



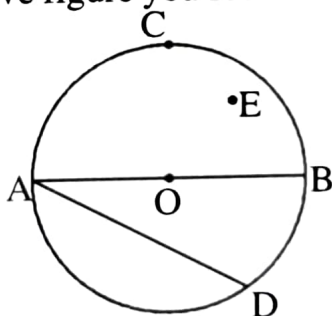
- Experimental verification of the following properties of circle—
- Angle of the semi-circle is right angle.
- The angle made at the centre of the arc is twice the angle formed on the rest part of the circle by same arc.
- The angles made in same segment of circle are equal.

Introduction

In previous class we have studied the concept of circle. Together with this we have got the knowledge about some defined words as centre, Radius, Diameter, Chord, Arc, Semicircle, Sector of a circle, Segment of a Circle, Which we can recall from the following example.

Do this

Take any point O, as a centre and draw a circle of radius 3.0 cm. Increase the radius OA up to any point B of the Circle. Draw the chord AD from point A of the circle. Now think at the following points. In the above figure you seen that-



In this figure AB is diameter. you know that diameter is double of radius. In the figure, radius OA, of circle is given 3.0 cm. $AB = 2 \times OA$. Therefore the measurement of diameter AB will be 6 cm you know also that longest chord of circle is diameter. Diameter divides circle in two equal parts. Every part is known as semi circle, curve BCA and ADB are semi circle. Any part of circle is chord. In figure curve BC is arc.

Now in this unit we will study the relation between angle subtended by chord or arc at the centre and the angle subtended at any point of the circle by same chord or arc.

11.1 Angle of Semi-Circle

According to fig 11.1 draw a circle in your answer book whose centre is O. Draw a diameter AOB of this circle. Take a point C at the circle. Join C from the point A and B. In semi circle ACB, Angle made by diameter AB is $\angle ACB$. $\angle ACB$ is called Angle of semi circle.

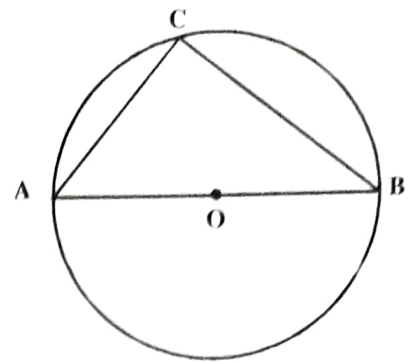


Fig 11.1

Do these and find out the result-

Construct three circles, like figure 11.2 on your exercise book, whose centre is O. Draw diameter AOB of this circle. Take a point C on the semi circle. Draw line segment AC and BC. Thus we get Angle $\angle ACB$ of semi circle. Measure $\angle ACB$ and find out difference $90^\circ - \angle ACB$. Repeat this activity with other semicircle and write down the results in the following table.

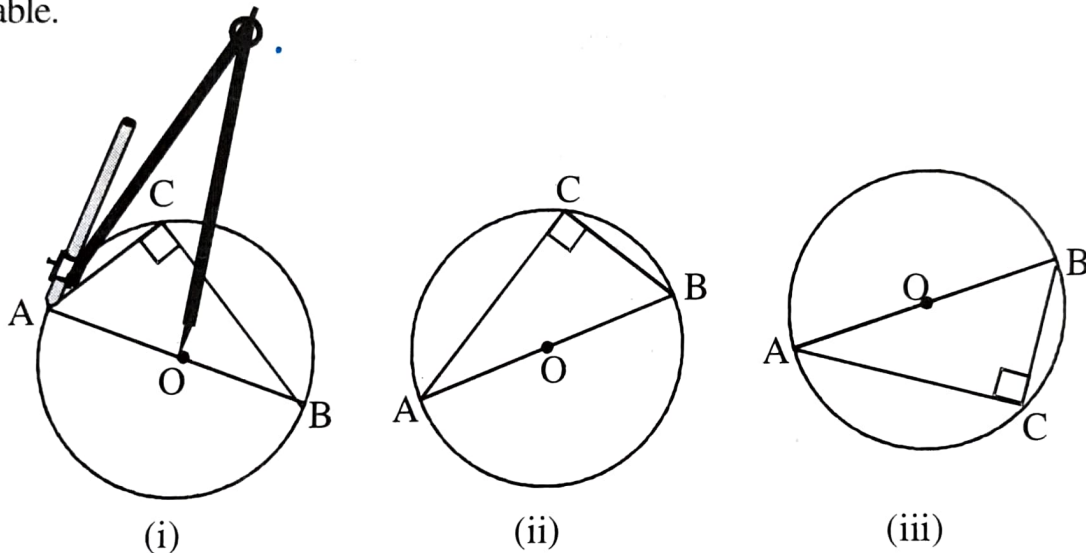


Fig. 11.2

S.No. of Semi Circle	$\angle ACB$	$90^\circ - \angle ACB$
1.		
2.		
3.		

We will find that every where the value of $90^\circ - \angle ACB$ is zero or Approximate zero. So we can say that $\angle ACB = 90^\circ$,

Conclusion:

Angle made by diameter at any point of semi circle is right angle.

Try it :

In your exercise book draw five different Circle of different radius. Measure the Angle by protector made by diameter at the different point of semi circle.

11.2 Verify by discussing on this

Angle of circle made by diameter at any point of semi circle becomes right angle.

Taking mid point O, of diameter AB as a Centre and OA radius. Construct a Semi circle at diameter AB. Take a point C on semi circle. Draw line segment AC and BC. Thus we get $\angle ACB$ of Semi Circle.

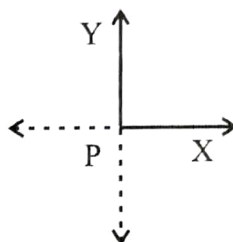
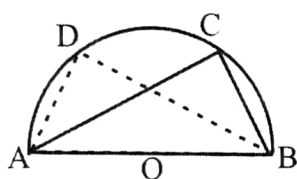


Fig. 11.3

Now take a tracing paper and make a right angle XPY with the help of set-square and cut this Angle XPY as shown in the fig. 11.3 (ii) Overlap $\angle XPY$ on $\angle ACB$. So that point P at point C. and side PX overlap side CA. Now do the side PY Overlaps at side CB ? We see that in real side P if CP. Thus $\angle XPY$ completely overlap $\angle ACB$.

So $\angle ACB = \angle XPY$

But $\angle XPY = 90^\circ$

So $\angle ACB = 90^\circ = 1 \text{ Right angle}$

Similarly another point D taken in a semicircle. Join AD and BD to make a semicircle.

So, Angle made by diameter at any point of semi circle is right angle.

Exercise 11 (a)

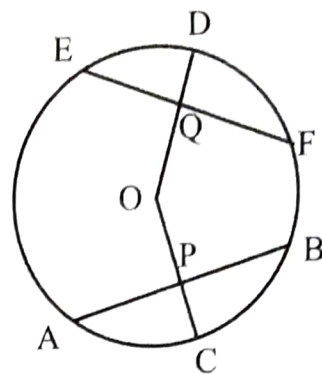
1. In adjacent figure O is Centre of Circle. Say true/ false in the following statement.

(i) Line segment AB is Chord.

(ii) QF is radius.

(iii) OD is radius.

(iv) PC is Chord.



2. Angle of Semi Circle is

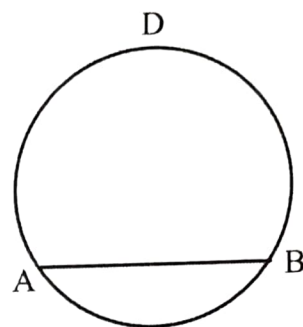
(i) 30°

(ii) 60°

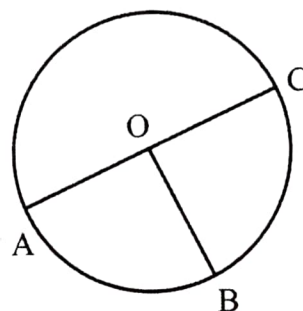
(iii) 180°

(iv) 90°

3. According fig 11.5 draw a figure on your exercise book and shade its major segment of the Circle.

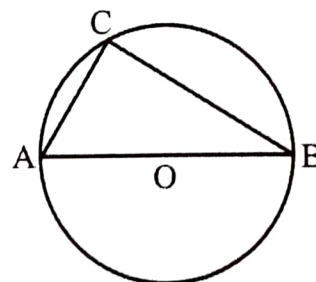


4. In fig. 11.6, O is centre of Circle. Give the name of any two sector of Circle.



5. Draw a circle of radius 2.5 cm, whose centre is O. Divide this circle in two semi circle.

6. In fig. 11.7, O is Centre of Circle. What is the value of angle $\angle ACB$ in degree. Give reason for of your answer.



Degree measure of an Arc

Try

There are three circles below, whose centres are P, Q and R respectively. There is an angle in front of minor arc AB in each circle.

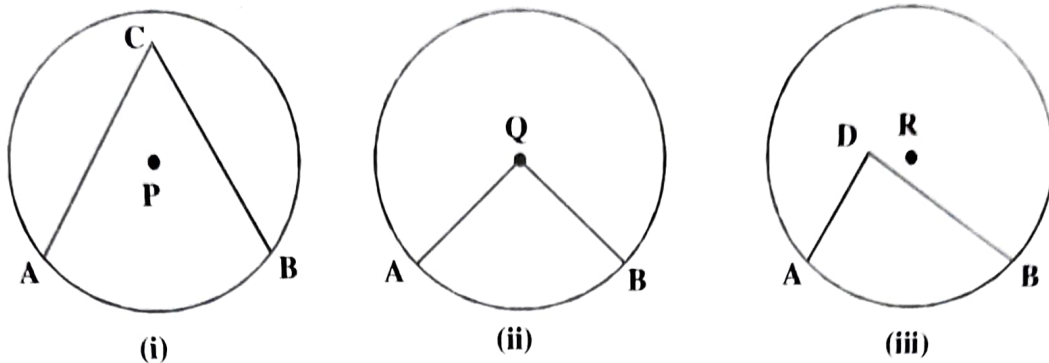


Fig. 11.8

In which above figure there is an angle made at the centre in front of arc AB?

In fig (ii) there is an angle AQB made at the centre in front of arc AB. Measure of $\angle AQB$ is called Degree measure of an arc AB.

Angle made by any arc at the centre is called degree measure of that arc.

Remember it

In fig. 11.9, O is centre of a circle. Degree measure of the arc AB of this circle is measure of the angle AOB which is made at the centre in front of the arc AB. Degree measure of minor arc AB is written as $m \widehat{AB}$.

In figure Degree measure of major arc AB is $360^\circ - m \widehat{AB}$. There $m \widehat{AB}$ is degree measure of corresponding minor arc. The length of any arc AB is proportional to interior angle made by arc AB at the centre of the circle.

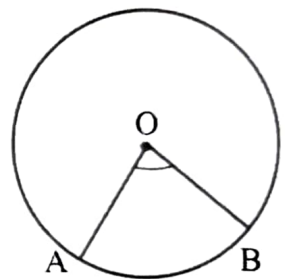


Fig. 11.9

11.4 Degree measure of circle and semicircle-

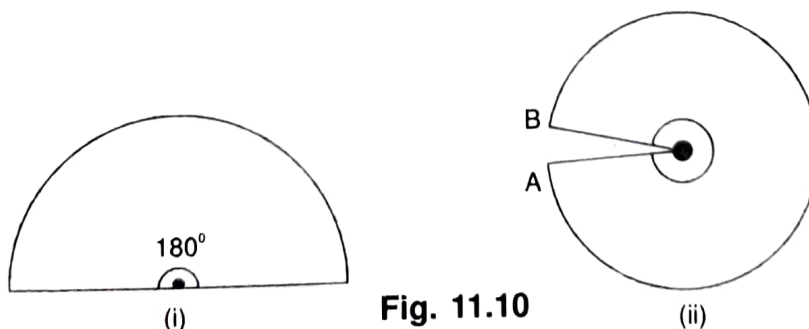


Fig. 11.10

Degree of measure of semi circle and Circle have been seen in above figure. If the point A and B of the circle be coincide to each other then the Degree of major of circle will be 360° .

So, Degree measure of semi circle be 180° and Degree measure circle be 360° .

Try it :

Say true/false statement in the following.

- (i) Degree measure of Circle be 180° .
- (ii) Angle of major segment of Circle be acute angle.
- (iii) Angle of minor segment be right angle.
- (iv) The biggest Chord of Circle be Diametre.

11.5 Inscribed angle

Remember

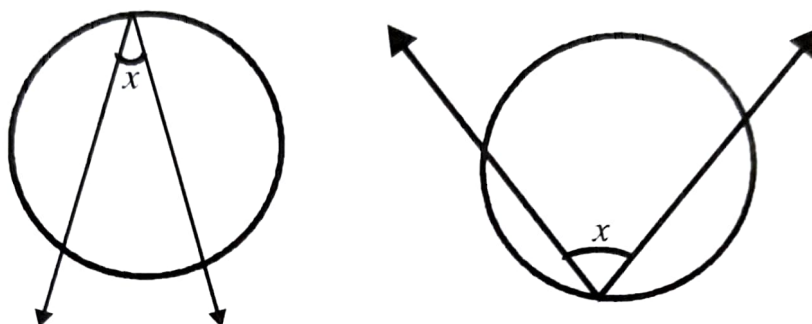


Fig 11.11

See in above figure, the vertex of $\angle x$ is a point on the circle and both sides of angle intersect the circle at two different points. The angle made like this is called inscribed angle. Thus we can say that

Any angle whose vertex is a point at circle and sides of that angle intersect two different points at the circle is called inscribed angle.

Remember

In adjacent figure 11.12, O is centre of the circle. C is a point at major arc AXB of this Circle. It has been drawn line segment CA and CB. Thus $\angle ACB$, is Inscribed angle of major arc AXB. We can say like this also that Inscribed angle of major arc AXB is a angle ACB, which is made by minor arc AB at point C of another part of the circle.

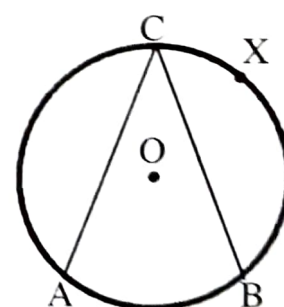


Fig. 11.12

11.6 Relation between angle made at centre by an arc and angle subtended at any point in another part of that circle-

Do this, think and find out Result:

According fig. 11.13 draw three circle whose centre is O.

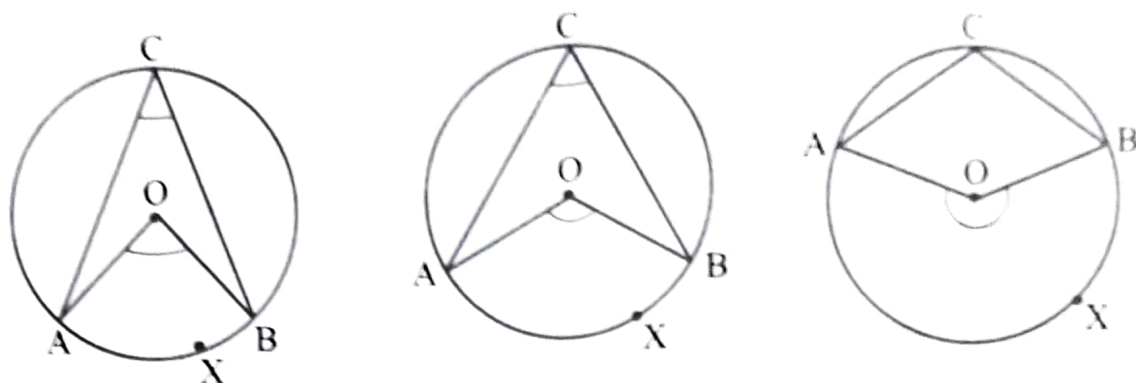


Fig. 11.13

Draw a Circle in your exercise book according to fig (i) whose centre is O. Take two point A and B at the Circle. Take Point X at minor arc AB and point C on measure arc AB. Join line segment AC, BC, AO and BO. In which Inscribed angle ACB and $\angle AOB$ has been made at the Centre.

On your exercise book repeat above process according to fig (2) and (3). Write down the result in the following table.

Serial of Circle	$\angle ACB$	$\angle AOB$	$\angle AOB - 2 \angle ACB$
(i)			
(ii)			
(iii)			

We see that in every situation $\angle AOB - 2 \angle ACB$ is zero or approximately zero. So we can take $\angle AOB = 2 \angle ACB$ Fig. 11.9.

Do this also and think

Draw circles according to fig. 11.14 at your exercise book and take centre O. Take two points A and B at the circle. Take a point at minor arc AB and point C at rest part of the circle. Draw line segment AC, BC, AO and BO. Thus angle ACB is inscribed angle and $\angle AOB$ is angle at the centre made by arc AXB.

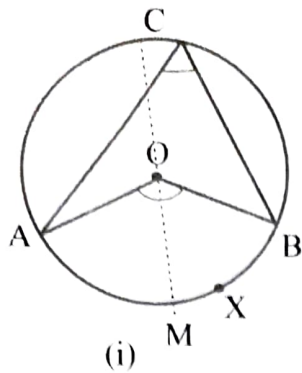
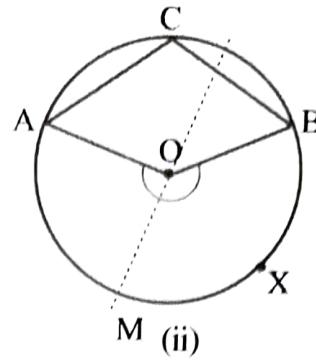


Fig. 11.14



Draw fig (i) on tracing paper. Like this we get Angle AOB and $\angle ACB$ on tracing paper also. Here also the centre of circle is O. Fold this paper as point A lie at point B and we get the mid point m of arc AXB. Thus $\angle AOB$ is divided into equal angle $\angle AOM$ and $\angle MOB$.

Thus $\angle AOM = \angle MOB$ because folding the figure along OM, side OA covers the side OB.

so $\angle AOM = \angle MOB = \frac{1}{2} \angle AOB$. Now $\angle AOM$ which is made on the paper put on the angle $\angle ACB$ which is made on your exercise book.

We see that both angle cover to each other.

So $\angle ACB = \angle AOM$

But $\angle AOM = \frac{1}{2} \angle AOB$

So $\angle ACB = \frac{1}{2} \angle AOB$

Repeat this process for figure (ii) also

We will see that

$$\angle ACB = \frac{1}{2} \angle AOB$$

So we get the result that-

An angle which is made on the centre by any arc is double of the angle subtended at any point in another part of the circle.

Example 1. Find the value of x in the fig. 11.15

Solution : Given $\angle AOB = 80^\circ$

and $\angle ACB = x ?$

Since $\angle AOB$, is a angle made by arc AB at the centre O and $\angle ACB$ is the angle subtended at the another part of the circle by same arc AB.

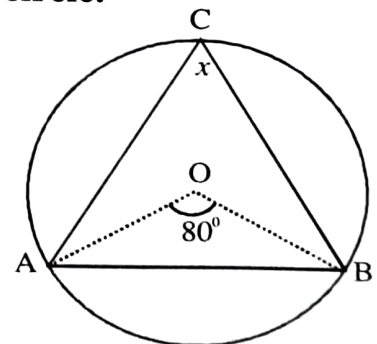


Fig. 11.15

$$\angle AOB = 2 \times (\angle ACB)$$

$$\text{or, } 80^\circ = 2x$$

$$\text{or, } x = 80^\circ \times \frac{1}{2}$$

$$\text{or, } x = 40^\circ$$

Thus the Value of x is 40° ,

Example 2. In figure 11.16 the centre of circle is O. AOC is a diameter, B D is chord, OB and CD are joined. If $\angle AOB = 130^\circ$, then find the value of $\angle BDC$.

Solution : Given, AOC is diameter of circle,

and $\angle AOB = 130^\circ$

Then $\angle BDC = ?$

AOC is a line segment

$$\therefore \angle BOC + \angle AOB = 180^\circ$$

$$\text{or, } \angle BOC + 130^\circ = 180^\circ$$

$$\text{or, } \angle BOC = 180^\circ - 130^\circ$$

$$\text{or, } \angle BOC = 50^\circ$$

$\therefore \angle BOC$ is angle situated at centre by the circle made by arc BC circle made by same arc, So

$$\angle BOC = 2 (\angle BDC)$$

$$50^\circ = 2 (\angle BDC)$$

$$\text{or } 2 (\angle BDC) = 50^\circ$$

$$\text{or, } \angle BDC = \frac{50^\circ}{2}$$

$$\text{or, } \angle BDC = 25^\circ$$

$$\text{So } \angle BDC = 25^\circ$$

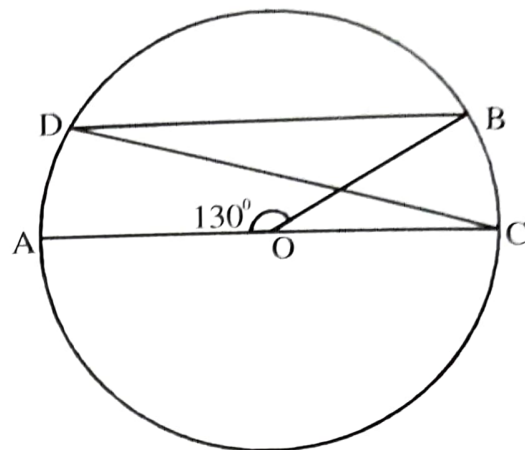


Fig. 11.16

Exercise 11 (b)

1. Degree measure of semi circle is :
(i) 45° (ii) 90° (iii) 180° (iv) 360°
2. If in any circle degree measure of minor arc is 70° , then How much will be its degree measure of major arc.
3. What is proportion between the angle made by any arc at centre and angle subtended at the another part of circle made by same arc?

4. In fig 11.17, O is the centre of circle. What is the degree measure of arc AXB.

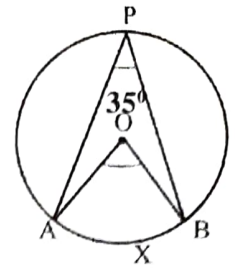


Fig. 11.17

5. In fig. 11.18 write the name of inscribed angle of minor arc BCD and major arc BAD.

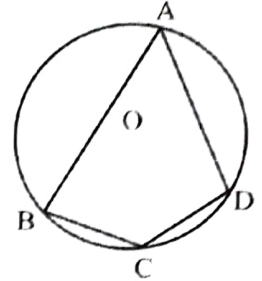


Fig. 11.18

6. In fig. 11.19 O is centre of circle. A, C and B are three points on the circle. $\angle AOB = 240^\circ$, find out the value of $\angle ACB$.

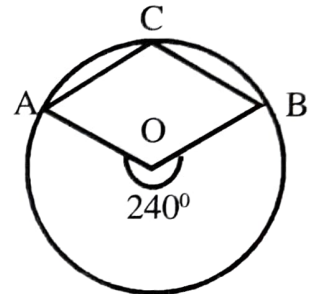


Fig. 11.19

Do this, Think and write:

Angle of same segment of a circle.

Draw a circle in your exercise book whose centre is O. Draw a Chord AB in it. Thus the circle are divided in two parts, AXB and AYB. Take Two points C and D on are AYD, draw line segment AC, BC, AD and BD.

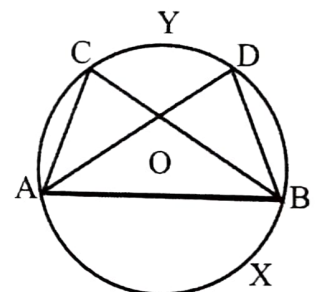


Fig. 11.20

Thus $\angle ACB$ and $\angle ADB$ are angle of same segment of circle, made by same arc AYB.

If the vertex of two angle are at the same arc of the circle then, that angles are called angle of same segment of a circle.

11.7 Relation between angles of same segment of a circle:

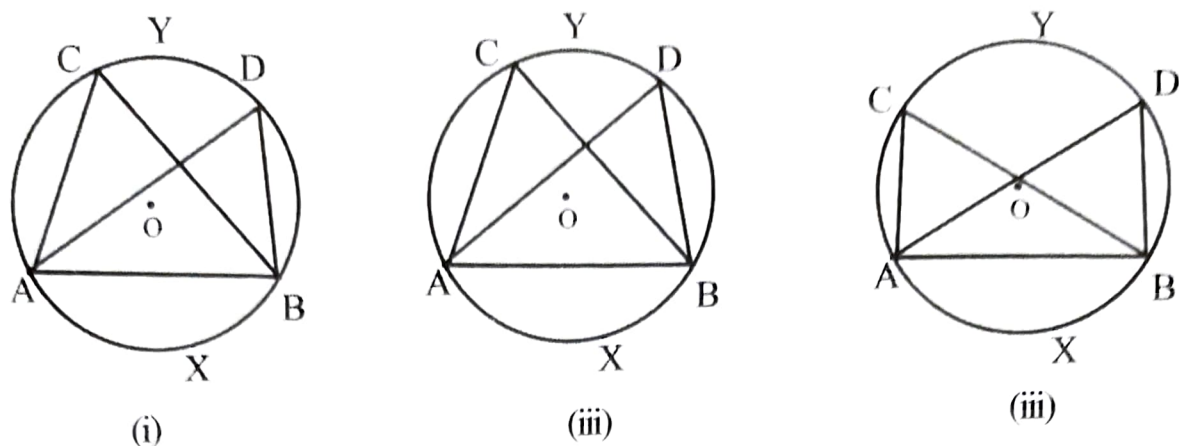


Fig. 11.21

As above according fig. (i) draw a circle whose centre be O at your exercise book. Draw a chord AB. This the circle is divided in two parts A X B and A Y B. Take two points C and D on the are A Y B. Draw line segment AC, AD, BC and BD. Thus $\angle ACB$ and $\angle ADB$ are the angles of same segment A Y B:

Measure $\angle ACB$ and $\angle ADB$ and find out the value of $\angle ACB - \angle ADB$. In the same way according to fig. (ii) and (iii) draw two other circle on your exercise book. and repeat the above activity and write down the results in the following table.

S.No. of Circle	$\angle ACB$	$\angle ADB$	$\angle ACB - \angle ADB$
(i)			
(ii)			
(iii)			

We will see that in every situation value of $\angle ACB - \angle ADB$ is zero or approximate zero.
 $\angle ACD = \angle ADB$.

Do these also, discuss on it and find result:

According to the fig draw a circle on your exercise book. Whose centre is O. Take two point A and B on the circle. Circle is divided in two arc A X B and A Y B. Take two point C and D at are A Y B. Draw line segment AC, BC, AD and BD.

Thus $\angle ACB$ and $\angle ADB$ are the angle of same segment A Y B of circle.

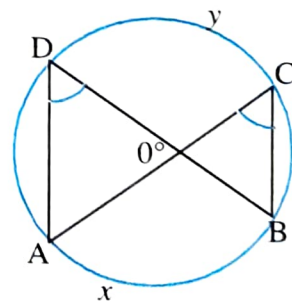


Fig. 11.22

Now on a tracing paper make a angle equal to angle ACB. Cut this angle and put this angle on $\angle ADB$ such a way that point C lies at point D. and side CA, lies on side DA. Now see, that does the side CB of angle ACB, lie on side DB?

We will see that side CB lies in the side DB.

Thus $\angle ACB = \angle ADB$

So $\angle ACB = \angle ADB$

Now rotate trace copy of $\angle ACB$ such a way that point C, lie on point E of chord AYB. and CA always go from point A. You will see that in every situation CB will go through the point B. So if any point E is on arc AYB then $\angle AEB = \angle ACB = \angle ADB$.

So we reach the result that,

The angle in a same segment of a circle are equal.

Example 3. In figure 11.23 there are two chords AB and CD, intersect at any point in the Circle. If $\angle ABC = 30^\circ$, then what will be the value of $\angle CDA$?

Solution :

Given $\angle ABC = 30^\circ$

then $\angle CDA = ?$

$\angle CDA$ and $\angle ABC =$ same angle of major arc AC

So $\angle CDA = \angle ABC$

or $\angle CDA = 30^\circ$

Thus $\angle CDA = 30^\circ$

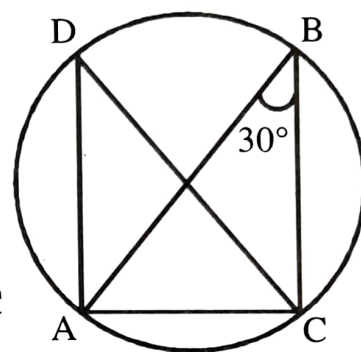


Fig. 11.23

Example 4. In fig. 11.24 $\triangle ABC$ is situated in the circle. The bisector of $\angle BAC$, intersect BC at point D and circle at point E. If $\angle BAC = 30^\circ$.

Then what is the value of $\angle ABC$?

Solution :

Given $\angle BCE = 30^\circ$ and AE is

bisector of $\angle BAC$, then $\angle BAC = ?$

$\angle BAD$ and $\angle BCE =$ Angle in a same segment BE of a circle.

So $\angle BAD = \angle BCE$

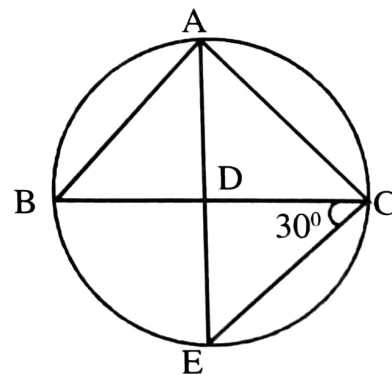


Fig. 11.24

$$\text{or } \angle BAD = 30^0$$

AE is bisector of $\angle BAC$

$$\text{so } \angle BAC = 2 \angle BAD$$

$$\text{or } \angle BAC = 2 \times 30^0$$

$$\text{or } \angle BAC = 60^0$$

$$\text{Thus } \angle BAC = 60^0$$

Exercise 11 (c)

1. In fig 11.25, write the name of the angles of same segment of a Circle.

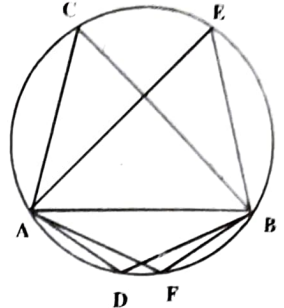


Fig. 11.25

2. In fig. 11.26 if $\angle PRQ = 45^0$ then find out the value of $\angle PSQ$.

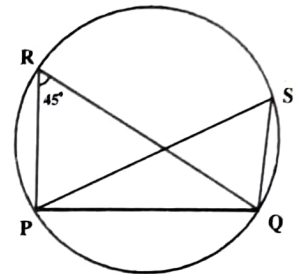


Fig. 11.26

3. In fig. 11.27 if $\angle ACB = x$ then find out the value of $\angle ADB$ and $\angle AEB$.

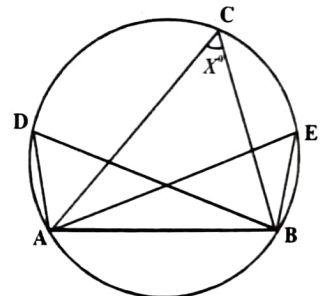


Fig. 11.27

4. In fig. 11.28 write down true/ false statement in the following statement in reference to the angle

- (i) $\angle BDC = \angle BAC$
- (ii) $\angle BDC = \angle BCA$
- (iii) $\angle ACB = \angle ADB$
- (iv) $\angle BDA = \angle CDB$
- (v) $\angle ACD = \angle DBA$

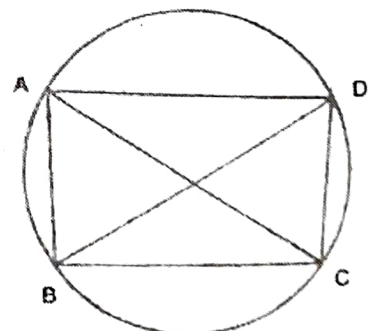


Fig. 11.28

5. In fig. 11.28 who is equal to $\angle ACB$ in the following?

- (i) $\angle ABD$
- (ii) $\angle ADB$
- (iii) $\angle DBC$
- (iv) $\angle BAD$

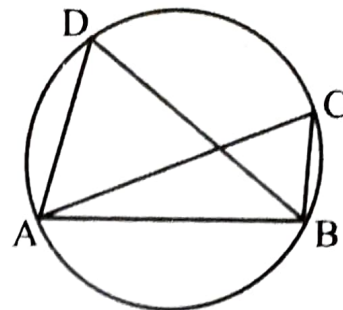


Fig. 11.29

Consolidate Example

Example 5. There are three point A, B and C situated on circles. Point O is Centre of Circle. if $\angle ABC = 100^\circ$ then find the $\angle AOC$.

Solution : Since reflex angle AOC is made on the centre O by arc AYC and $\angle AOC$ is a angle made on the point B in another part of Circle.

Hence reflex Angle $AOC = 2 \angle ABC$

But $\angle ABC = 100^\circ$

So reflex angle $AOC = 2 \times 100^\circ = 200^\circ$

or Obtuse Angle $AOC = 360^\circ - 200^\circ = 160^\circ$

or $\angle AOC = 160^\circ$

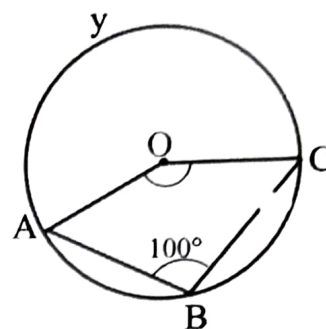


Fig. 11.30

Example 6. In fig. 11.31, O is the centre of circle. A point C is at the major arc ABC. If $\angle OAB = 30^\circ$ then find the value of angle ACB.

Solution : In triangle OAB,

Since, $OA = OB$ (because these are radius of same circle)

So $\angle OBA = \angle OAB = 30^\circ$

or $\angle AOB = 180^\circ - \angle OBA - \angle OAB$

$= 180^\circ - 30^\circ - 30^\circ$

$= 180^\circ - 60^\circ$

$= 120^\circ$

$\angle AOB$ is an angle at the centre made by minor arc CAB and $\angle ACB$ is angle made at the point C in another part of the Circle.

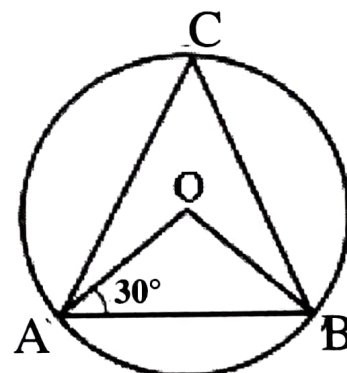


Fig. 11.31

$$\text{So } \angle ACB = \frac{1}{2} \angle AOB$$

$$= \frac{1}{2} \times 120^\circ$$

$$= 60^\circ$$

Ability Exercise 11

1. In fig 11.32, O is the Centre of circle. Line BOD is the bisector of $\angle AOC$ and $\angle COD = 50^\circ$ then measure of $\angle ABC$ will be :

- (i) 50° (ii) 25°
(iii) 100° (iv) 120°

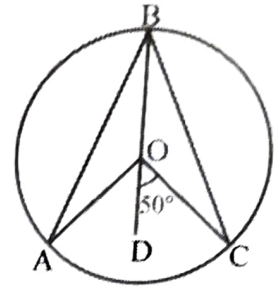


Fig. 11.32

2. In fig. 11.33, AB is Chord of circle and point C and D are on the Circle, if $\angle ADB = 45^\circ$, then measure of $\angle ACB$ will be :

- (i) 90° (ii) 135°
(iii) 45° (iv) $22\frac{1}{2}^\circ$

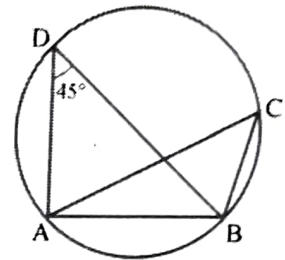


Fig. 11.33

3. In fig 11.34, O is the centre of circle and $\angle AOB = 60^\circ$ then the measure of $\angle ADB$ will be:

- (i) 120° (ii) 150°
(iii) 140° (iv) 30°

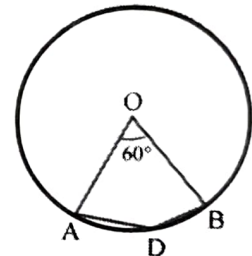


Fig. 11.34

4. In fig. 11.35, O is the centre of circle. A, B, C are three points on the circle if $\angle ACB = 40^\circ$ then measure of $\angle AOB$ will be:

- (i) 20° (ii) 40°
(iii) 60° (iv) 80°

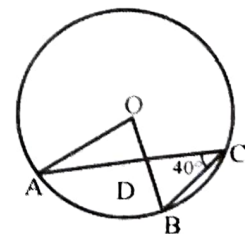


Fig. 11.35

5. In fig. 11.36, O is the centre of circle.
AC and BC are equal chord. Find the value of angle $\angle ABC$.

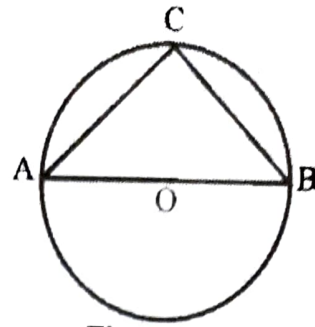


Fig. 11.36

6. In the following fig. O is the centre of each circle, find the value of x in each circle.

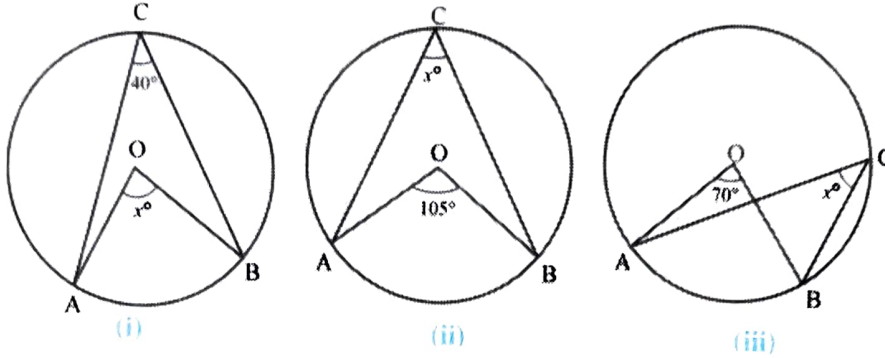


Fig. 11.37

7. The chord of a circle is equal to the radius of that circle. Find the Value of the angle inscribed by this chord in the minor segment of the circle.
8. Draw a circle of radius 3.0 cm. Divide this circle in two parts on drawing a chord of this circle.
9. Define a semi-circle. Explain with a diagram.
10. In fig. 11.38, O is the centre of the Circle.
AOB is a diameter. $\angle COB = 40^\circ$, find out
(i) Degree measure of major arc BC.
(ii) Degree measure of major arc AC.
(iii) Degree measure of minor arc AC.
(iv) Degree measure of semi circle ACB.
11. In fig. 11.39, O is the centre of circle.
A triangle ABC inscribed in a circle.
if $\angle ACB = 30^\circ$, then find the $\angle A$ and $\angle B$.

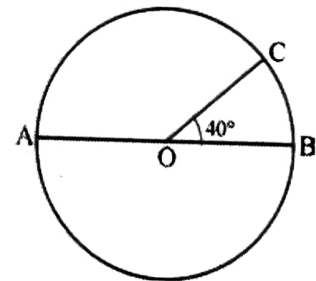


Fig. 11.38

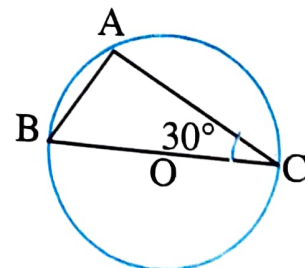


Fig. 11.39

12. The length of the chord of a circle is equal to the length of radius of that circle. Find out the angle in major segment of circle inscribed by this chord.

13. In fig 11.40 O is the centre of circle, $\angle AEB = 130^\circ$ and $\angle EBC = 20^\circ$, then find the value of $\angle BDA$.

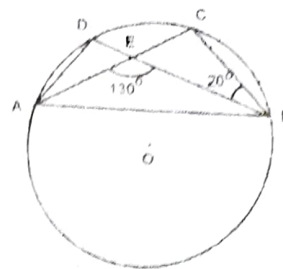


Fig. 11.40

14. In fig 11.41, O is centre of circle $\angle ABC = 40^\circ$ and $\angle CAB = 80^\circ$. then find out the value of $\angle ADB$.

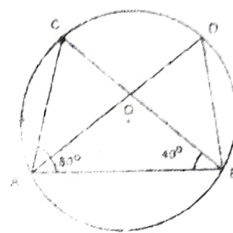


Fig. 11.41

15. In fig. 11.42, O is the centre of circle and $\triangle ABC$ is a equilateral triangle then find out the value of $\angle BOC$.

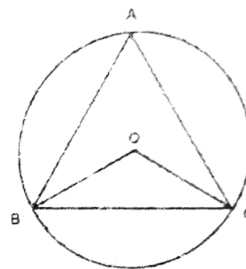


Fig. 11.42

16. In fig. 11.43, O is the centre of the circle and $\angle BOD = 130^\circ$, find the measure of angle BCD.

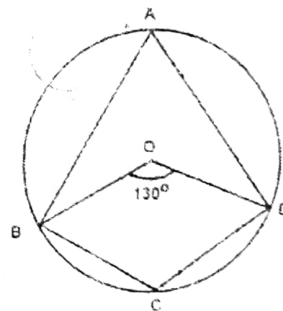


Fig. 11.43

17. In fig 11.44, O is the centre of circle. If $\angle OBC = 40^\circ$, then find out value of $\angle BAC$.

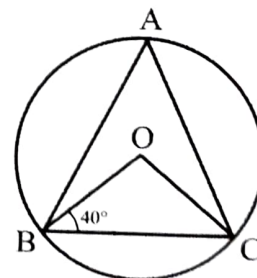


Fig. 11.44

We have learnt:

1. The angle made by the diameter at any point of the circle is called angle of semi circle.
2. The angle of semi circle be Right angle.
3. The angle formed between the radii joining the end points of an arc to the centre is called the degree measure of an arc.
4. The degree measure of semi circle is 180^0 and degree measure of circle is 360^0 .
5. Any angle of circle is called inscribed angle if circle. If vertex of that angle is a point of circle and sides intersect circle at different point.
6. In any arc there may be more than one Inscribed angle.
7. An angle at the centre of circle made by an arc is double of the angle, subtended at any point in another part of the circle.
8. If vertex of two angle are on the same arc of the circle. Then that angle are called angle of same segment of circle. The value of these two angle are equal.

ANSWERS

Exercise 11 (a)

1. (i) yes (ii) No (iii) yes (iv) No. 2. 90^0 4. AOB and BOC 6. $\angle ACB = 90^0$

Exercise 11 (b)

1. (iii) 180^0 ; 2. 290^0 ; 3. 2 : 1; 4. Arc of degree measure = 70^0 ; 5. $\angle BCD$ and $\angle BAD$
6. $\angle ACB = 120^0$

Exercise 11 (c)

1. $\angle ACB$, $\angle AEB$ angle are same major segment $\angle ADB$, $\angle AFB$ are same minor segment of circle 2. $\angle PRQ = \angle PSQ = 45^0$; 3. $\angle ADB = x^0$ and $\angle AEB = x^0$; 4. (i) true (ii) false (iii) true (iv) false (v) true 5. (ii) $\angle ADB$

Ability Exercise 11

1. (i) 50^0 ; 2. (iii) 45^0 ; 3. (ii) 150^0 ; 4. (iv) 80^0 ; 5. 45^0 ; 6. (i) 80^0 ; (ii) $52\frac{1}{2}^0$; (iii) 35^0 ; 7. 150^0 ; 10. (i) 320^0 , (ii) 220^0 , (iii) 140^0 , (iv) 180^0 ; 11. $\angle A = 90^0$, $\angle B = 60^0$; 12. 30^0 ; 13. 110^0 ; 14. 60^0 ; 15. 120^0 ; 16. 115^0 ; 17. 50^0

