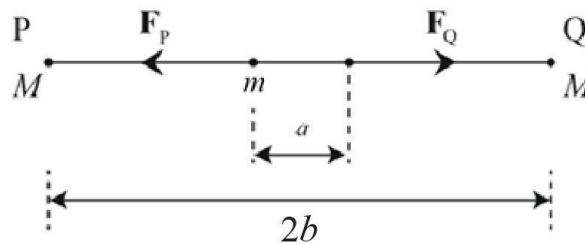


Fig. A.8.1.2

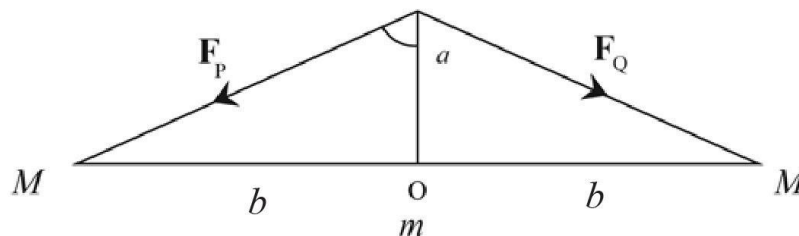


Fig. A.8.1.3

Refer to Fig. A.8.1.2 (for (i))

And Fig A.8.1.3 (for (ii))

The resultant force experienced by m , is given by

$$\mathbf{F}_1 = \mathbf{F}_Q - \mathbf{F}_P$$

$$\begin{aligned}\therefore \mathbf{F}_1 &= GMm \left\{ \frac{1}{(b-a)^2} - \frac{1}{(b+a)^2} \right\} \\ &= GMm \left\{ \frac{(b+a)^2 - (b-a)^2}{(b^2 - a^2)^2} \right\} \\ &= GMm \frac{4ab}{(b^2 - a^2)^2} = \frac{4GMma}{b^2} \times \frac{1}{\left(1 - \frac{a^2}{b^2}\right)^2} \\ \frac{a^2}{b^2} &\rightarrow 0, \quad \text{so, } \mathbf{F}_1 = \frac{4GMma}{b^2}\end{aligned}$$

In case (ii)

The resultant force experienced by m is given by

$$\begin{aligned}\mathbf{F}_2 &= 2\mathbf{F} \cos \theta, \text{ where } \mathbf{F}_P = \mathbf{F}_Q = \mathbf{F} \\ &= \frac{2GMm}{(a^2 + b^2)} \cdot \frac{a}{\sqrt{a^2 + b^2}} = \frac{2GMma}{(a^2 + b^2)^{\frac{3}{2}}} \\ \frac{2GMma}{b^3 \left(1 + \frac{a^2}{b^2}\right)^{\frac{3}{2}}} &= \frac{2GMma}{b^3} \left(\because \frac{a^2}{b^2} \rightarrow 0, \right)\end{aligned}$$

$$\therefore \mathbf{F}_1 = \mathbf{F}_2 = 4:2 = 2:1$$

A.8.1.4 (D)

The speed of the planet increases from minimum at aphelion (A) to maximum at perihelion (P).

A.8.1.5 (A)

According to Kepler's second law, areal velocity is constant in time.

A.8.1.6 (D)

This is according to Kepler's first law.

A.8.1.7 (A)

As per law of conservation of angular momentum

$$\therefore md_1 v_1 = md_2 v_2$$

$$v_2 = \frac{d_1 v_1}{d_2}.$$

A.8.1.8 (B)

We use the algebraic identity, $ab = \frac{1}{4}[(a + b)^2 + (a - b)^2]$

So for $(a + b) = a$ constant, ab will be maximum, when $a = b$. Therefore, in this case $x = (1 - x)$.

Or, $x = \frac{1}{2}$.

A.8.1.9 (A)

The planet moves faster when it is closer to the Sun than when it is farther from the Sun to keep areal velocity constant.

A.8.1.10 (A)

At aphelion, the satellite has the lowest speed. In circular orbit we need constant speed. To make orbit circular we may increase speed at aphelion.



A.8.2.1 (A)

The gravitational force acting on the upper part of the object will be greater than the gravitational force acting on the lower most part. Thus the centre of gravity will lie above the centre of mass.

A.8.2.2 (C)

Since $T^2 \propto r^3$

A.8.2.3 (C)

$$g_h = g\left(1 - \frac{2h}{R}\right) = g_d = g\left(1 - \frac{d}{R}\right)$$

$$d = 2h$$

A.8.2.4 (A)

$$g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2} = \frac{4}{3}G\pi\rho R$$

$$\therefore \frac{g_1}{g_2} = \frac{r_1}{r_2}$$

A.8.2.5 (C)

$$v = \sqrt{\frac{2GM}{R}}$$

$$= \sqrt{\frac{2G\frac{4}{3}\pi R^3\rho}{R}}$$

$$= R\sqrt{\frac{8}{3}\pi G\rho}$$

$$\frac{v_p}{v_E} = \frac{R_p}{R_E} = 2$$

A.8.2.6 (C)

If force suddenly disappears, the satellite will continue to move in a straight line with the velocity having the same magnitude along tangential direction with its orbit.

A.8.2.7 (D)

For a satellite in circular orbit of radius r , total energy is, $-\frac{GMm}{2r}$

The work done is the difference in the values of total energy at $r = R$ and $r = 2R$.

$$\text{The required difference} = -\frac{GMm}{4R} - \left(-\frac{GMm}{2R}\right) = GMm\left(\frac{1}{2R} - \frac{1}{4R}\right) = \frac{GMm}{4R}$$

A.8.2.8 (B)

$$M = \frac{4\pi}{3}R^3\rho \text{ in } v_e = \sqrt{\frac{2GM}{R}} \text{ gives the answer.}$$

A.8.2.9 (C)

Potential energy on the earth's surface is $-\frac{GMm}{R} = -mgR$. Thus kinetic energy needed to send it to infinity is mgR .

A.8.2.10 (B)

With symbol having usual meanings,

$$KE = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$PE = -\frac{GMm}{r},$$

$$\text{So, } E = -\frac{GMm}{2r}$$

$$\therefore PE = -2E$$

A.8.3.1 (C)

It goes around the earth in a west-east direction to synchronize with earth's motion because earth rotates from west to east.

A.8.3.2 (B)

4 times that of earth as $r^3\omega^2 = \text{constant}$.

A.8.3.3 (B)

Communication needs geostationary equatorial orbit.

A.8.3.4 (B)

Apparent weight = $m(g - a)$.

A.8.3.5 (A)

At the space station $g' = \frac{GM}{r^2} \neq 0$.

A.8.3.6 (D)

$$T^2 \propto r^3$$

$$\left(\frac{T_s}{T_E}\right)^2 = \left(\frac{r_s}{r_E}\right)^3 \Rightarrow n^2 = \frac{r_s^3}{r_E^3} \Rightarrow n = \frac{r_s^{\frac{3}{2}}}{r_E^{\frac{3}{2}}} = n^{\frac{2}{3}} r_E$$

A.8.3.7 (B)

Presence of earth alters the central character of the Sun's force. So, the path will not be elliptical, but it will be closed as the moon is bound by the attraction of the earth.

A.8.3.8 (D)

In a freely falling frame, apparent weight is zero.

A.8.3.9 (B)

As the astronaut gently drops the object out, the object continues to have the same speed as a satellite around the earth as before.

A.8.3.10 (D)

With symbols having their usual meanings, in an elliptic orbit $mvr = \text{constant}$ but $\frac{mv^2}{r} \neq \text{constant}$. Since the radius vector is not always normal to the ellipse, the direction of the weight keeps changing.



Development Team

Atul Mody, Former Associate Professor, VES College, Mumbai

V.S. Upadhaya, Former Associate Professor, G.N. Khalsa college, Mumbai

Review Team

C.K. Ghosh, Former Director, NCIDE, IGNOU, New Delhi

Naresh Kumar Sharma, Physics Teacher, Govt. Boys Sr. Secondary School, Bawana, Delhi

Nidhi Kalucha, PGT, Sanskriti School, Chanakyapuri, New Delhi

Vikrant Bhardwaj, PGT, Kendriya Vidyalaya, Gole Market, New Delhi

Member- Coordinator

Shashi Prabha, Professor, CIET, NCERT, New Delhi

Technical support

Riya Rani, JPF, CIET, NCERT, New Delhi

Bhawana Bhatt, Technical Content Developer, CIET, NCERT, New Delhi

Rajat Kumar, Graphic Designer, CIET, NCERT, New Delhi