

UNIT-9 FACTOR OF EXPRESSIONS



- Factor of expression like $(ax + ay)$
- Factor of expression like $(ax^2 + ay^2 + bx^2 + by^2)$

9.1 Introduction

You have learn prime factorization of natural number in class 6. You know that a natural number can be written in the form of multiple of many natural number in different ways.

For example take a natural number 30.

Natural number 30 can be written in many form.

$$30 = 1 \times 30, 30 = 2 \times 15$$

$$30 = 3 \times 10, 30 = 5 \times 6$$

$$30 = 2 \times 3 \times 5$$

So 1,2,3,5,6,10,15,30, are factors of number 30, number 2,3 and 5 are prime factors of 30.

So we write factor of 30 as $2 \times 3 \times 5$

Similarly prime factor of 70 written as $2 \times 5 \times 7$

In previous chapter we have learnt to write the terms of algebraic expression as products of numbers and variables

In this chapter we will learn factorization of algebraic expressions like in natural numbers.

9.2 Factor of algebraic expressions.

When an algebraic expression can be written as the product of two or more expressions, then each of these expression is called a factor of the given expression.

Example : One algebraic expression $5xy$

$$\begin{aligned}\text{So } 5xy &= 5 \times (xy) \\ &= (5x) \times y \\ &= (5y) \times x \\ &= 5 \times x \times y\end{aligned}$$

Observe, that on writing $5xy$ in the form of $5 \times x \times y$; we get prime factor. we can not further factorize these ie non factorizable in algebraic expression we write non factorizable at the place of prime while $5xy$, like $5(xy)$, $5(y)x$, and $5(xy)$ are non factorizable because (xy) , $(5x)$, $(5y)$ can be refactorize.

What is factorization?

The process of writing an algebraic expression as the product of two or more expressions is called factorization. These factor are algebraic variable or algebraic expression.

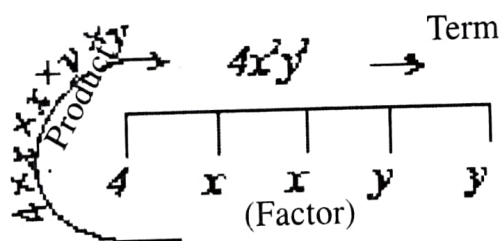
Some algebraic expressions are in the form of product of factor only. On observation its factor can be found.

e.g. $5x(y+3)$ and $7(y+2)(z+9)$

or $5x(y+3) = 5 \times x \times (y+3)$

and $7(y+2)(z+9) = 7 \times (y+2) \times (z+9)$

Here 5 , x and $(y+3)$ are non factorizable factors and $5x(y+3)$ is product. Let see others example $4x^2y^2$

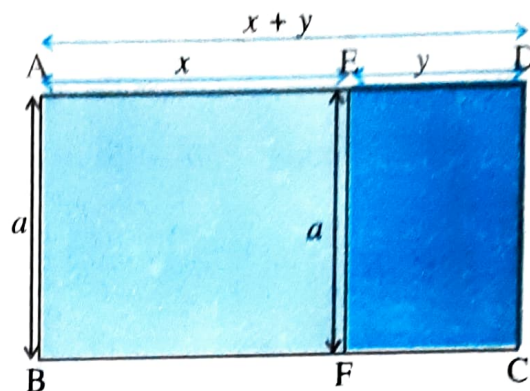


Here $4x^2y^2$ is a monomial expression, in this 4 , x , x , y and y are factors, where as again multiply 4 , x , x , y and y then product $4x^2y^2$ is obtained.

Till now we have learned the factor of expression in which the factor are visible, But there are binomial or polynomial expression in which the factors are not clear. For example $5x+5y$, x^2+3x , x^2+6x+5 etc. For factorization such expressions following methods are used.

9.3 Factor of expression like $(ax + ay)$ (where are given common factor)

There are two parts of rectangle $ABCD$ in figure. whose length AD is divided in two parts AE & ED such that $AE = x$ and $ED = y$ In this way $AD = (x + y)$ breadth of rectangle $ABCD$ is $AB = a$.



So area of rectangle $AEFB$ + area of rectangle $EDCF$ = area of rectangle $ABCD$.

$$\text{i.e. } (AE \times AB) + (ED \times EF) = AD \times AB$$

$$\text{or } x \times a + y \times a = (x + y) a$$

$$\text{or } ax + ay = a (x + y)$$

In this way

factor of $ax + ay$ are a and $(x + y)$

We have read in previous chapter about its inverse operation as multiplication.

$$a (x + y) = ax + ay$$

Learn following example.

Example 1. Factorize $5xy + 10x^2$

Find the Common factor by writing both sides of given expression in non factorizable form

$$\text{Soluton : } 5xy + 10x^2 = 5 \times x \times y + 5 \times 2 \times x \times x$$

Here $5x$ is common factor

$$\begin{aligned} 5xy + 10x^2 &= 5 \times x (y) + 5x \times (2x) \\ &= 5x (y + 2x) \end{aligned}$$

$$\text{Here } 5x (y + 2x) = (5x \times y) + (5x \times 2x)$$

(Distributive law of multiplication over addition)

$$\text{So } 5xy + 10x^2 = 5x (y + 2x)$$

Similarly

$$by + bz = b(y + z)$$

and $cm + cn = c(m + n)$

and $da - db = d(a - b)$

Example 2. Factorize $10xy + 5y$

Solution : $10xy = 5 \times 2 \times x \times y$

$$5y = 5 \times y$$

In Both term common factor is $5y$

$$\begin{aligned} 10xy + 5y &= 5y \times 2x + 5y \times 1 \\ &= 5y(2x + 1) \end{aligned}$$

Note: Here 1 is a factor has to be shown

Example 3. Factorize $ax + ay + az$

Solution : Since in all terms the common factor is a

$$ax + ay + az = (x + y + z)$$

Example 4. Factorize $12a^2b + 15ab^2$

Solution : First method

$$\begin{aligned} 12a^2b + 15ab^2 &= 3 \times 4 \times a \times (ab) + 3 \times 5 \times (ab) \times b \\ &= 3ab \times 4a + 3ab \times 5b \\ &= 3ab(4a + 5b) \text{ (by commutative law)} \end{aligned}$$

Second method : Write non-factorizable multiple factors in each term.

$$\begin{aligned} 12a^2b + 15ab^2 &= 2 \times 2 \times 3 \times a \times a \times b + 3 \times 5 \times a \times b \times b \\ &= 3 \times a \times b(2 \times 2 \times a + 5 \times b) \\ &= 3ab(4a + 5b) \end{aligned}$$

Example 5. Factorize $10x^2 - 18x^3 + 14x^4$

Solution : $10x^2 - 18x^3 + 14x^4 = (2 \times 5 \times x \times x) - (2 \times 3 \times 3 \times x \times x \times x) + (2 \times x \times x \times 7 \times x \times x)$

In above three terms, the common factor are 2 and x^2

$$\begin{aligned} &= (2 \times x \times x) [5 - (3 \times 3 \times x) + 7 \times x \times x] \\ &= 2x^2 \times (5 - 9x + 7x^2) \\ &= 2x^2 \times (7x^2 - 9x + 5) \end{aligned}$$

Try it

Factorize the following expressions

(i) $12x + 36$

(ii) $14pq + 35pqr$

(iii) $3a + ba$

9.3 Factor of expression $a(x + y) + b(x + y)$ (when binomial expression are same)

Expression $a(x + y) + b(x + y)$, (Putting $x + y = p$)

$$\begin{aligned} a(x + y) + b(x + y) &= ap + bp \\ &= p(a + b) && \text{(Replacing value of } p\text{)} \\ &= (x + y)(a + b) \end{aligned}$$

So

$$a(x + y) + b(x + y) = (x + y)(a + b)$$

Example 6. Factorize $5x(y + 9) + 7(y + 9)$

$$\begin{aligned} 5x(y + 9) + 7(y + 9) &= 5xp + 7p && \text{(where } p = (y + 9)\text{)} \\ &= p(5x + 7) \\ &= (y + 9)(5x + 7) && \text{(Replacing value of } p\text{)} \end{aligned}$$

Example 7. Factorize $(x - 3)^2 + 8(x - 3)$

$$\begin{aligned} \text{Solution : } (x - 3)^2 + 8(x - 3) &= p^2 + 8p && \text{(where } x - 3 = p\text{)} \\ &= p(p + 8) \\ &= (x - 3)(x - 3 + 8) && \text{(Replacing value of } p\text{)} \\ &= (x - 3)(x + 5) \end{aligned}$$

Second Method

$$\begin{aligned} (x - 3)^2 + 8(x - 3) &= (x - 3)(x - 3) + 8(x - 3) \\ &= (x - 3)\{(x - 3) + 8\} && \text{(where } x - 3 \text{ is common)} \\ &= (x - 3)(x + 5) \end{aligned}$$

3.2 Factorize $a(x - y) + b(y - x)$ (when common binomial expression are opposite in sign)

Expression $a(x - y) + b(y - x)$ Replacing $(x - y) = p$ and second inverse expression is $(y - x) = -p$

$$\begin{aligned}a(x - y) + b(y - x) &= ap + b(-p) \\&= ap - bp \\&= p(a - b) && \text{(Replacing value of P)} \\&= (x - y)(a - b)\end{aligned}$$

Example 8. Factorize $9(a - 3) + b(3 - a)$

Solution : $9(a - 3) + b(3 - a) = 9p + b(-p)$ (where $p = a - 3$)

$$\begin{aligned}&= 9p - bp \\&= p(9 - b) && \text{(Replacing value of p)} \\&= (a - 3)(9 - b)\end{aligned}$$

Exercise 9 (a)

- Write the common factors following terms :
(i) $7xy, 35x^2y^2$ (ii) $4m^2, 6m^2, 8m^3$ (iii) $3a, 21ab$
- The common factor in expression $7pq + 8qr + 3qs$ will be—
(i) q (ii) r
(iii) $p + q + r$ (iv) $3s$
- The common factor in expression $b(6a - b) + 2c(6a - b)$
(i) b (ii) $2c$ (iii) $(6a - b)$ (iv) $(a - b)$
- Factorize the following-
(i) $5x^2 - 25xy$ (ii) $9a^2 - 6ax$ (iii) $36a^2b - 60a^2bc$
(iv) $6p + 8p^2 - 4p^3$ (v) $3a^2bc + 6ab^2c + 9abc^2$
- Factorize the following-
(i) $x(x - 2) + 3(x - 2)$ (ii) $7(a - 4) + 7(4 - a)$
(iii) $2y(y + 5) - 3(y + 5)$ (iv) $(d - 7)^2 + 7(d - 7)$
(v) $a(a - 5) + 9(5 - a)$ (vi) $(z - 2)^2 - 3(z - 2)$
(vii) $17(a + 3) + 17(3 - a)$

4. Factor of the expression like $ax^2 + ay^2 + bx^2 + by^2$ (By grouping)

Above expressions has four terms i.e. given expression is called four terms or polynomial expression. So no one common factor is in the four terms, but in $ax^2 + ay^2$ First two terms have common factor a and factor in $bx^2 + by^2$ last two term have common factor b

$$\begin{aligned}\text{So } ax^2 + ay^2 + bx^2 + by^2 &= a(x^2 + y^2) + b(x^2 + y^2) \\ &= (x^2 + y^2)(a + b)\end{aligned}$$

It can also be written in other form.

$$\begin{aligned}\text{So } ax^2 + ay^2 + bx^2 + by^2 &= ax^2 + bx^2 + ay^2 + by^2 \\ &= x^2(a + b) + y^2(a + b) \\ &\quad [(a + b) \text{ is common factor}] \\ &= (a + b)(x^2 + y^2)\end{aligned}$$

Example 1. $x^2 + ab + bx + ax$

Solution : In first and third term in expression $x^2 + ax + bx + ab$ the common factor is x and a is common in second and fourth term. The common factor is a show by grouping.

$$\begin{aligned}x^2 + ax + bx + ab &= x^2 + bx + ab + ax \\ &= x(x + b) + a(b + x) \\ &= x(x + b) + a(x + b) \quad \{\because (x + b) = (b + x)\} \\ &= (x + b)(x + a)\end{aligned}$$

Example 2. Factorize $7a^2 - xa^2 + xb^2 - 7b^2$

Solution : In first and fourth term in expression $7a^2 - xa^2 + xb^2 - 7b^2$ the common factor is 7 and x is common in second and third terms. The common factor is a show by grouping.

$$\begin{aligned}7a^2 - xa^2 + xb^2 - 7b^2 &= 7a^2 - 7b^2 - xa^2 + xb^2 \\ &= 7(a^2 - b^2) - x(a^2 - b^2) \\ &= (a^2 - b^2)(7 - x) \\ &\quad [\text{identity } x^2 - y^2 = (x + y)(x - y)] \\ &= (a + b)(a - b)(7 - x)\end{aligned}$$

Example 3. Using factor method find the value of $90 \times 46 + 90 \times 54$

$$\begin{aligned}\text{Solution : } 90 \times 46 + 90 \times 54 &= 90 \times (46 + 54) \\ &= 90 \times 100 \\ &= 9000\end{aligned}$$

Conclusion:

While factorizing a quadrimomial, we divide in two groups such that a factor is common in both groups. This common factor in the groups is taken as one of the factor and other factor is taken by adjusting the rest at appropriate place.

Application

If binomial or polynomial is given as area of rectangle, area of square and area of right triangle then by factorizing of given polynomial we can find their sides.

Example 4. Find the adjacent side of rectangle, whose area is given by $6ab - b^2 - 2bc + 12ac$

Solution : Given area of rectange = $6ab - b^2 - 2bc + 12ac$

To find sides of rectangle factorize the polynomial by grouping method.

Forming the first group $6ab - b^2 = b(6a - b)$... (1)

Forming the second group $12ac - 2bc = 2c(6a - b)$... (2)

By adding equation 1 and 2

$$6ab - b^2 - 2bc + 12ac = b(6a - b) + 2c(6a - b)$$

Common factor $(6a-b)$ are both side

$$\text{So } 6ab - b^2 - 2bc + 12ac = (6a - b)(b + 2c)$$

So sides of rectangle will be respectively $(6a - b)$ and $(b + 2c)$

Exercise 9 (b)

1. Factorize the given expression where as there is one common factor in each term of expression

(i) $x(y - z) + 4(y - z)$

(ii) $2(5 + b) - 7(5 + b)$

(iii) $y(a^2 + x) - (a^2 + x)$

(iii) $a(a + 3) - 5(3 + a)$

2. Find out the factor of the following

(i) $x^3 + 2x^2 + 5x + 10$

(ii) $3ax - 6xy + 8by - 4ab$

(iii) $ax^2 + cx^2 + ay^2 - by^2 - bx^2 + cy^2$

(iv) $ab^2 - (a - c)b - c$

(v) $p^2q - pr^2 - pq + r^2$

3. Factorize the expression $50x^2y + 10y^2x + 30xy + 6y^2$. If $x = 1$, $y = 2$, then find the value of factor given expression.
4. Find out the sides of right triangle whose area is $\frac{b^2 + ab}{2}$

Ability Exercise 9

1. Factorise the following expressions.

(i) $4a - 12$

(ii) $ac - bc + c + cb$

(iii) $36p + 45p^3$

(iv) $y^2 - 8ay$

(v) $7a + 7b$

(vi) $3a^3x - 45a^2x - 18a$

2. Simplify the following by factor

(i) $\frac{3x - 15yx}{5y - 1}$

(ii) $\frac{a^2b + b^2a}{a + b}$

(iii) $\frac{6x^4 + 10x^3 + 8x^2}{2x^2}$

3. Factorize the following expressions

(i) $xy(z^2 + a^2) - x^2za - y^2za$

(ii) $p^3 + p + q - 1 - p^2 - pq$

(iii) $2ab^2 - aby + 2cby - cy^2$

(iv) $x^2 + y^3 + xy(y + 1)$

4. Find the value of the following by using factor

(i) $23 \times 72 + 77 \times 72$

(ii) $56 \times 25 - 25 \times 39 - 25 \times 17$

(iii) $27 \times 47 + 55 \times 8 + 27 \times 53 + 45 \times 8$

5. What should be added in expression $(m^2 - mn + 4m)$ so get common factor $(m - n)$?

6. Fill in the blank :

(i) $ut + (\dots) = (u + at) (\dots)$

(ii) $a^3 \dots ab + b^3 = (\dots) (a^2 - b)$

(iii) $3x^2 + 6x^2y + 9xy^2 = (\dots) (x + 2xy + 3y^2)$

We have learnt:

1. To write down in the form of product of non factorizable algebraic expression of any two or more than two algebraic expression is factorization.

2. Factor of algebraic expression of the following type.

(i) $(ax + ay) = a(x + y)$

$$(ii) ax^2 + ay^2 + bx^2 + by^2 = (x^2 + y^2) (a + b)$$

3. If an algebraic expression is given for the area of a rectangle, then after factorizing that algebraic expression, we get length and breadth of the rectangle.
4. While factorizing a quardinomial we devide it in two groups so that a factor is common in both. This common factor in the group is taken as the factor and other factor in obtain by adjusting the rest at appropriate place.

ANSWERS

Exercise 9 (a)

1. (i) $7xy$, (ii) $2m^2$, (iii) $3a$; 2. (i); 3. (iii); 4. (i) $5x(x - 5y)$, (ii) $3a(3a - 2x)$, (iii) $12a^2b(3 - 5c)$, (iv) $2P(3 + 4p - 2p^2)$, (v) $3abc(a + 2a + 3c)$; 5. (i) $(x - 2)(x + 3)$ (ii) 0, (iii) $(y + 5)(2y - 3)$ (iv) $d(d - 7)$ (v) $(a - 5)(a - 9)$, (vi) $(z - 2)(z - 5)$ (vii) 102

Exercise 9 (b)

1. (i) $(y - z)(x + 4)$, (ii) $-5(5 + b)$, (iii) $(a^2 + x)(y - 1)$, (iv) $(a + 3)(a - 5)$; 2. (i) $(x^2 + 5)(x + 2)$, (ii) $(a - 2y)(3x - 4b)$, (iii) $(a - b + c)(x^2 + y^2)$, (iv) $(b - 1)(ab + c)$, (v) $(pq - r^2)(p - 1)$; 3. $2y(5x + y)(5x + 3)$, 224; 4. $b(a + b)$

Ability Exercise 9

1. (i) $4(a - 3)$, (ii) $c(a + 1)$, (iii) $9p(4 + 5p^2)$, (iv) $y(y - 8a)$, (v) $7(a + b)$, (vi) $3a(a^2x - 15ax - 6)$; 2. (i) $-3x$, (ii) ab , (iii) $3x^2 + 5x + 4$; 3. (i) $(yz - xa)(zx - ay)$, (ii) $(p - 1)(p^2 - q + 1)$, (iii) $(2b - y)(ab + cy)$, (iv) $(x + y)(x + y^2)$; 4. (i) 7200, (ii) 0, (iii) 3500, 5. $-4n$; 6. (i) at^2 , t (ii) $-a^2b^2$, $(a - b^2)$ (iii) $3x$

