



- Quadrilateral and its various parts.
- Type of quadrilateral
- Experimental verification of properties of quadrilateral.

## 10.1 Introduction

In your daily life, you see such types of many objects whose plane surface is surrounded by four line segments, such as book, blackboard, gate etc. The plane of all these things are closed by four line segments (rectangle, square).

In class VI you have studied that simple closed figure formed by line segments is called a polygon, and you have also studied that figure closed by three line segments is triangle. In this Unit we will read figure closed by four line segments and its properties.

We know the figure formed (closed) by three line-segment is a triangle. Similarly the figure closed by four line-segments is called a quadrilateral.

In Figure 10.1 the quadrilateral have:

- Four sides  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$  and  $\overline{DA}$
- Four vertices A, B, C and D.
- Four angles  $\angle A$ ,  $\angle B$ ,  $\angle C$  and  $\angle D$ .
- Sides AB, AD ; sides AB, BC ; sides BC, CD and sides CD, AD are attached to each other, which are called adjacent sides.
- Side CD in  $\angle C$  and  $\angle D$ ; side AD in  $\angle A$  and  $\angle D$ ; side AB in  $\angle A$  and  $\angle B$  and side BC in  $\angle B$  and  $\angle C$  are common.

Therefore, the pair of angles in which one side is common are called adjacent angles.

- Sides AB and DC; sides AD and BC are parallel to each other, these sides are called corresponding sides.

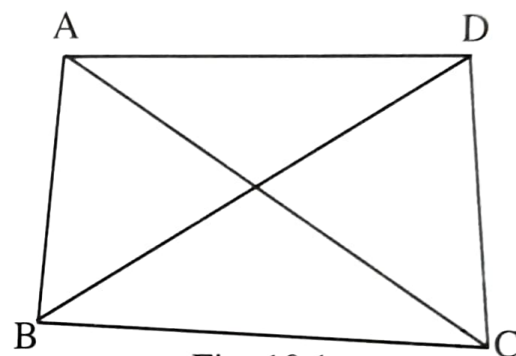
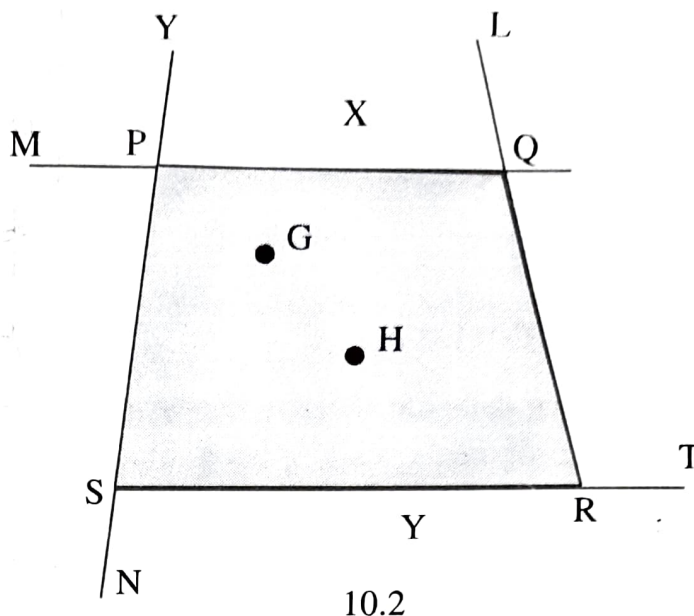


Fig. 10.1

- (vii) Vertical angles  $\angle A$  and  $\angle C$  and  $\angle B$  and  $\angle D$  are called vertically opposite angles.
- (viii) The sides (line segments) AC and BD which join the vertices A and C; vertices B and D; are called diagonals.

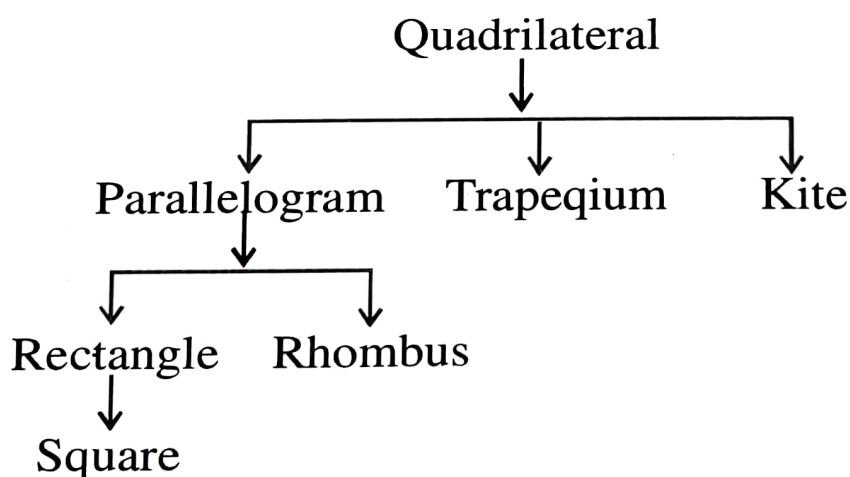
Now we consider a quadrilateral PQRS in which two adjacent sides SP and QP meet at P.

- (i) The angles ( $\angle QPS$ ,  $\angle PSR$ ,  $\angle SRQ$  and  $\angle RQP$ ) formed by the adjacent sides of quadrilateral PQRS are called internal angles.
- (ii) To extend sides SR to T, side RQ to L, side QP to M and side PS to N, get exterior angles  $\angle QRT$ ,  $\angle LQR$ ,  $\angle NSR$  and  $\angle MPS$ .



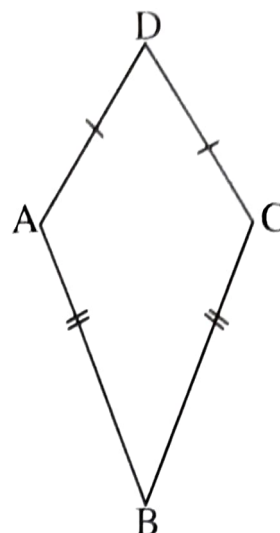
- (iii) The interior part of quadrilateral PQRS is called interior and the exterior part of quadrilateral PQRS is called exterior part. The points H and G are interior points and X and Y are exterior points of the quadrilateral PQRS.

The quadrilaterals have different names on the basis of angles and sides of the quadrilaterals.



|                                                                                                                 |                                                                                                                                               |
|-----------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| <p>1. Parallelogram: The parallel sides are equal in a quadrilateral is called parallelogram.</p>               | <p><math>AB \parallel DC</math><br/><math>AD \parallel BC</math></p>                                                                          |
| <p>2. Rectangle: A parallelogram in which all four interior angles are <math>90^\circ</math> (right angle).</p> | <p><math>AB = DC</math><br/><math>AD = BC</math><br/><math>\angle A = \angle B =</math><br/><math>\angle C = \angle D = 90^\circ</math></p>   |
| <p>3. Rhombus: A parallelogram in which all four sides are equal.</p>                                           | <p><math>AB = BC =</math><br/><math>CD = DA</math><br/><math>AB \parallel CD</math><br/><math>AD \parallel BC</math></p>                      |
| <p>4. Square: A parallelogram in which each sides are equal and all four interior angles are right angle.</p>   | <p><math>AB = BC =</math><br/><math>CD = DA</math><br/><math>\angle A = \angle B =</math><br/><math>\angle C = \angle D = 90^\circ</math></p> |
| <p>5. Trapezium: A quadrilateral in which opposite sides are equal.</p>                                         | <p><math>AB \parallel CD</math><br/><math>DL = CM = h</math></p>                                                                              |

6. Kite: Kite is a quadrilateral in which pair of two adjacent sides are equal.



$$\begin{aligned} AB &= BC \\ AD &= DC \\ AB &\neq CD \\ BC &\neq AD \end{aligned}$$

### Exercise 10 (a)

1. See fig 10.18 (quadrilateral ABCD)

- How many sides are there?
- How many internal angles are there?
- How many pair of opposite sides are there?
- How many pair of adjacent sides are there?
- How many diagonals will be there?
- Is there any diagonal among AB, BC, CD and DA?
- How many vertices are there?

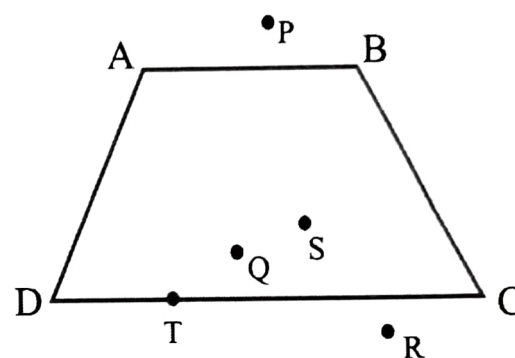


Figure. 10.18

- On the basis of fig. 10.18, fill up the blanks in your exercise book.
  - Q is situated in ..... region.
  - R is situated in ..... region.
  - T is situated .....
- Related with any quadrilateral ABCD fill up the blanks in the following statement.
  - On joining two..... vertices we get diagonals.
  - Vertices A and ..... joined to form a diagonal.
  - Vertices D and ..... joined to form a diagonal.
  - A diagonal of a quadrilateral divides the quadrilateral.

4. Fill up the blanks in your exercise book.

(a) Four sides of rhombus are .....

(b) Angle of rectangle are .....

(c) ..... sides of square is equal and ..... angle be right angle.

(d) One pair of ..... of trapezium be parallel.

## 10.4 Experimental verification of the properties of a Quadrilateral.

### 10.4 1 Sum of internal angles of a quadrilateral.

#### Activities-

Construct a quadrilateral ABCD on a thick paper sheet. Cut it in four pieces so that different pieces contain different angles A, B, C and D. Now according fig. 10.19 draw a ray OP. Arrange four angles on point O in a order. Now we see that, four internal  $\angle A$ ,  $\angle B$ ,  $\angle C$  and  $\angle D$

of quadrilateral, arranging in order at point O, the side of angle D, comes again at side OP. We know that at any point a complete angle  $360^\circ$  is formed. So, sum of four internal angles of quadrilateral i.e  $\angle A$ ,  $\angle B$ ,  $\angle C$  and  $\angle D$  is  $360^\circ$ .

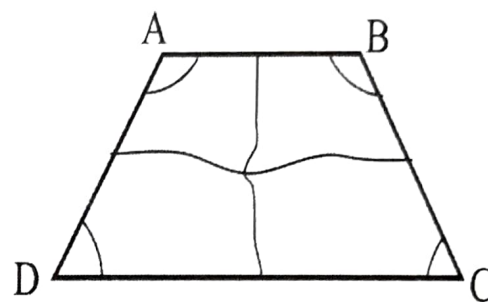
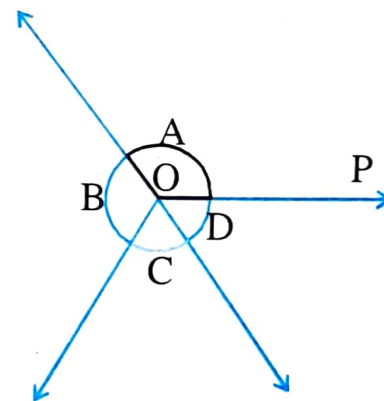


Figure. 10.19

2. Now you draw three different quadrilateral in your exercise book. Give the name of every quadrilateral ABCD. Measure every angle of quadrilateral and write the measurement of angle in the following table.



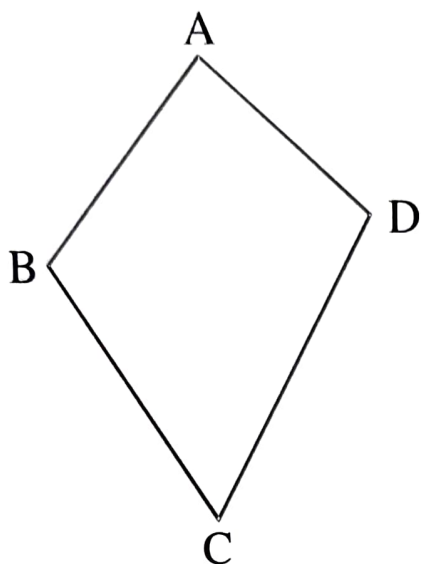
| Serial of quadrilateral | $\angle A$ | $\angle B$ | $\angle C$ | $\angle S = \angle A + \angle B + \angle C + \angle D$ | $360^\circ - S$ |
|-------------------------|------------|------------|------------|--------------------------------------------------------|-----------------|
| 1.                      |            |            |            |                                                        |                 |
| 2.                      |            |            |            |                                                        |                 |
| 3.                      |            |            |            |                                                        |                 |

Now we see that the value  $360^\circ - S$  is zero or so small that we can overlook this. i.e.  $\angle A + \angle B + \angle C + \angle D = 360^\circ$ .

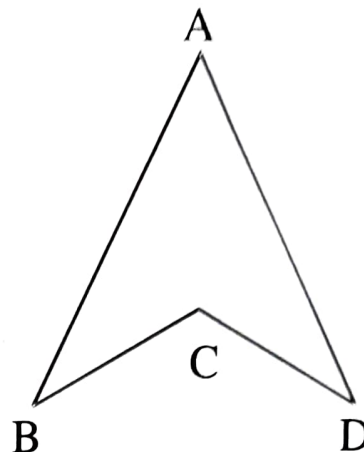
**The sum of internal angle of a quadrilateral is  $360^\circ$ .**

From the above activities we get the result that the sum of internal angles of a quadrilateral is  $360^\circ$ , and each angle is less than  $180^\circ$ . Then it is a convex quadrilateral. If any internal angle of a quadrilateral is greater than  $180^\circ$ , then that quadrilateral is concave quadrilateral.

**Note :** In geometry we study only Convex quadrilaterals.



Convex quadrilateral.



Concave quadrilateral

Figure 10.20

### Exercise 10 (b)

1. One angles of a quarilateral is  $60^\circ$  and remaining three angles are equal. Find out the measurement of remaining angles.
2. Two angles of a quadrilateral are  $60^\circ$  and  $120^\circ$ , Remaining two angle are equal. Find out the value of remaining two angles.
3. Internal angles of a triangle are equal find the value of all the angle.
4. If two angles of a quadrilateral are supplementary angles, then find out the sum of remaining two angles.
5. Construct an angle BAC of  $45^\circ$ , draw a perpendicular from PN and PM on line segment BA and AC from point P which is situated in interior of angle BAC. Find the value of angle NPM.
6. The ratio of internal angle of a quadrilateral is 3 : 4 : 5 : 6. find the value of each angle.

7. If three external angle of a quadrilateral are  $80^\circ$ ,  $100^\circ$  and  $120^\circ$ , find out the value of fourth internal angle.
8. If the internal angle A, B, C and D are such that  $\angle A : \angle B = 1 : 2$ ,  $\angle B : \angle C = 2 : 3$ ,  $\angle C : \angle D = 3 : 4$  then find out the value of each angle.
9. Vertex angle of an isosceles triangle is  $40^\circ$ , Mid point of the sides AB and AC are M and N. Join point M and N. Find out the sum of internal angle  $\angle BMN$  and  $\angle CNM$  of quadrilateral BMNC, also find the value of  $\angle BMN$  and  $\angle CNM$  intersecting.

### 10.4.2 Opposite sides of Parallelogram.

#### Activity

Draw a pair of parallel line l and m. Draw two more parallel lines p and q intersecting line l and m. According to fig. 10.21, write the name of the intersecting points A, B, C and D. Like this we get parallelogram ABCD. Draw next two parallelogram in your exercise book. Write these three parallelogram ABCD, Now, measure the side AB, BC, CD and DA and complete the following table.

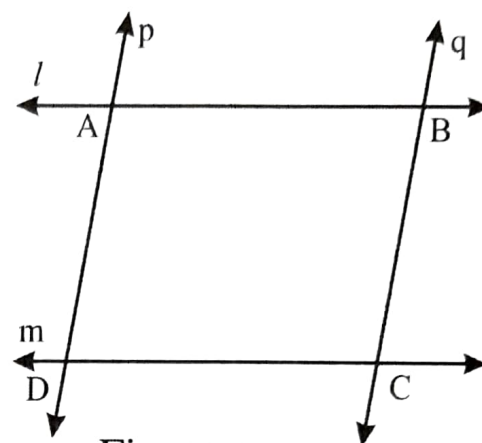


Fig 10.21

| S.N. | Pair of opposite side<br>AB and DC |    |       | Pair of opposite<br>sides BC and AD |    |       |
|------|------------------------------------|----|-------|-------------------------------------|----|-------|
|      | AB                                 | DC | AB-DC | BC                                  | AD | BC-AD |
| 1.   |                                    |    |       |                                     |    |       |
| 2.   |                                    |    |       |                                     |    |       |
| 3.   |                                    |    |       |                                     |    |       |

We see in the Table AB-DC and BC-AD is zero or difference of these are so small that we can ignore this value. i.e. in every situation  $AB = DC$  and  $BC = AD$ . So we see that

**Opposite sides of parallelograms are equal.**

### 10.4.3 Opposite angles of parallelogram-

#### Activities

Draw a pair of parallel line  $l$  and  $m$ . Draw an other pair of parallel line  $p$  and  $q$  which intersect the line  $l$  and  $m$ . Like this name the parallelogram  $ABCD$ .

Draw two more parallelogram  $ABCD$ . Now measure the internal angle of these three parallelograms and complete the following table.

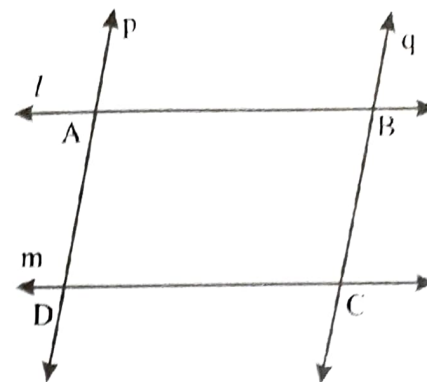


Fig 10.22

| S.No. | Pair of opposite angles A and C |            |                       | Pair of opposite angles B and D |            |                       |
|-------|---------------------------------|------------|-----------------------|---------------------------------|------------|-----------------------|
|       | $\angle A$                      | $\angle C$ | $\angle A - \angle C$ | $\angle B$                      | $\angle D$ | $\angle B - \angle D$ |
| 1.    |                                 |            |                       |                                 |            |                       |
| 2.    |                                 |            |                       |                                 |            |                       |
| 3.    |                                 |            |                       |                                 |            |                       |

We see in the table that  $\angle A - \angle C$  and  $\angle B - \angle D$  are zero or difference is so small that we can ignore this i.e.  $\angle A = \angle C$  and  $\angle B = \angle D$

So, we see that

**Opposite angles of parallelogram are equal**

#### 10.4.4 Diagonals of parallelogram

Draw a pair of parallel line  $l$  and  $m$ . Draw an other pair of parallel line  $p$  and  $q$  which intersect the line  $m$  and  $n$ . Thus we get a parallelogram  $ABCD$ . Join vertex  $A, C$  and  $B, D$ . Diagonal  $AC$  and  $BD$  intersect each other at point  $O$ . Construct two more parallelogram like this. Name these parallelogram  $ABCD$ . Diagonals of these also intersect at point  $O$ .

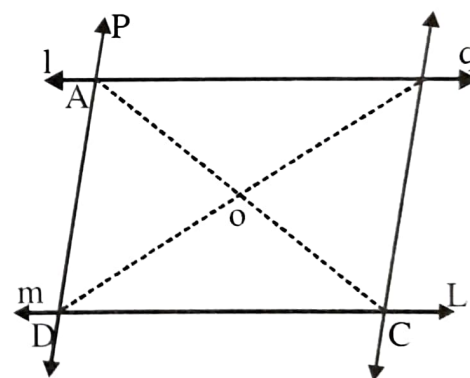


Fig 10.23

Now measure  $AO$ ,  $OC$  and  $DO$ ,  $OB$  then fill up the following table.

| S.No. | Diagonal AC |    |       | Diagonal BD |    |       |
|-------|-------------|----|-------|-------------|----|-------|
|       | AO          | OC | AO-OC | DO          | OB | DO-OB |
| 1.    |             |    |       |             |    |       |
| 2.    |             |    |       |             |    |       |
| 3.    |             |    |       |             |    |       |

We see that  $AO-OC$  and  $DO-OB$  is zero or difference of these are so small that we can ignore it i.e.  $AO = OC$  and  $DO = OB$ . So,

**Diagonals of parallelogram bisect each other**

### Exercise 10 (c)

- One internal angle of a parallelogram is  $30^\circ$ , find the value of remaining angles.
- Difference of the two angles of a parallelogram which is made at any side is  $40^\circ$ , Find the value of each angle.
- If the ratio of the angles which is made on any side of parallelogram is  $1 : 3$ , find out the value of every angle.
- Adjacent side of parallelogram are 4 cm and 6 cm. Find out the measurement of other two sides of quadrilateral.
- Adjacent sides of parallelogram are 8 cm and 6 cm. Find out the perimeter of quadrilateral.
- The ratio of two adjacent sides of a parallelogram is  $1 : 2$ , If the perimeter is 30 cm then find out measurement of all the side.
- In fig. 10.24 PQRS is a parallelogram after measuring line segment OP, OQ, OR and OS. Verify that  $OP = OR$  and  $OQ = OS$

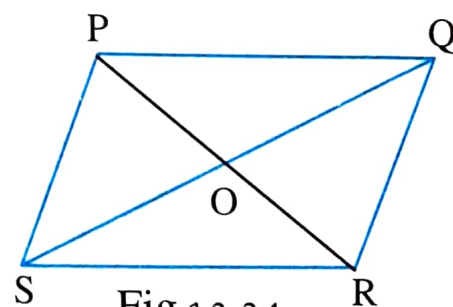


Fig 10.24

8. In fig. 10.25, PQRS is a trapezium, find measured value of Angle P and S.

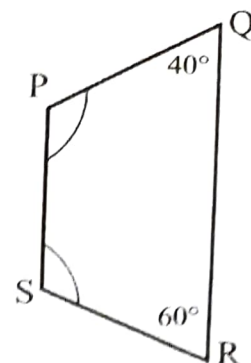


Fig 10.25

### 10.4.5 Diagonals of rectangle

Draw a pair of parallel line  $l$  and  $m$ . Similarly draw next pair of parallel lines which is perpendicular to previous line. Now name the rectangle as ABCD. Construct two more rectangle like this. Now name them also as ABCD. Now measure the diagonal AC and BD and complete the following table.

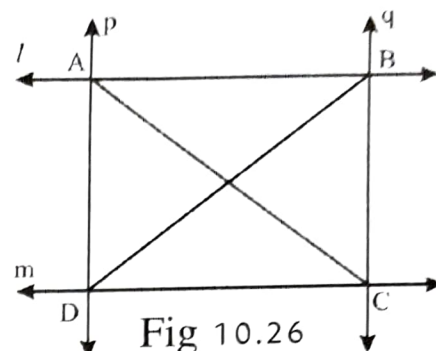


Fig 10.26

| S.No. | Length of the diagonals |    |         |
|-------|-------------------------|----|---------|
|       | AC                      | BD | AC - BD |
| 1.    |                         |    |         |
| 2.    |                         |    |         |
| 3.    |                         |    |         |

We see that the difference  $AC - BD$  is zero or this is so small that we can ignore it. i.e.  $AC = BD$  So.

**The diagonals of rectangle are equal. Remember that, rectangle is a special type of parallelogram. So its diagonals bisect each other.**

### 10.4.6 Diagonals of a Rhombus

Draw a pair of parallel line  $l$  and  $m$ . Take a point A on the line, taking centre at A draw an arc C which intersect both lines. Let intersection point are D and B. Join point A, with point B and draw a line DC parallel to AB from point

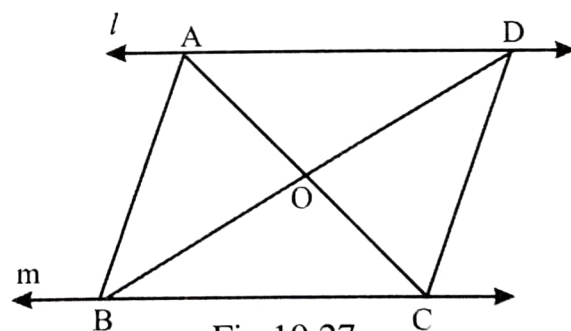


Fig 10.27

D. Figure ABCD is a Rhombus, whose four sides are equal. AC and BD are diagonals.

Let these intersect at point D. Construct two more rhombus like this. Give name of these rhombus ABCD.

Now measure the Angle  $\angle AOB$  and  $\angle COB$  and fill up the following table.

| S.No. of rhombus | $\angle AOB$ | $90^\circ - \angle AOB$ | $\angle COD$ | $90^\circ - \angle COD$ |
|------------------|--------------|-------------------------|--------------|-------------------------|
| 1.               |              |                         |              |                         |
| 2.               |              |                         |              |                         |
| 3.               |              |                         |              |                         |

We see that the value of  $90 - \angle AOB$  and  $90^\circ - \angle COB$  is zero or this is so small that we can ignore it, ie.  $90^\circ = \angle AOB$  and  $90^\circ = \angle COB$  or  $\angle AOB = \angle COB = 90^\circ$ .

**The diagonal of rhombus bisect each other at right angle.**

#### 10.4.7 Diagonals of a square

Construct a square ABCD. Diagonals of this intersect at point O. Construct two more square like this. Name these by ABCD. Measure diagonal AC and BD and write down in following table.

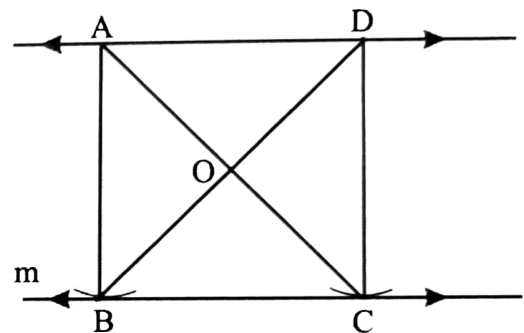


Fig 10.28

| S.No. of square | AC | BD | AC - BD |
|-----------------|----|----|---------|
| 1.              |    |    |         |
| 2.              |    |    |         |
| 3.              |    |    |         |

We see that the value of AC - BD is zero or so small that we can ignore it.  
i.e.,  $AC = BD$

Again measure  $\angle AOB$  and  $\angle COB$  in above three square and write down in following table.

| S.No.<br>of square | $\angle AOB$ | $90^\circ - \angle AOB$ | $\angle COB$ | $90^\circ - \angle COB$ |
|--------------------|--------------|-------------------------|--------------|-------------------------|
| 1.                 |              |                         |              |                         |
| 2.                 |              |                         |              |                         |
| 3.                 |              |                         |              |                         |

We see that the value of  $90^\circ - \angle AOB$  and  $90^\circ - \angle COB$  are zero or so small that we can ignore it. i.e.  $90^\circ = \angle AOB$  and  $90^\circ = \angle COB$  or  $\angle AOB = \angle COB = 90^\circ$ .

So

**Diagonals of square bisect each other at right angle.**

### Exercise 10 (d)

1. ABCD is a parallelogram in fig 10.30.

Give that condition when this

- (i) becomes rhombus
- (ii) becomes rectangle
- (iii) becomes square

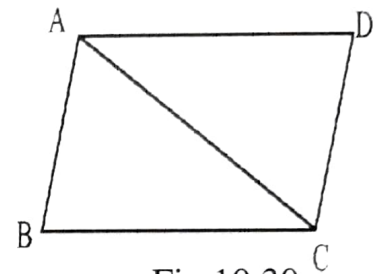


Fig 10.30

2. What will be the name of parallelogram ABCD when the following statement will be true?
- (i)  $AB = BC$
  - (ii)  $\angle ABC = 90^\circ$
  - (iii)  $\angle ABC = 90^\circ$  and  $AB = BC$
3. In square
- (i) Length of sides are .....
  - (ii) Diagonals are .....
  - (iii) Each angle is .....
  - (iv) Diagonals are ..... to each other.
4. If the square of any diagonal of a square is  $50 \text{ cm}^2$ , then what will be its perimeter.
5. One diagonal of your books page is smaller than its other diagonal. Is the book rectangular?
6. Construct a rectangle whose adjacent sides are 6 cm and 8 cm. Measure the diagonal and verify the measurement by Pythagoras theorem.
7. Construct a rectangle whose adjacent sides are 5 cm and 12 cm. Measure the diagonal of this rectangle and verify the measurement by Pythagoras theorem.

8. If one diagonal of a rhombus is equal to its one side, then find out all the internal angle of this rhombus.

### We have learnt:

1. Vertices, sides, diagonals, adjacent sides, opposite sides and its internal and external angle of a quadrilateral.
2. Various type of quadrilateral- Parallelogram, Rhombus, Rectangle and square.
3. Experimental Verification of the following properties of a rectangle.
  - (i) The sum of each Internal angle of a quadrilateral becomes  $360^\circ$ .
  - (ii) Opposite sides of Parallelogram are equal.
  - (iii) Opposite angles of parallelogram are equal.
  - (iv) Diagonals of parallelogram bisect each other.
  - (v) Diagonals of rectangle are equal and bisect each other.
  - (vi) Diagonals of Rhombus bisect each other at right angle.
  - (vii) Diagonals of square are equal and bisect each other at right angle.

## ANSWERS

### Exercise 10 (a)

1. (a) four, (b) four, (c) two, (d) four pair, (e) two, (f) No, (g) four, 2. (a) Internal, (b) external (c) On side DC, 3. (a) Opposite, (b) C (c) B, (d) two, 4. (a) equal, (b) four, (c) four four, (d) Opposite side

### Exercise 10 (b)

1.  $100^\circ$ ,  $100^\circ$ ,  $100^\circ$ , 2.  $90^\circ$ ,  $90^\circ$ , 3.  $90^\circ$ ,  $90^\circ$ ,  $90^\circ$ ,  $90^\circ$ ; 4.  $180^\circ$ , 5.  $135^\circ$ , 6.  $60^\circ$ ,  $80^\circ$ ,  $100^\circ$ ,  $120^\circ$ , 7.  $120^\circ$ , 8.  $36^\circ$ ,  $72^\circ$ ,  $108^\circ$ ,  $144^\circ$ , 9.  $220^\circ$ ,  $110^\circ$ ,  $110^\circ$

### Exercise 10 (c)

1.  $150^\circ$ ,  $30^\circ$ ,  $150^\circ$ ; 2.  $110^\circ$ ,  $70^\circ$ ,  $110^\circ$ ,  $70^\circ$ ; 3.  $135^\circ$ ,  $45^\circ$ ,  $135^\circ$ ,  $45^\circ$ ; 4. 4 cm, 6 cm. 5. 28 cm, 6. 5 cm, 10 cm, 5. 10 cm, 10 cm, 7.  $\angle P = 140^\circ$ ,  $\angle S = 120^\circ$

### Exercise 10 (d)

1. (i) Adjacent side equal (ii) One Angle  $90^\circ$ , (iii) One Angle  $90^\circ$ , Adjacent side are equal, 2. (i) Rhombus (ii) Rectangle (iii) Square 3. (i) equal (ii) equal (iii) Right Angle (iv) equal 4. 20 cm. 5. No. 6. 10 cm; 7. 13 cm; 8.  $60^\circ$ ,  $120^\circ$ ,  $60^\circ$ ,  $120^\circ$

