



- Concept of rational numbers
- Rational numbers between any two consecutive integers
- Equivalent rational numbers
- Ordering of rational numbers

1.1 INTRODUCTION

To begin with, we came to learn about numbers by counting several distinct objects present in our vicinity here and there around. The numbers used for purpose of counting are called counting numbers or natural numbers namely 1, 2, 3, — — —. After discovery and inclusion of zero to collection of natural numbers we encounter to have another collection 0, 1, 2, 3, 4, 5 called a set of whole numbers. The collection so obtained was further extended to another different class of elements called the group of integers by adjoining all of the negative of natural numbers to the set of whole numbers. The set of integers is -3, -2, -1, 0, 1, 2, 3, In brief, what we have achieved so far is a superior number system starting from set of natural numbers to set of whole numbers and further up to the set of integers in the end. At the stage, all of us know about to what fractions are, and how mathematical operations are defined over there.

In this chapter, we will try our best to make ourselves understand well about the concept and mains involved mathematical operations they follow up.

1.2 CONCEPT OF RATIONAL NUMBERS

Indeed, we have felt the necessity and need of whole number system to apply to our day today life very well. Time and again, we also require negative fractional numbers to deal with too many counter mathematical situations arising so often.

Example

We can understand and express the 600m height above sea level as $\frac{3}{5}$ km where as the depth 600m below sea level as $-\frac{3}{5}$ km to our mind.

Interestingly we could convince ourselves that $-\frac{3}{5}$ is neither an integer nor a fractional number of course. Thus to our concern, it is now required that the number system know at

present be extended to include and understand such more numbers to our existing number system so far.

Activities

The teachers now will try to make their students understand about rational numbers through telling subjective tales related to stories from our day to day life. For example, allocation of legal shares by some one to his family members by virtue of a will led down during his life time. Distribution of revenue land among his family members by a farmer through will deed is a clear example of such type of mathematical concept where half of the property allocated to the elder son, one third of remaining to the second son and remaining property to the youngest son. Explain well and serve our purpose well.

Note:-

Consider $a \div b$ (a divided by b) where a, b are integers and $b \neq 0$ in the following table-1.

TABLE-1

$a \div b$	$a \div b = \text{value of } c$	Quotient c is integer or not?
$-15 \div 5$	-3	Integer
$-12 \div 2$	-6	Integer
$0 \div 5$	0	integer
$12 \div 5$	$\frac{12}{5}$	It is a fraction not an integer
$-12 \div 7$	$?$	neither fraction nor an integer.
$13 \div -3$	$?$	neither fraction nor an integer.

The table-1 makes it clear that $-\frac{12}{7}$ & $\frac{13}{-3}$ are neither an integer nor a fraction. Fractions are never negative.

Consider the table-2

TABLE-2

Number	Is integer or not
-3	Integer
$+6$	Integer
0	Integer

$\frac{12}{5}$	not an integer but a fraction
$\frac{11}{7}$	not an integer, but a fraction.
$\frac{1}{-3}$	neither an integer nor a fraction.
$-\frac{13}{6}$	neither integer nor a fraction.

The above mentioned facts in table-2, could be re-expressed in the form of solution of mathematical equation of following type.

(i) $4x = -12$

or $x = \frac{-12}{4} = -3$ (negative integer)

Thus x is an integer.

(ii) $-3x = -18$

or $3x = +18$

or $x = \frac{18}{3} = 6$ (a whole number)

Thus again x may be understood to be an integer (a positive integer)

(iii) $-5x = 0$

or $x = \frac{0}{-5} = 0$ (whole number)

Thus x is again an integer.

(iv) $5x = 12$

$x = \frac{12}{5}$ is a fraction but not an integer.

(v) $7x = 11$

$x = \frac{11}{7}$ is a fraction but not an integer.

(vi) $-3x = 1$

the solution $x = -\frac{1}{3}$, which is neither an integer nor a fraction.

(vii) Consider, the equation

$6x = -13$

Equation	Value of variable x
$4 \times x = -12$	$x = -3$
$(-3) \times x = -18$	$x = 6$
$-5 \times x = 0$	$x = 0$
$5 \times x = 12$	$x = \frac{12}{5}$
$7 \times x = 11$	$x = \frac{11}{7}$
$(-3) \times x = 1$	$x = \dots ?$
$6 \times x = -13$	$x = \dots ?$

or $x = \frac{-13}{6}$ which is neither an integer nor a fraction.

Again the existing solution to the equation is neither an integer nor a fraction.

Clearly, here we fine to admit that during above discussion all the time we do not have integer like solution to the given equation. Even otherwise, we also do not have solutions to these equations as rational numbers obviously all the time. But we know also that a linear equation in one variable has always a proper solution. Thus, the present number system is extended for this purpose.

Try the following

- $4 \times \boxed{} = -16$
- $(-3) \times \boxed{} = -24$
- $(-3) \times \boxed{} = 0$
- $5 \times \boxed{} = 12$
- $7 \times \boxed{} = 21$
- $(-3) \times \boxed{} = -1$
- $6 \times \boxed{} = -15$

What are rational numbers?

The word rational may be understood to be generated by the word ratio. We are known that actually $3 : 5$ and $\frac{3}{5}$ are same. In fact, here 3 and 5 both are natural numbers. Importantly the number $\frac{-3}{5}$ do not correspond to any ratio of two number because symbols like $-3 : 5$ does not represent any meaning to the mathematical aspects.

Consider the following sequence of numbers :

$$\frac{1}{1}, \frac{1}{2}, \frac{2}{2}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \dots$$

we refine our understanding about numbers

at this stage, if we reconsider the following.

We observe that the numbers $\frac{-1}{1}, \frac{1}{-1}, \frac{-1}{-1}$ are

the different numbers derived with the help of

fractional number $\frac{1}{1}$

also that, $\frac{-1}{2}, \frac{1}{-2}, \frac{-1}{-2}$ are different type of numbers obtained through the fractional

number $\frac{1}{2}$.

Similarly $\frac{-2}{3}, \frac{2}{-3}, \frac{-2}{-3}$ are different numbers obtained with the help of $\frac{2}{3}$

and in the same way the numbers $\frac{0}{-1}$ and $\frac{0}{1}$ are different other numbers corresponding to '0'.

Try to do the following—

- Write out other differently deduced with the help of $\frac{3}{4}$ but other than this.
- Find a sequence of numbers different from $\frac{4}{5}$ and $\frac{5}{7}$ with the help of these numbers.

Thus, we understand now to put forward the ratio of any two integers $\frac{p}{q}$ or $(p : q)$ definitely $q \neq 0$, to a different class of new numbers whole by different from $\frac{p}{q}$ but actually desired from $\frac{p}{q}$ itself.

Definition:

The number's that can be expressed in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$ is called a rational number for example $\frac{-2}{5}$ is rational number, here $p = -2$ and $q = 5$ which is non-zero and positive.

Fractions and Rational Numbers

Consider the fractions like $\frac{7}{11}, \frac{4}{9}, \frac{5}{13}, \dots$ and compare it with its general form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

Let $\frac{7}{11}$, then $p = 7$ and $q = 11$

$\frac{4}{9}$, then $p = 4$ and $q = 9$

$\frac{5}{13}$, then $p = 5$ and $q = 13$

Thus considering several types of fractions we find that fractions can be expressed in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$ always.

conclusion that :

All fractions are rational number.

Try the following :

Write down five different rational numbers such that :

- 1. Numerator is a negative integer and denominator is a positive integer.**
- 2. Numerator is a positive integer and denominator is a negative integer.**
- 3. Numerator as well as denominator both are negative integers.**

Are Integers rational numbers ?

Integers are rational numbers because for any integer 5 or -5. We have $5 = \frac{5}{1}$ and $-5 = \frac{-5}{1}$ is trivially true format for every rational number satisfying expressed standard form as $\frac{p}{q}$.

The number 0 can be written as

$$0 = \frac{0}{3} \text{ or } \frac{0}{5} \text{ or } \frac{0}{7} \dots\dots$$

Thus number '0' is again a rational number.

0 is a rational number.

The number zero ('0') is not a positive or negative rational number.

Identify which of the following rational numbers are fractions

$$-\frac{3}{7}, \frac{3}{7}, \frac{5}{-11}, \frac{5}{11}, \frac{-2}{-9}$$

comparing the above rational number with the standard form $\frac{p}{q}$ for fractions where p and q both are integers.

$$\text{for } \left. \begin{matrix} -\frac{3}{7} & p = -3 \\ & q = 7 \end{matrix} \right\} \dots\dots(1) \text{ and for } \left. \begin{matrix} \frac{3}{7} & p = 3 \\ & q = 7 \end{matrix} \right\} \dots\dots(2)$$

$$\left. \begin{matrix} \frac{5}{-11} & p = 5 \\ & q = -11 \end{matrix} \right\} \dots\dots(3) \text{ in } \frac{5}{11} \quad p = 5, q = 11 \quad \dots\dots(4)$$

$$\left. \begin{matrix} \frac{-2}{-9} & p = -2 \\ & q = -9 \end{matrix} \right\} \dots\dots(5)$$

clearly rational numbers $\frac{-3}{7}, \frac{5}{-11}$ and $\frac{-2}{-9}$ are in $\frac{p}{q}$ form but either p or q or both are not non-negative integers. Therefore, these numbers are not fractions.

Moreover, for $\frac{3}{7}, \frac{5}{11}$ in $\frac{p}{q}$ form and both p and q are positive integers at a time therefore $\frac{3}{7}$ and $\frac{5}{11}$ are the two fraction numbers among the given collection of rational numbers.

Now, it is important to note that in rational numbers $\frac{-3}{7}, \frac{5}{-11}$ and $\frac{-2}{-9}$ the stage of $\frac{-2}{-9}$ is of special. If be multiply (-1) in its numerator and denominator, then

$$\frac{-2}{-9} = \frac{(-2) \times (-1)}{(-9) \times (-1)} = \frac{2}{9}$$

Thus $\frac{-2}{-9}$ has reduced form as $\frac{2}{9}$ which is fraction because in $\frac{2}{9}$, 2 and 9 both are non-zero, non-negative integers. Thus, we find that, though $\left(\frac{-2}{-9}\right)$ is not a fraction but it can be equivalently reduced to a fractional number $\frac{2}{9}$, but $\frac{-3}{7}$ and $\frac{5}{-11}$ can never be equivalently reduced to fractional number.

Conclusion :

Thus any rational number having numerator as well as denominator as negative integers can be reduced to an equivalent fraction by re-multiplying its numerator and denominator to (-1) .

Try it

Is $\frac{-5}{-7}$ a rational number?

Is -8 a negative rational number?

Pay attention :

We have enough examples now of fractions like $\frac{7}{11}, \frac{4}{9}$ and $\frac{5}{13}$ and their resemblance to the standard form of fraction of general type $\frac{p}{q}$, with p and q as positive integers and $q \neq 0$.

Conclusion :

Rational numbers include integers as well as fractions in its collection.

Positive and Negative Rational Numbers

Let us consider a rational number $\frac{2}{3}$ where the numerator as well as denominator both are positive integers. Such rational numbers are called positive rational numbers. $\frac{3}{7}, \frac{5}{8}, \frac{2}{9}$ etc. are positive rational numbers too.

Again consider the number $\frac{-5}{7}$ where numerator is a negative integers. Such type of numbers are called negative rational numbers.

Thus arguments advanced on same grounds confirm that the numbers $\frac{-3}{5}, \frac{-3}{8}, \frac{-9}{5}$ are also negative integers.

We can do same thing for negative denominators $\frac{3}{-7}, \frac{4}{-5}, \frac{7}{-9}$ etc. to assert that these numbers are also some negative integers.

At this stage it is pretty inquisitive to pay our attention to the number observe about the numbers which are not of the form $\frac{p}{q}$, p and q integers $q \neq 0$. Such type of numbers are called irrational numbers.

All rational numbers are not fractions whereas, all fractions are rational numbers.

Note :

- The rational number $\frac{-2}{-9}$ is not a fraction whereas $\frac{-2}{-9}$ its equivalent $\frac{2}{9}$ is a proper fraction.
- If the numbers a and b are positive integers, then the two rational numbers $\frac{x}{y}$ and $\frac{-x}{-y}$ both are positive and $\frac{-x}{-y} = \frac{x}{y}$
- Both the rational numbers namely $\frac{-a}{b}$ and $\frac{a}{-b}$ are negative integers and $\frac{-x}{y} = \frac{x}{-y} = -\frac{x}{y}$.

Integers and Rational numbers

An integer can be expressed in different form of rational number. For example integers like

11, - 5, 0, 13

may be re-expressed as follows to observe that :

$$11 = \frac{11}{1} = \frac{-22}{-2} = \frac{33}{3} = \frac{44}{4} = \dots$$

Then clearly $\frac{11}{1}, \frac{-22}{-2}, \frac{33}{3}, \frac{44}{4}, \dots$ all satisfy $\frac{p}{q}$ form where p and q are integers and $q \neq 0$,

also that $-5 = \frac{-5}{1} = \frac{-10}{2} = \frac{15}{-3} = \frac{-20}{4} = \dots$

Then definitely

$\frac{-5}{1}, \frac{-10}{2}, \frac{15}{-3}, \frac{-20}{4}, \dots$ again of the type $\frac{p}{q}$, where p and $(q \neq 0)$ are integers

The number '0' (zero) can be represented by equivalent ratio's like

$\frac{0}{1} = \frac{0}{2} = \frac{0}{3} = \frac{0}{-3} = \frac{0}{4} = \dots$ all $\frac{p}{q}$ form where p and q are integers such that $q \neq 0$.

Attempt the following:

- Represent 13 in different forms of rational numbers.
- Represent integers like $-7, 6, -9, \dots$ as a rational number in fractional form such that their denominator is 1.
- Express $-\frac{2}{3}$ in different forms of equivalent fractions.

As we have discussed the representation of some of integers in different form as ratio of two integers p and q where $q \neq 0$. We could generalize the concept for a generate integer p .

i.e., $p = \frac{p}{1} = \frac{2p}{2} = \frac{3p}{3} = \frac{-p}{-1} = \frac{-2p}{-2} = \dots$

Thus we can assert that every integer is a rational number.

Two important properties of rational numbers

(1) The property of being equivalent.

Activity:

- The teacher is supposed to divide a class into five groups and start to provide each group a paper sheet of same size.
- The groups are instructed to fold and divide the given papers into different parts as follows:

Group 1—Two parts , Group 2—Four parts , Group 3—Six parts,

Group 4—Eight parts , Group 5—Ten parts

- Different groups are again asked to colour up differently as follows :

Group 1—One parts , Group 2 —Two parts , Group 3—Three parts ,

Group 4—Four parts , Group 5—Five parts

- Now the teacher is collect the coloured papers from different groups and convince students that coloured parts have the same value even though they may look different. The size of the collected papers of colour part is same.

We know that :

If we multiply the numerator and denominator of fraction $\frac{1}{2}$ by 2, 3, 4, 5 respectively the value $\frac{1}{2}$ is unchanged. Thus $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$ all are equivalent fractions corresponding to fraction $\frac{1}{2}$.

Similarly expressed the fraction $\frac{4}{5}$ and $\frac{2}{3}$ are as follows.

$$\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \frac{16}{20} = \frac{20}{25} = \frac{24}{30} \dots\dots \text{are equivalent fractions.}$$

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \dots\dots \text{are equivalent fractions.}$$

Similarly, equivalent fractions corresponding to a given rational number may be obtained by multiplying it's numerator and denominator both by a non-zero integer.

Thus

$$\frac{3}{-5} = \frac{6}{-10} = \frac{9}{-15} = \frac{12}{-20} = \frac{15}{-25} \dots\dots \text{are equivalent rational number of } -3/5.$$

Attempt the following :

$$\begin{array}{l} \frac{3}{5} = \frac{\boxed{}}{15} = \frac{12}{\boxed{}} = \frac{-15}{\boxed{}} \\ \frac{-5}{4} = \frac{\boxed{}}{16} = \frac{-25}{\boxed{}} = \frac{15}{\boxed{}} \end{array}$$

Consider a general fraction $\frac{p}{q}$ then $\frac{p}{q} = \frac{2p}{2q} = \frac{3p}{3q} = \dots = \frac{(\pm n)p}{(\pm n)q}$ where $(\pm n)$ is a non-zero integer. Therefore, since number of non-zero integers is infinite we have infinite equivalent rationals corresponding to a given rational number. Now, since we fail to produce a rational number if its denominator become zero by one way or other. Therefore, multiplied by zero to the numerator or denominator of a rational number is always prohibited (undefined). That's why we avoid fully multiply by zero the numerator and denominator when finding equivalent fraction of a given rational number.

Conclusion

If $\frac{x}{y}$ is a rational number and m be a non-zero integer, then $\frac{x}{y} = \frac{x \times m}{y \times m}$

Attempt the following :

Write down the five equivalent rational numbers of $\frac{-3}{7}$ and $\frac{5}{-6}$

Example 1. Express $\frac{-7}{5}$ in the form of another rational equivalent, having

- (a) numerator -14 (b) denominator 20
 (c) denominator -30 (d) numerator 35

Solution :

(a) $\frac{-7}{5} = \frac{-7 \times 2}{5 \times 2} = \frac{\boxed{-14}}{10} \rightarrow \text{numerator } -14$

(b) $\frac{-7}{5} = \frac{-7 \times 4}{5 \times 4} = \frac{-28}{\boxed{20}} \rightarrow \text{denominator } 20$

(c) $\frac{-7}{5} = \frac{(-7) \times (-6)}{5 \times (-6)} = \frac{42}{\boxed{-30}} \rightarrow \text{denominator } -30$

(d) $\frac{-7}{5} = \frac{(-7) \times (-5)}{5 \times (-5)} = \frac{\boxed{35}}{-25} \rightarrow \text{numerator } 35$

2. Fractions in simplest form.

We have already learnt to simplify fractions like $\frac{8}{12}, \frac{35}{40}, \frac{49}{63}$.

Taking HCF of both numerator and denominator and divide numerator and denominator by the highest common factor (H.C.F) of numerator and denominator.

A fraction is in simplest form if its numerator and denominator are prime (they have no common factors other than natural number 1). This can also be done though deviding numerator and denominator by their HCF.

Now consider

$$\frac{8}{12} = \frac{8 \div \text{HCF } (8,12)}{12 \div \text{HCF } (8,12)} = \frac{8 \div 4}{12 \div 3} = \frac{2}{3}$$

$$\frac{35}{40} = \frac{35 \div \text{HCF } (35,40)}{40 \div \text{HCF } (35,40)} = \frac{35 \div 5}{40 \div 5} = \frac{7}{8}$$

Similarly, other rational numbers can also be reduced to their simplified form equivalently by keeping denominator positive and deviding each of numerator and denominator simultaneously by the HCF of the absolute values of their numerator and denominator altogether.

For example :

$$\frac{8}{-14} = \frac{8 \div \text{HCF } (8, \text{absolute value of } -14)}{-14 \div \text{HCF } (8, \text{absolute value of } -14)}$$

$$= \frac{8 \div \text{HCF of } (8,14)}{-14 \div \text{HCF of } (8,14)} = \frac{8 \div 2}{-14 \div 2} = \frac{4}{-7} = \frac{-4}{-(-7)} = \frac{-4}{7}$$

and $-\frac{25}{40} = \frac{-25 \div 5}{40 \div 5} = \frac{-5}{8}$

Thus we have observe that

If $\frac{x}{y}$ be a rational number. Also suppose $m = \text{HCF of } (x, y)$.

Then, $\frac{x}{y} = \frac{x \div m}{y \div m}$ giving simplified form of $\frac{x}{y}$ as its true value $\frac{(x/m)}{(y/m)}$.

The rational number $\frac{x}{y}$ is taken to be of simplest form when $\text{HCF } (x, y) = 1$, where y is positive x and y relatively prime to each other.

The simplified form or standard form of a rational number.

Suppose Amit knew very well that $\frac{60}{72}$ is a rational number. The HCF (Highest Common Factor) of its numerator 60 and denominator 72 is 12.

Now,
$$\frac{60}{72} = \frac{60 \div HCF(60, 72)}{72 \div HCF(60, 72)} = \frac{60 \div 12}{72 \div 12} = \frac{5}{6}$$

Now, again GCD (Greatest Common Divisor) of 5 and 6 is not other than 1. Hence, Amit simplified $\frac{60}{72}$ to an equivalent simplified form $\frac{5}{6}$.

Attempt the following:

Find simplified form of $\frac{18}{45}$.

Again consider $\frac{80}{-112}$ having numerator denominator as the rational numbers. Here one of 80 and denominator -112 . Highest common factor of the absolute values of numerator and denominator is 8.

So,
$$\frac{80}{-112} = \frac{80 \div 8}{-112 \div 8} = \frac{10}{-14}$$

clearly $\frac{10}{-14}$ is one of the simplified form of the given rational number $\frac{80}{-112}$ but 10 and 14 again endowed with a common factor like the 2.

Therefore,
$$\frac{10}{-14} = \frac{10 \div 2}{-14 \div 2} = \frac{5}{-7}$$

In order to standarize the given form of a rational number denominator showed be positive. Therefore, multiply numerator and denominator again by (-1) . Simultaneously *i.e.*

$$\frac{5}{-7} = \frac{5 \times (-1)}{(-7) \times (-1)} = \frac{-5}{7}$$

also that 5 and 7 are relatively prime numbers ($GCD(5, 7) = 1$).

Thus the simplest form of rational number $\left(\frac{80}{-112}\right)$ is $\left(\frac{-5}{7}\right)$.

Attempt the following:

Express the following rational numbers in the simplest form.

(a) $\frac{18}{45}$

(b) $\frac{-18}{27}$

(c) $\frac{-45}{36}$

(d) $\frac{32}{-72}$

Conclusion

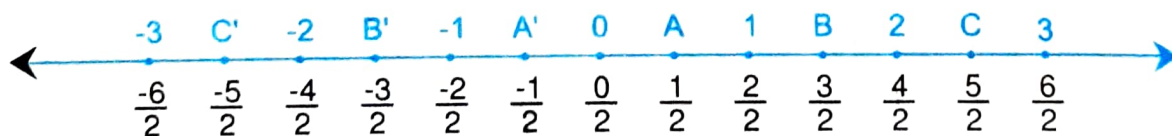
- A rational number is of the $\frac{p}{q}$ form where q is a positive integer and GCD of absolute values of p and q is non other than first natural number 1.
- The standard form of a rational number is the most simplest simplified form of a rational number.

Exercise-1(a)

- Attempt to write the following integers in the form of a rational number with denominator as 1— $-7, 11, 27, -45, 71$
- Express $\frac{-4}{5}$ equivalent in a rational number form having numerator as—
 (a) 8 (b) -16 (c) 20 (d) -24
- Express $\frac{-5}{-7}$ equivalently in a rational number form having denominator as :
 (a) 7 (b) -14 (c) 21 (d) -35
- Convert equivalently the following rational number so that their denominator is positive:
 (a) $\frac{-9}{-11}$ (b) $\frac{11}{-17}$ (c) $\frac{-4}{-19}$ (d) $\frac{7}{-13}$
- Convert the numerator to a positive integer of the following rational numbers:
 (a) $\frac{-7}{13}$ (b) $\frac{-11}{-19}$ (c) $\frac{-18}{23}$ (d) $\frac{-19}{-23}$
- Which one of these following rational number are positive?
 (a) $\frac{-9}{-13}$ (b) $\frac{11}{-19}$ (c) $\frac{-7}{-23}$ (d) $\frac{8}{-13}$
- Which of these following rational numbers are negative?
 (a) $\frac{-7}{11}$ (b) $\frac{-6}{-13}$ (c) $\frac{8}{-35}$ (d) $\frac{-21}{-23}$
- Simplify the following rational numbers in the simplest form :
 (a) $\frac{-9}{21}$ (b) $\frac{-18}{-27}$ (c) $\frac{21}{-36}$ (d) $\frac{-36}{64}$
- Fill in the blanks in your exercise book :
 (a) $\frac{-3}{4} = \frac{\square}{-20} = \frac{\square}{28}$ (b) $\frac{-5}{-8} = \frac{\square}{24} = \frac{25}{\square}$ (c) $\frac{7}{9} = \frac{14}{\square} = \frac{35}{\square}$ (d) $\frac{-8}{\square} = \frac{4}{15} = \frac{\square}{-60}$
- Find three equivalent rational numbers corresponding to each one of the following:
 (a) $\frac{2}{5}$ (b) $\frac{7}{-11}$ (c) $\frac{-8}{-5}$

1.3 Determination of rational numbers between any two integers

The integers ..., $-3, -2, -1, 0, 1, 2, 3$ are indicated on the following number line.



Divide the equally the distance between any two consecutive integers on the above number line. Mark the middle points between any two consecutive integer by $A, B, C, \dots$ towards right to point O indicating '0' (zero) on the rational numbers represented by $A, B, C, \dots$ correspond $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$ respectively.

Similarly mark the middle points in between any two consecutive integers by $A', B', C', \dots$ towards left of O respectively.

Clearly

$$\begin{aligned} OA &= OA' \\ OB &= OB' \\ OC &= OC' \\ &\dots \dots \dots \\ &\dots \dots \dots \end{aligned}$$

Thus we find that points $A, B, C, \dots$ and $A', B', C', \dots$ are equidistant from O but lie opposite on the number line with respect to O respectively.

Therefore, A', B' and $C', \dots$ represent rational numbers $\frac{-1}{2}, \frac{-3}{2}, \frac{-5}{2}, \dots$ reasonably well and respectively.

As we know that

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3}, \dots, 2 = \frac{2}{1} = \frac{4}{2} = \frac{6}{3}, \dots \text{ and } 3 = \frac{3}{1} = \frac{6}{2} = \frac{9}{3}$$

Thus rational numbers $\frac{2}{2}, \frac{4}{2}, \frac{6}{2}, \dots$ are equal to integers $1, 2, 3, \dots$. So we write negative rational numbers of $1, 2, 3, 4, \dots$ i.e. $-1, -2, -3$ respectively in the form of

$$-\frac{2}{2}, -\frac{4}{2}, -\frac{6}{2}, \dots$$

Thus we observe that on the number-line right to O . Rational numbers are represented as $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{6}{2}, \dots$.

Conclusion :

The rational numbers which are represented by the points left to 0 are

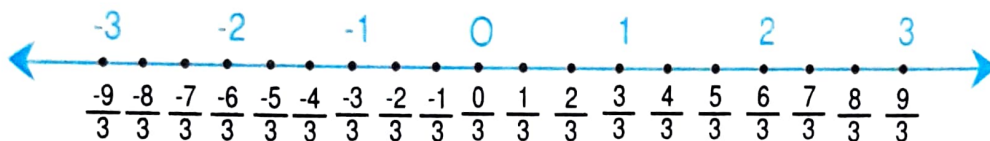
$$\frac{-1}{2}, \frac{-2}{2}, \frac{-3}{2}, \frac{-4}{2}, \frac{-5}{2}, \frac{-6}{2}, \dots \text{ respectively.}$$

$$0 \text{ can be written as } \frac{0}{2}.$$

Over all we find that rational numbers from left to right herein represented on the number line are:

$$\dots\dots\dots \frac{-6}{2}, \frac{-5}{2}, \frac{-4}{2}, \frac{-3}{2}, \frac{-2}{2}, \frac{-1}{2}, \frac{0}{2}, \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{6}{2} \dots\dots\dots$$

Again divide in three the portion available between any two consecutive integers on number line.



What are the rational numbers respectively on denoted points from O to right?

The rational numbers respectively are : $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3} \dots\dots\dots$

What are the rational numbers respectively on denoted points from O to left?

The rational numbers respectively are : $\frac{-1}{3}, \frac{-2}{3}, \frac{-3}{3}, \frac{-4}{3}, \frac{-5}{3}, \frac{-6}{3} \dots\dots\dots$

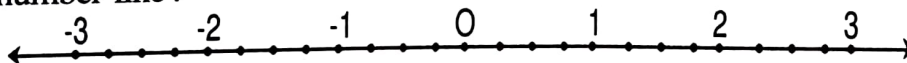
Attempt the following.

All denoted rational numbers of above number-line are-

$$\dots\dots\dots \frac{-6}{3}, \frac{-5}{3}, \frac{-4}{3}, \frac{-3}{3}, \frac{-2}{3}, \frac{-1}{3}, \frac{0}{3}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3} \dots\dots\dots$$

Do as follows :

Divide the distance between two consecutive integers in to four equal parts on the following number-line :



Find the rational numbers represented by consecutive integers between 0 and 1 on number-line.

Thus the rational numbers represented by the points denoted between them with integers

0 and 1 are respectively- $\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$

Thus the rational numbers represented by the points denoted between them with integers

-1 and 0 are respectively- $\frac{-4}{4}, \frac{-3}{4}, \frac{-2}{4}, \frac{-1}{4}, \frac{0}{4}$



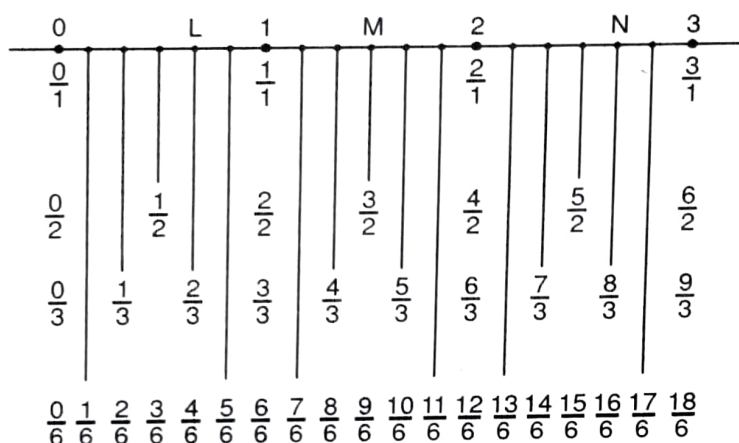
Do it:

Write down all the numbers represented by the points lie between -3 to +3 on appropriate number-line in your exercise book

We have found new points by dividing the distance between two integers of five, six, seven.....equal parts on its number-line by above method. The rational number on presented by these new points.

• **Presentation of infinite rational numbers by infinite points between any two whole numbers**

See the picture of the following number-line



The consecutive whole numbers 0,1,2..... are represented on the number line in the above figure. The distance between each two consecutive whole numbers is divided into six equal parts. It is clear that between two consecutive whole numbers such as 1 and 2, there are an infinite number of rational numbers. Infinitely many rational numbers lie between two rational numbers, which are even very near to each other.



Do it :

Give the answer to see the above figure:

- (1) Which equivalent rational numbers are represented by point L?
- (2) Equivalent rational numbers $\frac{8}{3}$ and $\frac{16}{6}$ are represented by which point on the number-line?
- (3) Equivalent rational numbers $\frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \frac{12}{4}, \frac{15}{5}, \frac{18}{6}$ are equivalent to which whole number?
- (4) Which equivalent rational numbers are represented by point O?

Conclusion

- One point of Number line denotes infinite equivalent rational numbers .
- There are infinite rational numbers between any two consecutive integers.

Representation of given rational number, which is not an integer on the number line between any two consecutive integers.

If $\frac{6}{5}$ be the rational number then :

$$\frac{6}{5} = \frac{5+1}{5} = \frac{5}{5} + \frac{1}{5} = 1 + \frac{1}{5}$$

Clearly $\frac{6}{5} > 1$.

$$\text{again } 2 = \frac{2}{1} = \frac{10}{5}$$

$$\text{therefore } \frac{6}{5} < \frac{10}{5}$$

$$\text{hence } 1 < \frac{6}{5} < 2$$

The point A corresponds to rational number $\frac{6}{5}$ on the number line.

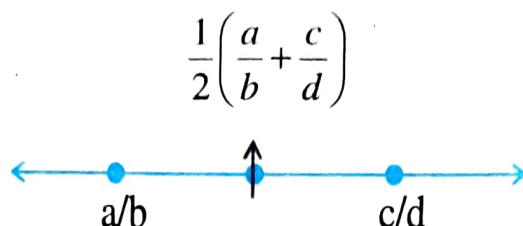


Attempt the following:

- Represent $\frac{8}{3}$ on the number line in between some consecutive integers.
- Represent $\frac{-7}{3}$ on the number line in between some consecutive integers.

1.3.1 Infinitely many rational numbers lie between any two rational

numbers $\frac{a}{b}$ and $\frac{c}{d}$, $b \neq 0$, $d \neq 0$.



The rational number exactly between two rational numbers is half of the sum of these two rational numbers and it is equidistant from rational numbers $\frac{a}{b}$ and $\frac{c}{d}$. By using this we can find infinite rational numbers between any two rational numbers.

Example : Find seven rational numbers in between the number 1 and 2.

Solution : The number in the middle of number 1 and 2. $= \frac{1+2}{2} = \frac{3}{2}$

Again the middle number in between the rational number 1 and $\frac{3}{2}$.

$$= \frac{1 + \frac{3}{2}}{2} = \frac{5}{4} = \frac{a + \frac{a+b}{2}}{2} = \frac{3a+b}{4}$$

The middle number between $\frac{3}{2}$ and 2.

$$= \frac{\frac{3}{2} + 2}{2} = \frac{7}{4}$$

Again the middle number between 1 and $\frac{5}{4}$

$$= \frac{1 + \frac{5}{4}}{2} = \frac{9}{8}$$

Middle number between $\frac{5}{4}$ and $\frac{3}{2}$

$$= \frac{\frac{5}{4} + \frac{3}{2}}{2} = \frac{11}{8}$$

Middle number between $\frac{3}{2}$ and $\frac{7}{4}$

$$= \frac{\frac{3}{2} + \frac{7}{4}}{2} = \frac{13}{8}$$

Middle number between $\frac{7}{4}$ and 2

$$= \frac{\frac{7}{4} + 2}{2} = \frac{15}{8}$$

Thus the required seven rational numbers between number 1 and 2 are :

$$\frac{9}{8}, \frac{5}{4}, \frac{11}{8}, \frac{3}{2}, \frac{13}{8}, \frac{7}{4}, \frac{15}{8}.$$

Note that all numbers are equi-distant.

Alternative method to find 7 equidistant numbers between the number 1 and 2.

That is $8 \div 8 = 1$ and again we can add 1 in numerator thus the seven equidistant numbers between 1 and 2 respectively.

$$1 = \frac{1 \times 8}{8} \text{ and } 2 = \frac{2 \times 8}{8} \text{ ie } \frac{8}{8} \text{ and } \frac{16}{8}, \text{ difference between numerator} = 16 - 8 = 8$$

$$\frac{8+1}{8} = \frac{9}{8}, \frac{8+2}{8} = \frac{10}{8}, \frac{8+3}{8} = \frac{11}{8}, \frac{8+4}{8} = \frac{12}{8}, \frac{8+5}{8} = \frac{13}{8}, \frac{8+6}{8} = \frac{14}{8}, \frac{8+7}{8} = \frac{15}{8}$$

Thus required numbers are :

$$\frac{9}{8}, \frac{10}{8}, \frac{11}{8}, \frac{12}{8}, \frac{13}{8}, \frac{14}{8} \text{ and } \frac{15}{8}.$$

$$\frac{9}{8}, \frac{5}{4}, \frac{11}{8}, \frac{3}{2}, \frac{13}{8}, \frac{7}{4} \text{ and } \frac{15}{8}.$$

To find required number of equidistant numbers between any two rational numbers.

Example : Find 8 equidistant rational numbers between $\frac{-1}{2}$ and $\frac{5}{4}$.

Solution : We put $\frac{-2}{4}$ and $\frac{5}{4}$ as equal denominator ration numbers for $\frac{-1}{2}$ and $\frac{5}{4}$..

Since we want to put 8 equidistant 8 rational numbers between $\frac{-2}{4}$ and $\frac{5}{4}$. So we multiply $8 + 1 = 9$ in the numerator and denominator of both numbers and put it as :

$$\frac{-2 \times 9}{4 \times 9}, \frac{5 \times 9}{4 \times 9}$$

that is, $\frac{-18}{36}, \frac{45}{36}$

To find desired number of equidistant numbers between any two rational numbers.

Example : Find 8 equidistant rational numbers between $\frac{-1}{2}$ and $\frac{5}{4}$.

Solution : We put $\frac{-2}{4}$ and $\frac{5}{4}$ as equal denominator rational numbers for $\frac{-1}{2}$ and $\frac{5}{4}$..

Since we want to put 8 equidistant 8 rational numbers between $\frac{-2}{4}$ and $\frac{5}{4}$. So we multiply $8 + 1 = 9$ in the numerator and denominator of both numbers and put it as :

$$\frac{-2 \times 9}{4 \times 9}, \frac{5 \times 9}{4 \times 9}$$

that is, $\frac{-18}{36}, \frac{45}{36}$

Now, the difference of their numerators $45 - (-18) = 63$, which is divided by 9 and we find quotient is $63 \div 9 = 7$.

Now we can find required 8 equidistant rational numbers by adding 7 in each successive numerator starting with first rational number as $\frac{-18}{36}$ and so on :

First number $= \frac{-18 + 7}{36} = \frac{-11}{36}$

Second number $= \frac{-11 + 7}{36} = \frac{-4}{36}$

Third number $= \frac{-4 + 7}{36} = \frac{3}{36}$

Fourth number $= \frac{3 + 7}{36} = \frac{10}{36}$

Fifth number $= \frac{10 + 7}{36} = \frac{17}{36}$

Sixth number $= \frac{17 + 7}{36} = \frac{24}{36}$

Seventh number $= \frac{24 + 7}{36} = \frac{31}{36}$

Eighth number $= \frac{31 + 7}{36} = \frac{38}{36}$

First method :

Suppose, Savita is required to simplify two rational numbers $-\frac{8}{12}$ and $\frac{30}{-45}$ to their simplest (standard) form and interpret finally.

Savita knows that the HCF of absolute values of numerator and denominator of $-\frac{8}{12}$ is

4. Therefore,

$$\frac{-8}{12} = \frac{-8 \div 4}{12 \div 4} = \frac{-2}{3}.$$

again HCF of absolute values of numerator and denominator. Corresponding to $\frac{30}{-45}$

is 15. Therefore,

$$\frac{30}{-45} = \frac{30 \div 15}{-45 \div 15} = \frac{2}{-3}$$

Therefore Savita finds that the simplest form $-\frac{8}{12}$ and $\frac{30}{-45}$ are same. Thus $-\frac{8}{12} = \frac{30}{-45}$.

Example 1. Verify the equality of $\frac{-16}{20}$ and $\frac{20}{-25}$

HCF of absolute values of numerator and denominator of $\frac{-16}{20}$ is 4.

$$\text{Therefore } \frac{-16}{20} = \frac{-16 \div 4}{20 \div 4} = \frac{-4}{5}$$

Again, HCF of absolute value of numerator and denominator of $\frac{20}{-25}$ is 5.

$$\text{Therefore, } \frac{20}{-25} = \frac{20 \div 5}{-25 \div 5} = \frac{4}{-5} = \frac{-4}{5}$$

Similarly, the simplest form of $\frac{-16}{20}$ and $\frac{20}{-25}$ is $\frac{-4}{5}$.

$$\text{So, } \frac{-16}{20} = \frac{20}{-25}$$

It is clear from above :

To verify the equality of two rational number, we obtain the simplest form of these rational numbers. If their simplest forms (standard forms) are same then the rational numbers otherwise they are different.

Second Method:

(By method of changing equivalently to the same denominator)

Consider the two given rational numbers $\frac{-8}{-14}$ and $\frac{12}{21}$ change both $\frac{-8}{-14}$ and $\frac{12}{21}$ to equal denominator maintaining then into equivalent form.

Now $\frac{-8}{-14}$ and $\frac{12}{21}$ are written as $\frac{8}{14}$ and $\frac{12}{21}$

L.C.M. of 14 and 21 is 42.

Therefore,

$$\frac{-8}{-14} = \frac{8}{14} = \frac{8 \times 3}{14 \times 3} = \frac{24}{42} \quad \dots(i)$$

$$\text{and} \quad \frac{12}{21} = \frac{12 \times 2}{21 \times 2} = \frac{24}{42} \quad \dots(ii)$$

therefore, from (i) and (ii)

$$\frac{-8}{-14} = \frac{12}{21}$$

Similarly we may take the pair of numbers $\frac{-12}{15}$ and $\frac{36}{-45}$.

Consider the L.C.M. of absolute values of 15 and -45.

i.e. L.C.M. (15, 45) = 45

$$\text{Then,} \quad \frac{-12}{15} = \frac{(-12) \times 3}{15 \times 3} = \frac{-36}{45} \quad \dots(i)$$

$$\text{also that,} \quad \frac{36}{-45} = \frac{36 \times (-1)}{45 \times (-1)} = \frac{-36}{45} \quad \dots(ii)$$

from (i) and (ii) we ensure that : $\frac{-12}{15} = \frac{36}{-45}$

Attempt the following :

(i) Verify the equality of $\frac{8}{-18}$ and $\frac{-32}{72}$.

(ii) Find out $\frac{-16}{20}$ and $\frac{24}{-30}$ are equal ?

The condition for two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ to be equal.

Consider a pair of equal rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, such that

(i) $\frac{a}{b} = \frac{3}{-4}$ and $\frac{c}{d} = \frac{-9}{12}$

(ii) $\frac{a}{b} = \frac{-5}{9}$ and $\frac{c}{d} = \frac{10}{-18}$

(iii) $\frac{a}{b} = \frac{-7}{-11}$ and $\frac{c}{d} = \frac{14}{22}$

Observe the pair of rational numbers in the next table. Now obtain the values ad and bc corresponding to both rational numbers and ensure mutual comparison.

Equal rational numbers $\frac{ac}{bd}$	$ad = a \times d$	$bc = b \times c$	relation between ad and bc
$\frac{39}{-412}$	$3 \times 12 = 36$	$(-4) \times (-9) = 36$	equal
$\frac{-510}{918}$	$(-5) \times (-18) = 90$	$9 \times 10 = 90$	equal
$\frac{-714}{-1122}$	$(-7) \times (22) = -154$	$(-11) \times 14 = -154$	equal

Try it to complete the table :

$\frac{a}{b}, \frac{c}{d}$	$ad = a \times d$	$bc = b \times c$	relation among ad and bc
$\frac{1}{2}, \frac{-4}{-8}$			
$\frac{-5}{9}, \frac{15}{-27}$			
$\frac{3}{-4}, \frac{-12}{16}$			

Thus the evaluation table assess that—

The two rational number $\frac{a}{b}$ and $\frac{c}{d}$ are same then $ad = bc$.

Now, we can apply this specific property to verify about the equality of two given rational numbers.

Example 2. Verify about the equality of $\frac{-15}{25}$ and $\frac{-12}{20}$

If in the given rational numbers, $\frac{-15}{25} = \frac{a}{b}$ and $\frac{-12}{20} = \frac{c}{d}$, then $a = -15$, $b = 25$,

$c = -12$ and $d = 20$

Thus $ad = a \times d = (-15) \times (20) = -300$

$bc = b \times c = (25) \times (-12) = -300$

Thus $ad = bc$

$\therefore \frac{a}{b} = \frac{c}{d}$ or $\frac{-15}{25} = \frac{-12}{20}$.

Attempt the following :

Find which of the following pairs are unequal.

(a) $\frac{-18}{24}$ and $\frac{7}{-21}$

(b) $\frac{0}{-7}$ and $\frac{0}{4}$

(c) $\frac{-6}{10}$ and $\frac{9}{-15}$

(d) $\frac{-15}{20}$ and $\frac{25}{-30}$

Exercise-1(b)

1. Find the simplified form of following rational numbers :

(a) $\frac{-8}{10}$

(b) $\frac{15}{20}$

(c) $\frac{25}{45}$

(d) $\frac{-14}{77}$

2. Fill in the blanks by using proper notations of = or $\neq$.

(a) $\frac{-4}{5} \square \frac{-5}{7}$

(b) $\frac{7}{-11} \square \frac{-7}{11}$

(c) $\frac{-8}{5} \square \frac{-7}{-4}$

(d) $\frac{14}{-16} \square \frac{-21}{16}$

3. Which of the following pairs of rational number are equal?

(a) $\frac{-9}{12}$ and $\frac{8}{-12}$

(b) $\frac{-15}{45}$ and $\frac{16}{-48}$

(c) $\frac{-7}{21}$ and $\frac{3}{-9}$

(d) $\frac{-8}{-14}$ and $\frac{13}{21}$

4. Represent the following rational number on the number line.

(a) $\frac{3}{4}$

(b) $\frac{3}{5}$

(c) $\frac{5}{8}$

(d) $\frac{3}{16}$

5. Which one of the following pairs of rational number is unequal?

(a) $\frac{-8}{24}$ and $\frac{7}{-21}$

(b) $\frac{-15}{20}$ and $\frac{25}{-30}$

(c) $\frac{0}{-7}$ and $\frac{0}{4}$

(d) $\frac{-6}{10}$ and $\frac{9}{-15}$

1.5 Ordering of rational numbers:

Rational numbers can also be compared like integers and fractions.

Example 1. Which one rational numbers $\frac{-5}{4}$ or $\frac{7}{-9}$ is greater one ?

Solution : The standard form of $\frac{7}{-9}$ is $\frac{-7}{9}$.

Now write both of the rational numbers equivalently so that they have same denominator.

Here, We apply a process to equal the denominator of both the rational numbers.

Now

$$\frac{-5}{4} = \frac{(-5) \times 9}{4 \times 9} = \frac{-45}{36} \quad \dots(i)$$

$$\text{and} \quad \frac{-7}{9} = \frac{(-7) \times 4}{9 \times 4} = \frac{-28}{36} \quad \dots(ii)$$

Clearly $45 > 28$

that is $-45 < -28$

$$\text{so,} \quad \frac{-45}{36} < \frac{-28}{36}$$

$$\text{Similarly,} \quad \frac{-5}{4} < \frac{-7}{9} \quad [\text{from (i) and (ii)}]$$

$$\therefore \quad \frac{-7}{9} > \frac{-5}{4}$$

Example 2. Which one of the rational numbers $\frac{6}{-5}$ or $\frac{-7}{-6}$ is greater one?

Solution : When we make the denominator of rational numbers positive.

$$\frac{6}{-5} = \frac{(6) \times (-1)}{(-5) \times (-1)} = \frac{-6}{5}$$

$$\text{and} \quad \frac{-7}{-6} = \frac{(-7) \times (-1)}{(-6) \times (-1)} = \frac{7}{6}$$

$\therefore \frac{7}{6}$ is a positive number while $\frac{-6}{5}$ is a negative one therefore $\frac{7}{6} > \frac{-6}{5}$

and hence, $\frac{-7}{-6} > \frac{6}{-5}$

Take the several pairs of rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ so that $\frac{a}{b} > \frac{c}{d}$ holds.

Attempt the following:

- Find which rational number $\frac{3}{2}$ or $\frac{3}{-2}$ is greater.
- Which rational number $\frac{1}{2}$ or $\frac{-3}{4}$ stands greater.
- Which rational number $\frac{0}{1}$ or $\frac{-1}{2}$ stands greater.

Observe to have three pairs of above rational numbers in the following table that of form $\frac{a}{b} > \frac{c}{d}$. Find the corresponding value of ad and bc to compare the two $\frac{a}{b}$ and $\frac{c}{d}$. Finally, fill in the boxes in the table given below —

$\frac{a}{b} > \frac{c}{d}$	$a \times d = ad$	$b \times c = bc$	compare ad and bc
$\frac{1}{2} > \frac{-3}{4}$	$1 \times 4 = 4$	$2 \times (-3) = -6$	$\therefore 4 > -6$ $\therefore ad \square bc$
$\frac{0}{1} > \frac{-1}{2}$	$0 \times 2 = \dots$	$1 \times (-1) = -1$	$\therefore 0 > -1$
$\frac{3}{2} > \frac{-3}{2}$	$3 \times 2 = \dots$	$2 \times (-3) = \dots$	$\therefore \dots > \dots$

From the above table it is concluded that

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational number such that $\frac{a}{b} > \frac{c}{d}$. Then $ad > bc$ with the condition that b and d are positive numbers.

Now we can use the above property to compare any two given, rational numbers. Further, we can take other two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ satisfying $\frac{a}{b} < \frac{c}{d}$. Convert both the denominator to positive integer if it is not so. Evaluate ad and bc to establish that $ad < bc$. This property of rational number may be used for simple comparison of any two rational numbers.

Note : Since $\frac{a}{b}$ and $\frac{c}{d}$ are any two arbitrary rational numbers so the condition $\frac{a}{b} > \frac{c}{d}$ itself includes. The other counter condition like $\frac{a}{b} < \frac{c}{d}$ by mutual interchange of role of each other. The required results follow on their own.

Example 3. Which is greater rational number $\frac{4}{-7}$ or $\frac{-3}{5}$?

Solution : $\frac{4}{-7} = \frac{-4}{7} = \frac{a}{b}$ (say)

$\frac{-3}{5} = \frac{c}{d}$ (say)

Then $ad = (-4) \times 5 = -20$

$bc = 7 \times (-3) = -21$

$\therefore -20 > -21$

$\Rightarrow ad > bc$

$\Rightarrow \frac{a}{b} > \frac{c}{d}$

$\Rightarrow \frac{4}{-7} > \frac{-3}{5}$

Conclusion :

It is always necessary to convert a rational number to an equivalent form having positive denominator if it is not so during the process of comparison of the two.

Example 4. List the rational numbers $\frac{-7}{9}$, $\frac{-5}{21}$ and $\frac{2}{-3}$ in descending order.

Solution : The equivalent rationals having positive denominators are :

$$\frac{-7}{9}, \frac{-5}{21} \text{ and } \frac{-2}{3} \quad \dots(i)$$

L.C.M. of 9, 21 and 3 = 63

Now convert rationals in (1) to another equivalent form of rationals having denominator as 63.

$$\text{Thus, } \frac{-7}{9} = \frac{(-7) \times 7}{9 \times 7} = \frac{-49}{63} \quad \dots(i)$$

$$\frac{-5}{21} = \frac{(-5) \times 3}{21 \times 3} = \frac{-15}{63} \quad \dots(ii)$$

$$\text{and } \frac{-2}{3} = \frac{(-2) \times 21}{3 \times 21} = \frac{-42}{63} \quad \dots(iii)$$

Now on the number line

$$-49 < -42 < -15$$

$$\Rightarrow \frac{-49}{63} < \frac{-42}{63} < \frac{-15}{63}$$

$$\Rightarrow \frac{-7}{9} < \frac{-2}{3} < \frac{-5}{21} \quad [\text{using (i), (ii) and (iii)}]$$

Ordering property of rational numbers:

(i) Compare any positive rational number with 0 (zero). For example which one is greater 0 or $\frac{5}{7}$?

$$\text{Obviously, } 0 = \frac{0}{7}$$

$$\text{and } 5 > 0$$

$$\Rightarrow \frac{5}{7} > \frac{0}{7} = 0$$

Thus, we arrive at the conclusion that

Every positive rational number is greater than zero.

(ii) Comparison of negative rational number and zero.

Let us consider any negative rational number to compare it to the number zero.

In particular now let us find which one of $\frac{-4}{9}$ and 0 is greater among each other?

Definitely $0 = \frac{0}{9}$ and $\frac{-4}{9}$ is in standard form.

Now, $\therefore 0 > -4$

$$= \frac{0}{9} > \frac{-4}{9}$$

$$= 0 > \frac{-4}{9} = -\frac{4}{9} < 0$$

Thus we arrive at a result that,

Every negative rational number is always less than zero.

Use summerise as follows :

Every non-zero rational number is either positive or negative and that :

- (i) For each rational number x only one statement is true
- (i) $x > 0$ (ii) $x = 0$ (iii) $x < 0$
- (ii) If x and y are two rational numbers the exactly one of following statement is true:
- (i) $x > y$ (ii) $x = y$ (iii) $x < y$

There are the following working terms to find a greater rational number between two rational numbers :

- (i) First of all reduce the given rational number to an equivalent rational number having its denominator positive, in case, its not like so.
- (ii) One can easily ensure that out of two given rational number the positive one is always greater than that of the other negative rational number.
- (iii) If the both rational number have same sign, then their denominator are made equal by finding L.C.M. of absolute values of the denominator of the two rational numbers and multiplying above and below by the L.C.M. , we obtain the rational number with same denominator.
- (iv) We compare the numerators of the given pair of rational numbers having same denominator the greater one correspond to greater numerator. Valued rational number with regard to identical denominator.

(v) If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers with positive denominator then:

(i) $\frac{a}{b} = \frac{c}{d}$ if $ab = bc$

(ii) $\frac{a}{b} > \frac{c}{d}$ if $ad > bc$

(iii) $\frac{a}{b} < \frac{c}{d}$ if $ad < bc$

Exercise-1(c)

1. Which is greater in each of the following :

(a) $\frac{-4}{9}, \frac{7}{9}$ (b) $\frac{-3}{4}, \frac{-5}{8}$ (c) $\frac{-7}{12}, \frac{5}{-8}$ (d) $\frac{-5}{9}, \frac{-3}{-13}$

2. Which is smaller in each of the following :

(a) $\frac{-4}{9}, \frac{5}{-9}$ (b) $\frac{6}{11}, \frac{-7}{-11}$ (c) $\frac{16}{-7}, 3$ (d) $\frac{4}{-3}, \frac{-8}{9}$

3 (a) The equivalent rational number corresponding to $\frac{-4}{18}$ is:

(a) $\frac{2}{-9}$ (b) $\frac{6}{27}$ (c) $\frac{2}{9}$ (d) $\frac{4}{18}$

(b) The rational number greater than rational number $\frac{3}{5}$ is:

(a) $\frac{4}{10}$ (b) $\frac{-4}{10}$ (c) $\frac{6}{10}$ (d) $\frac{7}{10}$

(c) The rational number smaller than the rational number $\frac{4}{9}$ is:

(a) $\frac{-11}{18}$ (b) $\frac{13}{18}$ (c) $\frac{12}{27}$ (d) $\frac{11}{18}$

(d) The rational number greater than $\frac{0}{-5}$ is :

(a) $\frac{1}{-5}$ (b) $\frac{-2}{-7}$ (c) $\frac{-1}{8}$ (d) $\frac{4}{-9}$

4. By copying the following on your exercise book, put down proper symbols like $>$, $=$ or $<$ in blank boxes :

(a) $\frac{-4}{7} \square \frac{6}{13}$

(b) $\frac{-4}{5} \square \frac{-5}{7}$

(c) $\frac{-7}{8} \square \frac{21}{-24}$

(d) $\frac{-9}{-10} \square \frac{8}{9}$

5. Write in ascending order to the following rational numbers :

$$\frac{3}{5}, \frac{-7}{10}, \frac{8}{-15}, \frac{-17}{-30}$$

6. Write in descending order the following rational numbers :

$$\frac{4}{7}, \frac{-5}{6}, \frac{-3}{-12}, \frac{11}{-24}$$

Ability Exercise-1

1. There are four alternatives to the given problem with one correct answer. Write the correct one in your answer book.

(a) There is number of equivalent fractions to a given rational number is :

(i) One

(ii) Two

(iii) Fifty

(iv) infinite

(b) The simplified form of $\frac{-16}{80}$ is :

(i) $\frac{1}{5}$

(ii) $\frac{-1}{-5}$

(iii) $\frac{-1}{5}$

(iv) $\frac{-2}{5}$

(c) The simplified form of the rational number $\frac{40}{-25}$ is :

(i) $\frac{8}{25}$

(ii) $\frac{-8}{5}$

(iii) $\frac{16}{-5}$

(iv) $\frac{-8}{25}$

(d) The number of rational numbers between any two rational numbers are :

(i) 10

(ii) 20

(iii) infinite

(iv) 111

2. Which one of the following statements are true or false, indicate accordingly.

(a) The standard form of two equal rational numbers is same.

(b) $\frac{-7}{-11}$ is a positive rational number.

(c) $\frac{4}{-9}$ is a positive rational number.

(d) Both $\frac{3}{-11}$ and $\frac{-3}{11}$ are same.

(e) $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \dots$ are different form of a rational number.

(f) $\frac{-1}{-3}, \frac{-2}{-6}, \frac{-3}{-9}$ are different rational numbers.

(g) $\frac{-4}{-5}$ and $\frac{4}{5}$ both are same rational numbers.

(h) All of integers are rational numbers.

(i) All of rational numbers are integers.

(j) $\frac{0}{5}$ and $\frac{0}{-3}$ are not the same rational numbers.

(k) There are infinite numbers of rational numbers between any two distinct rational numbers :

3. Arrange the following rational numbers in their descending order :

(a) $\frac{2}{5}, \frac{4}{7}, \frac{5}{9}, \frac{1}{6}$

(b) $\frac{-4}{11}, \frac{1}{-3}, \frac{-5}{7}, \frac{-3}{4}$

(c) $\frac{5}{-9}, \frac{-7}{12}, \frac{7}{-18}, \frac{-2}{3}$

(d) $\frac{-3}{4}, \frac{5}{-12}, \frac{-7}{16}, \frac{9}{-24}$

We have learnt :

1. A number that can be expressed in the form $\frac{p}{q}$, where p and q are integers such that $q \neq 0$, is called a rational number.
2. All fractions are rational numbers, but all rational numbers are not fractions. All integers(p) as well as fractioned are jointly rational numbers. $p = \frac{p}{1} = \frac{p}{2} = \dots = \frac{-p}{-1} = \frac{-2p}{-2} \dots$
3. The number '0' is neither a positive nor a negative number.

4. There are unlimited number of rational numbers between any two rational numbers.
5. When numerator and the denominator both of a rational number are of same sign, it is a positive rational number. Whereas if either of the numerator or the denominator being negative a natural number, it corresponds to a negative rational number.
6. A rational number is said to be in standard form if its denominator is positive integer and the numerator and denominator have no common factor other than 1.
7. The simplest form of a rational number $\frac{p}{q}$ is of the form $\frac{p/m}{q/m}$ where $m = \text{HCF of absolute values of denominator and numerator}$.
8. If $\frac{a}{b}$ and $\frac{c}{d}$ be any two rational numbers then either $\frac{a}{b} > \frac{c}{d}$ or $\frac{a}{b} < \frac{c}{d}$ or $\frac{a}{b} = \frac{c}{d}$ is true at a time.
9. Every positive number is greater than zero and zero is always greater than any given negative number.

ANSWERS

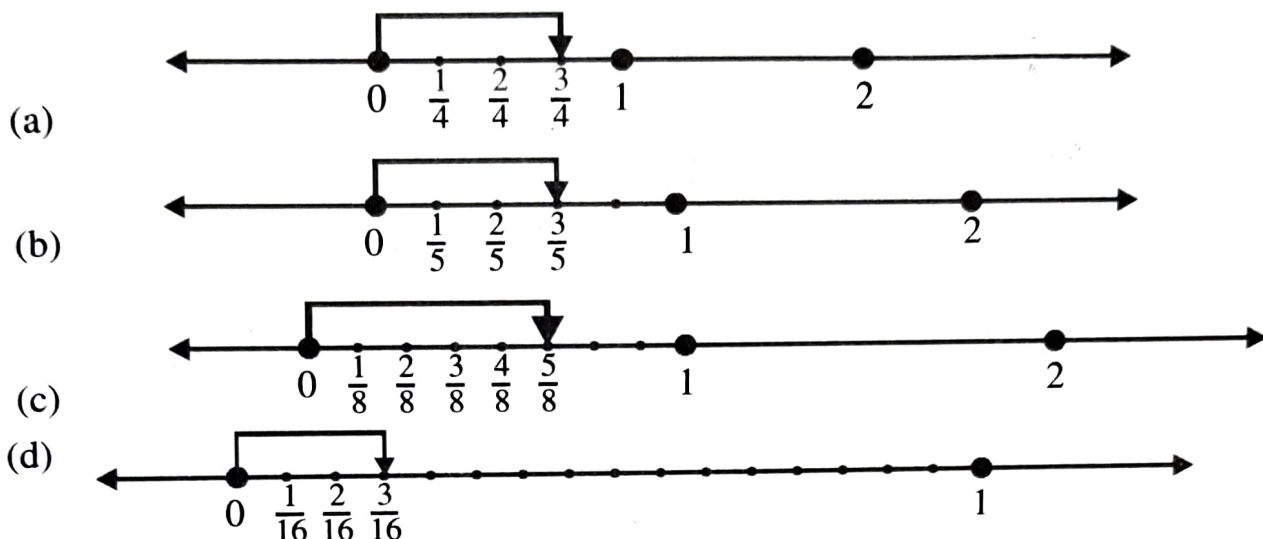
Exercise-1(a)

1. $\frac{-7}{1}, \frac{11}{1}, \frac{27}{1}, \frac{-45}{1}, \frac{71}{1}$ 2. (a) $\frac{8}{-10}$ (b) $\frac{-16}{20}$ (c) $\frac{20}{-25}$ (d) $\frac{-24}{30}$ 3. (a) $\frac{5}{7}$,
 (b) $\frac{-10}{-14}$ (c) $\frac{15}{21}$ (d) $\frac{-25}{-35}$ 4. (a) $\frac{9}{11}$ (b) $\frac{-11}{17}$ (c) $\frac{4}{19}$ (d) $\frac{-7}{13}$, 5. (a) $\frac{7}{-13}$ (b) $\frac{11}{19}$
 (c) $\frac{18}{-23}$ (d) $\frac{19}{23}$ 6. (a) positive, $\frac{9}{13}$ (c) positive $\frac{7}{23}$, 7. (a) negative, $\frac{-7}{11}$, (c) negative $\frac{-8}{35}$
 8. (a) $\frac{-3}{7}$ (b) $\frac{2}{3}$ (c) $\frac{-7}{12}$ (d) $\frac{-9}{16}$ 9. (a) 15, -21 (b) 15, 40 (c) 18, -45, (d) -30,
 -16 10. (a) $\frac{2}{5} = \frac{-20}{-50} = \frac{-6}{-15} = \frac{8}{20}$ so on (b) $\frac{7}{-11} = \frac{-7}{11} = \frac{-14}{22} = \frac{-21}{23}$ so on
 (c) $\frac{-8}{-5} = \frac{8}{5} = \frac{-16}{-10}, \frac{24}{15}$ so on

Note : We have considered here only few namely 3 equivalent rational numbers out of existing infinite number of rationals of equivalent nature.

Exercise-1(b)

1. (a) $-\frac{4}{5}$ (b) $\frac{3}{4}$ (c) $\frac{5}{9}$ (d) $-\frac{2}{11}$ 2. (a) $\neq$, (b) $=$, (c) $\neq$, (d) $\neq$, 3. (b) equal, (c) equal



5. (b) unequal

Exercise-1(c)

1. (a) $\frac{7}{9}$ (b) $-\frac{5}{8}$ (c) $-\frac{7}{12}$ (d) $-\frac{3}{-13}$ 2. (a) $\frac{5}{-9}$ (b) $\frac{6}{11}$ (c) $\frac{16}{-7}$ (d) $\frac{4}{-3}$ 3. (a) (i) $\frac{2}{-9}$
 (b) (iv) $\frac{7}{10}$ (c) (i) $-\frac{11}{18}$ (d) (ii) $-\frac{2}{-7}$; 4. (a) $<$, (b) $<$, (c) $=$, (d) $>$; 5. $-\frac{7}{10}, \frac{8}{-15}, \frac{-17}{-30}, \frac{3}{5}$
 6. $\frac{4}{7}, \frac{-3}{-12}, \frac{11}{-24}, \frac{-5}{6}$

Ability Exercise-1

1. (a) (iv) infinite (b) (iii) $-\frac{1}{5}$ (c) (ii) $-\frac{8}{5}$ (d) (iii) infinite, 2. (a) True (b) True
 (c) False (d) True (e) True (f) False (g) True (h) True (i) False (j) False (k) True
 3. (a) $\frac{4}{7}, \frac{5}{9}, \frac{2}{5}, \frac{1}{6}$ (b) $\frac{1}{-3}, \frac{-4}{11}, \frac{-5}{7}, \frac{-3}{4}$ (c) $\frac{7}{-18}, \frac{5}{-9}, \frac{-7}{12}, -\frac{2}{3}$ (d) $\frac{9}{-24}, \frac{5}{-12}, \frac{-7}{16}, \frac{-3}{4}$

