

UNIT-8 PRODUCT OF EXPRESSIONS AND IDENTITIES



- Product of expressions
- Identities
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$
- Difference between equation and identity
- Application of identities

8.1 Introduction

We are familiar with algebraic expression $2x$, $x + 5$, $y - 3$, $-3x + 4$, $6y - 12$. In this chapter we will study how algebraic expression are formed and how are they adjusted. Multiplication of monomial with monomial expression; multiplication of binomial with polynomial by row and column method. In algebra expressions are considered as central concept.

8.2 Multiplication of monomials expressions

(1) Multiplication of a number in an algebraic expression.

You think of some situations in which on algebraic expressions is to be multiplied.

Sabeena answered to find cost of any item she has to multiply the quantity of items by the rate of the item.

If 5 kg mangoes are to be purchased at the rate of ₹ x per kg then we have to multiply 5 kg with ₹ x .

$$\begin{aligned}\text{Cost} &= ₹5 \times x \\ &= ₹5x\end{aligned}$$

∴ Cost of 5 kg mangoes will be ₹ $5x$.

Like this

If any rectangle is of length $l = 3$ cm and breath $b = 2$ cm than area of the rectangle will be represented by geometrical figure

$$\begin{aligned}\text{Area} &= \text{length} \times \text{breath} = l \times b \\ &= 3 \text{ cm} \times 2 \text{ cm} \\ &= 6 \text{ cm}^2\end{aligned}$$

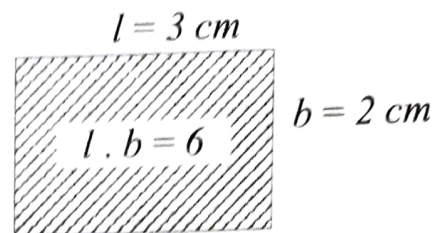


Fig. 8.1

To multiply a monomial by a monomial

$$x \times 3 = x + x + x = 3 \times x = 3x$$

Geometrically if above statement represents a rectangle whose length is 3 unit and breadth x unit then its area is $3x$ unit as shown in figure 8.2

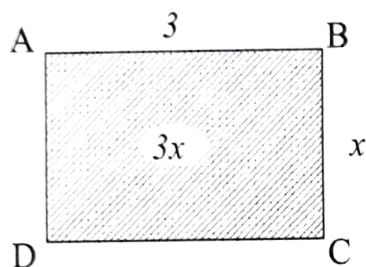


Fig. 8.2

So

- $2x \times 4 = 2x + 2x + 2x + 2x = 4 \times (2x) = 8x$
- $3y \times 5 = 3y + 3y + 3y + 3y + 3y = 5 \times (3y) = 15y$
- $4 \times 2x = (4 \times 2)x = 8x$
- $5 \times 3y = (5 \times 3)y = 15y$

Now note the following products

- (i) $x \times 3y = x \times 3 \times y = 3xy$ (iii) $5x \times (-3y) = 5 \times (-3) \times xy = -15xy$
- (ii) $3x \times 4y = 3 \times 4 \times x \times y = 12xy$

Note that product of monomial expressions also monomial.

Try it:

Multiply the following expressions and represent geometrically.

- (i) $2x \times 6$ (ii) $5 \times 4y$

So we find that

Multiplication of algebraic expression by any numbers we multiply the number by the coefficient of algebraic expression.

(ii) Multiplication of exponential algebraic expressions having same base :

We know that :

$$3x \times x = 3x^2$$

Similarly $4x^2 \times 5x^3 = (4 \times 5)x \times x \times x \times x \times x = 20x^5$

Look the index of the product of two terms having same base is equal to the sum of indices of the terms.

$$\text{i.e. } x^2 \times x^3 = x^5 = x^{2+3}$$

$$y^3 \times y^2 \times y = y \times y \times y \times y \times y \times y = y^6$$

$$\text{or } y^3 \times y^2 \times y = y^{3+2+1} = y^6$$

Similarly see by doing the multiplication of $z^3 \times z^2 \times z^3$

From above we find that

Multiplication of algebraic expressions with same base of is an algebraic expression having same base with index equal to the sum of their indices..

(iii) Multiplication of having different variables monomial expressions

Variable means that its value is not constant

i.e. $x \times y$ is written as xy *i.e.* $x \times y = xy$

Similarly $x \times y \times z = xyz$

and $a \times b \times c = abc$

or $a.b.c = abc$

We do not write (variable) the symbol of multiplication ($\times$) between the letters (variable) in the product. Similarly we do not write the symbol ($\times$) in the product of a constant and variable *i.e.* $2 \times x = 2x$.

Again $y \times x = yx = xy$ (commutative law by multiplication)

$$b \times a = ba = ab$$

$$d \times c = dc = cd$$

Look the following working of multiplying :

$$(i) 5x \times 4y = 5 \times x \times 4 \times y$$

Geometrically it is shown as in figure 8.3

$$\begin{aligned} (5 \times 4) \times (x \times y) \\ = 20 \times xy \\ = 20xy \end{aligned}$$

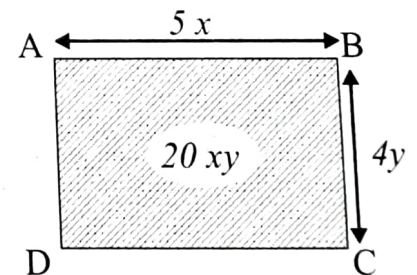


Fig. 8.3

$$\begin{aligned}
 \text{(ii)} \quad 7x \times 3y \times 2z &= 7 \times x \times 3y \times 2z \\
 &= 7 \times 3 \times 2 \times (x \times y \times z) \\
 &= 42 \times xyz \\
 &= 42xyz
 \end{aligned}$$

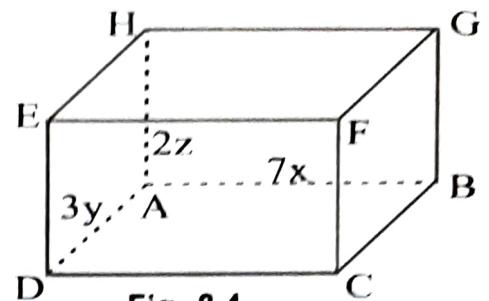


Fig. 8.4

Geometrically it represents a cuboid whose length $AB = 7x$ unit, breadth $AD = 3y$, height $AH = 2z$ in $42xyz$ cubic unit.

$$\begin{aligned}
 4x^2 \times (-2y) \times (-3x) &= 4 \times x^2 \times (-2) \times y \times (-3) \times x \\
 &= 4(-2) \times (-3) \times x^2 \times x \times y \\
 &= (-8) \times (-3) \times x^{2+1} \times y \\
 &= 24x^3y
 \end{aligned}$$

(iii)

Note it :

We know that the measurement is not a negative number so we can not represent geometrically each algebraic coefficient.

Try it :

Similarly

Find out the product of algebraic expressions in the following :

$$\text{(i)} \quad 3z^2 \times 2y^2 \times (-5x^2)$$

$$\text{(ii)} \quad 2x^2 \times (-4x^3) \times (-3y^2)$$

From above examples we see that :

(i) Numerical coefficient of product of algebraic expressions is equal to the product of numerical coefficient in the product of numerical coefficient of the expression.

(ii) Algebraic coefficient in the product of algebraic expressions is equal to the product of variable part in the expression.

Example 1. Multiply $5xy$ by $3x^2$.

Second method (multiplication by column)

Solution : First method (Multiplication by row)

$$3x^2 \times 5xy = 3 \times x^2 \times 5 \times x \times y$$

$$= 3 \times 5 \times x^2 \times x \times y$$

$$= 15 \times x^{2+1} \times y$$

$$= 15x^3y$$

$$\begin{array}{r}
 3x^2 \\
 \times 5xy \\
 \hline
 15x^3y
 \end{array}$$

Example 2. Find out product of $(-4x^2) \times (-3ay) \times \left(-\frac{2}{3}bx\right)$.

Solution :

$$\begin{aligned} (-4x^2) \times (-3ay) \times \left(-\frac{2}{3}bx\right) &= (-4x^2) \times (-3)a \times y \times \left(-\frac{2}{3}\right) \times b \times x \\ &= \left[(-4) \times (-3) \times \frac{-2}{3}\right] \times (x^2 \times a \times y \times b \times x) \\ &= [4 \times (-2)] \times (x^3 \times b \times a \times y) \\ &= -8abx^3y \end{aligned}$$

Example 3. If $x = 1$, $y = 2$. Find the product of $6xy \times \frac{2}{3}x^2$.

$$\begin{aligned} \text{Solution : } 6xy \times \frac{2}{3}x^2 &= \left(6 \times \frac{2}{3}\right) \times x \times y \times x^2 \\ &= 4 \times x \times x^2 \times y \\ &= 4 \times x^{1+2} \times y \\ &= 4x^3y \end{aligned}$$

On substituting the value of x and y

$$\begin{aligned} 4x^3y &= 4 \times 1^3 \times 2 \\ &= 4 \times 1 \times 2 \\ &= 8 \end{aligned}$$

Exercise – 8(a)

1. Find the value of following :

- (i) $4x \times (-7x)$
- (ii) $(-6x) \times (5x^2)$
- (iii) $3x^2y \times 7xy^2$

2. Find the products.

- (i) $4x^2 \times 3x^5$
- (ii) $3y^2 \times 5y^3 \times y$
- (iii) $5xy \times (-3x)$
- (iv) $(-4a^2y) \times (-bx^2)$
- (v) $(-3x^2y) \times (-5xy^2)$
- (vi) $\left(-\frac{3}{8}x^4yz\right) \times (-16yz^2)$
- (vii) $(2p) \times (-3q) \times (-4pq)$

3. Find the value of following product :

(i) $x^2 \times 7x^5 \times \frac{1}{7}x^3 \times (-6x)$ if $x = 1$

(ii) $2x \times (-10xy^2) \times 3x^2y$ if $x = 1, y = 2$,

4. In a garden there are $2x$ flower beds. Each flower bed has xy rows and each row containing y^2 rose plants. Find—

(i) How many plants are there in the garden.

(ii) If $x = 3, y = 2$ then find total number of plants?

8.3. Multiplication of monomial and polynomial expression

We know that

$$5 \times 16 = 5(10 + 6)$$

$$= 5 \times 10 + 5 \times 6$$

$$= 50 + 30 \quad \text{and}$$

$$= 80$$

$$7 \times 28 = 7 \times (30 - 2)$$

$$= 7 \times 30 - 7 \times 2$$

$$= 210 - 14$$

$$= 196$$

We use **distribution law** in similar calculation—

(i) $5 \times (x + 4) = 5 \times x + 5 \times 4$
 $= 5x + 20$

(ii) $8 \times (3x + 2y) = 8 \times 3x + 8 \times 2y$
 $= 24x + 16y$

(iii) $2x \times (4x - 3y) = 2x \times 4x - 2x \times 3y$
 $= 8x^2 - 6xy$

(iv) $3x \times (5x^2 - 4y) = 3x \times 5x^2 - 3x \times 4y$
 $= 15x^3 - 12xy$

Try it :

Find the product of the following expressions :

(i) $5y(3x^2 - 2z)$

(ii) $2y(5x + 2x^2)$

We have seen from above examples—

For multiplication polynomial expression by a monomial each term of the polynomial is multiplied by the monomial.

Geometrical representation and verification $a(x + y) = ax + ay$ the product a and $(x + y)$ is shown in following figure 8.5.

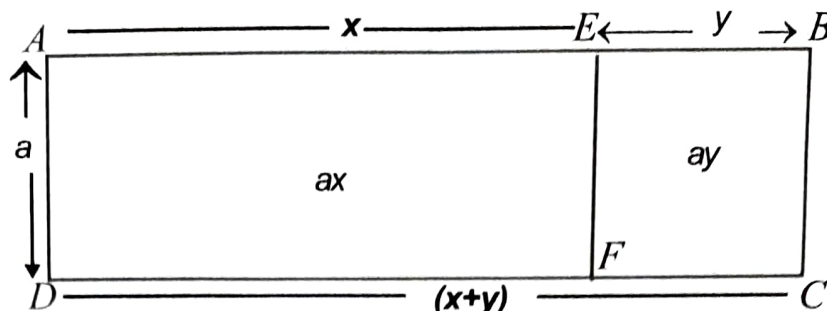


Fig. 8.5

Area of rectangle $ABCD$ = area of rectangle $AEFD$ + Area of rectangle $EBCF$

$$a(x + y) = ax + ay$$

Example 1. Multiply $5xy$ and $3x^2 + xy$.

Solution : First method (row method)

$$\begin{aligned} 5xy \times (3x^2 + xy) &= (5xy \times 3x^2) + (5xy \times xy) \\ &= (5 \times 3)x \times y \times x^2 + 5x \times y \times x \times y \\ &= 15x^{1+2}y + 5x^{1+1}y^{1+1} \\ &= 15x^3y + 5x^2y^2 \end{aligned}$$

Second method (column method)

$$\begin{array}{r} 3x^2 + xy \\ \times 5xy \\ \hline 15x^3y + 5x^2y^2 \end{array}$$

Example 2 : Simplify $x(y - z) + y(z - x) + z(x - y)$.

$$\begin{aligned} \text{Solution : } & x(y - z) + y(z - x) + z(x - y) \\ &= xy - xz + yz - yx + zx - zy \\ &= xy - xz + yz - xy + xz - yz \\ &= xy - xy - xz + xz + yz - yz = 0 \end{aligned}$$

Exercise – 8(b)

1. Multiply:

(i) $(-a - b)$ and $-x$

(ii) $2y$ and $(y^2 + 5y)$

(iii) $(5a - 7b + c)$ and $3y$

(iv) $3x$ and $(x^2 - 5x + 4)$

2. Simplify:

(i) $2x^2y(x - y + z)$

(ii) $3xy^2(2x - 3xy + 7y)$

(iii) $(y^2 - 8y)(-y)$

(iv) $2ab(5a - 7b + c)$

3. Simplify:
- $2x(3x + 5y) - 5y(2x - 3y)$
 - $x(y - z) + 2y(z - x) + z(x - y)$
 - $y^2(y^2 + 1) - y^3(y + 1) + y(y^2 - y)$
 - $x(1 + x^2) - x^2(x - 1) - (x + x^2)$
4. There are $2x$ classes in a school. Number of students in each class is $(x^3 + 2x + 2)$. Find this:
- How many students in the school?
 - If $x = 3$ then how many of student are there the school?
5. The speed of a train is $(2x^2 + x + 4)$ km/hour:
- How much distance it covers in $3x$ hours?
 - If $x = 5$ then find the distance travelled by train in above time.
6. In a Nyay Panchayat there are $(2x + 3)$ village committee. Each village committees has $(x^2 + 5x + 6)$ hand pumps _____. Find the total number of hand pumps in the Nyay Panchayat.
7. A Block has $(5x + 3)$ schools due to cleaners in the school the strength is increased by $(5x - 3)$ girls then how many girls are increased in the block.
8. On the occassion of children day student of $x + 5$ schools planted trees. Each school student planted $x^2 - 5x + 25$ trees then how many trees where planted by the students?

8.4 Multiplication by polynomial

We know that,

$$(a + b) \times p = ap + bp$$

$$\text{if } p = (c + d)$$

$$\text{so } (a + b) \times (c + d) = a(c + d) + b(c + d)$$

$$\text{Similarly } (x + y) \times (z + 2) = \overset{ac}{x(z + 2)} + \overset{ad}{y(z + 2)} + bc + bd$$

$$\text{and } (2x + 3)(y - z) = \overset{xz}{2x(y - z)} + \overset{yz}{3(y - z)} + 2y$$

$$\text{Try it : } = 2xy - 2xz + 3y - 3z$$

Simplify the following:

$$(i) (p + q) \times (r + y) \quad (ii) (2a - b) \times (3c + 2d)$$

Conclusion:

Two polynomials use multiply each term of first polynomial by each term of second polynomial.

Geometrical representation and verification : $(x + a)(x + b) = x^2 + ax + bx + ab$

Area of rectangle $ABCD = (x + a)(x + b)$

Area of rectangle $AMND = x(x + a)$

Area of rectangle $MBCN = b(x + a)$

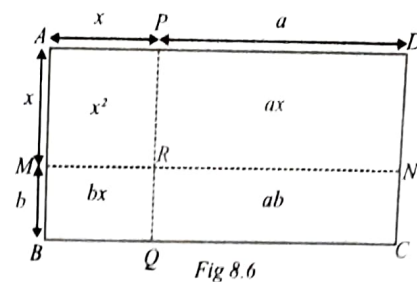
again rectangle $ABCD$

= rectangle $AMND$ + rectangle $MBCN$

= Square $AMRP$ + Rectangle $PRND$ + Rectangle $MBQR$ + Rectangle $RQCN$

$$\therefore (x + a)(x + b) = x(x + a) + b(x + a)$$

$$= x^2 + ax + bx + ab$$



Example 1. Multiply $(2x + 3y)$ and $(3x - 4y)$.

Solution :

Row Method

$$(2x + 3y)(3x - 4y)$$

$$= 2x(3x - 4y) + 3y(3x - 4y)$$

$$= 6x^2 - 8xy + 9xy - 12y^2$$

$$= 6x^2 + xy - 12y^2$$

Column Method

$$2x + 3y$$

$$\times (3x - 4y)$$

$$6x^2 + 9xy \quad (\text{multiply by } 3x)$$

$$- 8xy - 12y^2 \quad (\text{multiply by } -4y)$$

$$\hline 6x^2 + xy - 12y^2$$

Example 2. Multiply $(2x^2 + y^2)$ by $(3x - 5y^2)$. If $x = 2$, $y = 1$, so verify the product.

$$(2x^2 + y^2) \times (3x - 5y^2) = 2x^2(3x - 5y^2) + y^2(3x - 5y^2)$$

$$\text{Solution :} \quad = 6x^3 - 10x^2y^2 + 3xy^2 - 5y^4$$

Verification substitution $x = 2$, $y = 1$

$$(2x^2 + y^2) = 2 \times 2^2 + 1^2 = 8 + 1 = 9$$

$$(3x - 5y^2) = 3 \times 2 - 5 \times 1^2 = 6 - 5 = 1$$

$$\text{Left side} = (2x^2 + y^2) \times (3x - 5y^2) = 9 \cdot 1 = 9$$

$$\text{Right side} = 6x^3 - 10x^2y^2 + 3xy^2 - 5y^4$$

$$= 6 \times 2^3 - 10 \times 2^2 \times 1^2 + 3 \times 2 \times 1^2 - 5 \times 1^4$$

$$= 48 - 40 + 6 - 5$$

$$= 54 - 45$$

$$= 9$$

∴ Left side = Right side

Example 3. Multiply $(5x^2 - 6x + 9)$ by $(2x - 3)$.

$$\begin{array}{r} 5x^2 - 6x + 9 \\ \times 2x - 3 \\ \hline 10x^3 - 12x^2 + 18x \end{array}$$

Solution :

$$\begin{array}{r} -15x^2 + 18x - 27 \\ \hline 10x^3 - 27x^2 + 36x - 27 \end{array}$$

Exercise – 8(c)

Find the product

1. $(x + 2)(x + 5)$
2. $(x - 4)(x + 7)$
3. $(x - 3)(x - 8)$
4. $(x^2 + 5)(x^2 - 7)$
5. $(3x + 8)(4x - 7)$
6. $(5x - 3y)(3x + 4y)$
7. $(x^2 + 2xy + y^2)(x - y)$
8. $(2x^2 + 3x - 7)(5x + 4)$
9. Multiply $(x^2 - xy + y^2)$ by $(x + y)$. If $x = 3$, $y = 2$ then verify the answer.
10. Multiply $(x^2 + xy + y^2)$ by $(x^2 - xy + y^2)$. If $x = 2$, $y = 1$ verify the answer.
11. Kavita purchased $(3x + 7)$ copies from a book seller. If each copy cost rupees $(2x - 1)$, then :
 - (i) How much is the cost of total copies.
 - (ii) If $x = 5$ then, how much money kavita gave to the book seller.

8.5 Identities :

1. Difference between equation and Identity

Discuss and bring out the conclusion—

For variable x in algebraic statement $2x = x + 3$ different value statement observation the table.

x	(L.H.S.) $2x$	(R.H.S.) $x + 3$
1	2	4
2	4	5
3	6	6
4	8	7

- (i) What is the value of left hand side for $x = 1$.

(ii) If the value of left side is 8, then find the value of x .

(iii) How much value of right side for $x = 5$.

Such equality indicator algebraic statements which satisfied by certain values of variable (x) are called equation and thus fixed value of variable in called solution of the equation.

No in the algebraic statement $x(x + 1) = x^2 + x$ observe the table in both the sides.

For a variable x in the algebraic equation $2x = 3x$, observe the difference in the values of R.H.S. & L.H.S. of the table

x	(L.H.S.) $x(x + 1)$	(R.H.S.) $x^2 + x$
1	2	2
2	6	6
3	12	12
4	20	20

(i) What is the value of L.H.S. when $x = 1$?

(ii) For what value of x the value of both sides is 20?

(iii) What is the value of right side for $x = 3$?

(iv) For what value is x , the value both sides are not equal? Here observe that left side = right side for each value of x .

Such equality indicats algebraic statement which always hold for each values of the variable in called indentity.

Example 1. Check whether $x^2 + 3x - 5 = 1$. is an identify or not.

Solution : By $x = 0$ Right side $= x^2 + 3x - 5$
 $= 0 + 0 - 5$
 $= -5$

Right side $= 1$

\ By $x = 0$ L.H.S. $\neq$ R.H.S.
so $x^2 + 3x - 5 = 1$ is not an identity.

Exercise – 8(d)

1. Choose out the equation and identity and write in your exercise book from the following:

(i) $x + 1 = 4$ (ii) $2(x + 1) = 2x + 2$ (iii) $3x = 2x + x$ (iv) $2x - 1 = x$

2. Show that :

(i) $3x(x + 1) = 3x^2 + 3x -$ is an identity.

(ii) $x^2 - 1 = 8$ is not an identity.

(iii) $2x(x + 3) = 2x^2 + 6x$ is an identity.

(iv) $5x - 1 = 9$ is an equation.

8.5.1 (i) Identity $(a + b)^2 = a^2 + 2ab + b^2$

We know that $(a + b)^2 =$ multiplication of $(a + b)$ by $(a + b)$

Multiplying by row wise

Multiplying column wise

$$\begin{aligned} (a + b)^2 &= (a + b) \times (a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2 \end{aligned}$$

$$\begin{array}{r} a + b \\ \times (a + b) \\ \hline a^2 + ab \\ + ab + b^2 \\ \hline a^2 + 2ab + b^2 \end{array}$$

Numerical verification :

Let $a = 2$, $b = 3$ then

$$L.H.S. = (a + b)^2 = (2 + 3)^2 = (5)^2 = 25$$

$$\begin{aligned} R.H.S. &= a^2 + b^2 + 2ab = 2^2 + 3^2 + 2 \times 2 \times 3 \\ &= 4 + 9 + 12 \\ &= 25 \end{aligned}$$

$$\therefore L.H.S. = R.H.S.$$

Geometrical representation and verification :

In figure 8.7 $ABCD$ is square whose side $= a + b$

$$\therefore \text{Area of square } ABCD = (a + b)^2$$

$$\text{Area of square of } AEFM = a^2$$

$$\text{Area of rectangle } EBHF = ab$$

$$\text{Area of rectangle } MFGD = ab$$

$$\text{Area of square } FHCG = b^2$$

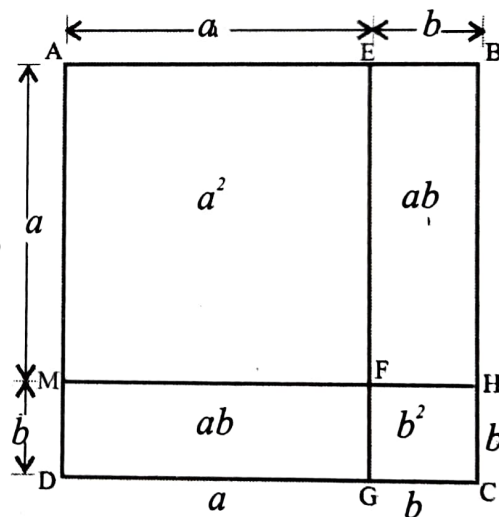


Fig 8.7

$\therefore$ Area of square $ABCD =$ Area of square $AEFM +$ Area of rectangle $EBHF +$ Area of rectangle $MFGH +$ Area of square $FHCG$

$$\therefore (a + b)^2 = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

$$\boxed{(a + b)^2 = a^2 + 2ab + b^2}$$

$$(\text{first term} + \text{second term})^2 = (\text{first term})^2 + 2(\text{first term}) \times (\text{second term}) + (\text{second term})^2$$

From above it is observed that :

The square of sum of two terms is equal to sum of the square of the given terms and adding twice of their product.

Example 1. Find the value of $(x + 5)^2$.

Solution : We know that $(a + b)^2 = a^2 + 2ab + b^2$

Substituting $a = x$ and $b = 5$

$$\begin{aligned}(x + 5)^2 &= x^2 + 2x \times 5 + 5^2 \\ &= x^2 + 10x + 25 \\ &= x^2 + 10x + 25\end{aligned}$$

Example 2. Find the value of $\left(x + \frac{1}{x}\right)^2$.

Solution : We know that $(a + b)^2 = a^2 + 2ab + b^2$

Substituting $a = x$ and $b = \frac{1}{x}$

$$\begin{aligned}\left(x + \frac{1}{x}\right)^2 &= x^2 + 2 \cdot x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 \\ &= x^2 + 2 + \frac{1}{x^2}\end{aligned}$$

$$\therefore \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

Exercise – 8(e)

Using the identity $(a + b)^2 = a^2 + 2ab + b^2$, find the value of the following :

1. $(x + 3)^2$ 2. $(2x + 1)^2$ 3. $(3x + 2)^2$
4. $(2x + y)^2$ 5. $(2y + z)^2$ 6. $(2 + x)^2$

7. In a garden there are $(x + 2y)$ row and each row has $(x + 2y)$ trees then find :

(i) How many trees are there in the garden.

(ii) If $x = 3, y = 2$ then find the number of tree in the garden.

8. The length of side of square shaped field is $(3x + y)$ metre. Find the area of field.

8.5.2. (ii) Identity $(a - b)^2 = a^2 - 2ab + b^2$

We know that $(a - b)^2 =$ multiplication of $(a - b)$ by $(a - b)$

Row method

$$\begin{aligned}(a - b)^2 &= (a - b) \times (a - b) \\ &= a(a - b) - b(a - b) \\ &= a^2 - ab - ba + b^2 \\ &= a^2 - ab - ab + b^2 \\ &= a^2 - 2ab + b^2\end{aligned}$$

Column method

$$\begin{array}{r}a - b \\ \times a - b \\ \hline a^2 - ab \\ - ab + b^2 \\ \hline a^2 - 2ab + b^2\end{array}$$

$$\therefore (a - b)^2 = a^2 - 2ab + b^2$$

Numerical verification :

Let $a = 5, b = 3$, so

$$L.H.S. = (a - b)^2 = (5 - 3)^2 = (2)^2 = 4$$

$$\begin{aligned}R.H.S. &= a^2 - 2ab + b^2 \\ &= 5^2 - 2 \times 5 \times 3 + 3^2 \\ &= 25 - 30 + 9 \\ &= 34 - 30 = 4\end{aligned}$$

$$\therefore L.H.S. = R.H.S.$$

Similarly, $(a - b)^2 = (a - b)(a - b)$

$$= a(a - b) - b(a - b)$$

$$= a^2 - ab - ba + b^2$$

$$= a^2 - 2ab + b^2, \because ab = ba$$

$$\text{So, } (a - b)^2 = a^2 - 2ab + b^2$$

So it is an identity

Try it :

Verify that $(a - b)^2 = a^2 - 2ab + b^2$ by $a = 4$, $b = 1$. We see that $(a - b)^2 = a^2 - 2ab + b^2$ is true for each value of a and b .

Geometrical representation and verification of this identity :

In figure 8.8

$$\text{Area of square } AMFE = (a - b)^2$$

$$\text{Area of square } ADCB = a^2$$

$$\text{Area of rectangle } MDCH = ab$$

$$\text{Area of rectangle } FHIG = ab$$

$$\text{Area of square } EBIG = b^2$$

$$\text{Square } AMFE = \text{square } ADCB + \text{square } EBIG - \text{rectangle } MDCH - \text{rectangle } FHIG$$

$$\begin{aligned} \therefore (a - b)^2 &= a^2 + b^2 - ab - ab \\ &= a^2 - 2ab + b^2 \end{aligned}$$

$$\therefore (a - b)^2 = a^2 - 2ab + b^2$$

$$(\text{first term} - \text{second term})^2 = (\text{first term})^2 - 2(\text{first term}) \times (\text{second term}) + (\text{second term})^2$$

It is observed from above that

Square of the difference of two terms = (sum of the squares of the given terms) - (twice their product)

Example 1. Find the value of $(x - 7)^2$.

Solution : We know that $(a - b)^2 = a^2 - 2ab + b^2$

Putting $a = x$ and $b = 7$ in it

$$\begin{aligned} (x - 7)^2 &= x^2 - 2 \times x \times 7 + 7^2 \\ &= x^2 - 14x + 49 \end{aligned}$$

Example 2. Find the value of $\left(x - \frac{1}{x}\right)^2$.

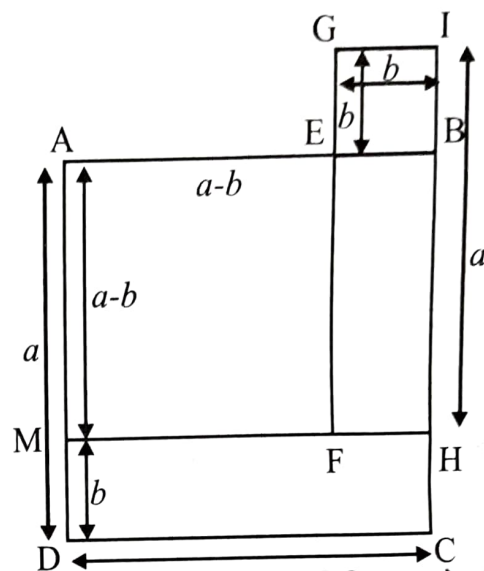


Figure. 8.8

Solution : We know that $(a - b)^2 = a^2 - 2ab + b^2$

Putting $a = x$ and $b = \frac{1}{x}$

$$\begin{aligned}\left(x - \frac{1}{x}\right)^2 &= x^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 \\ &= x^2 - 2 + \frac{1}{x^2}\end{aligned}$$

$$\therefore \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

Exercise – 8(f)

Using the identity $(a - b)^2 = a^2 - 2ab + b^2$, find the value of :

1. $(x - 5)^2$ 2. $(5x - 7)^2$ 3. $(2x - y)^2$
4. $(3x - 2y)^2$ 5. $(3 - x)^2$ 6. $(2y - z)^2$
7. The length of one side of the square field is $(3x - y)$ m. Find the area of the field.
8. In a field there are $(x - 2y)$ row. In each row there are $(x - 2y)$ papaya plants.
 - (i) How many papaya plants are there in the field?
 - (ii) If $x = 10$, $y = 1$, then the total number of plants?
9. (i) The value of a pen is $(2x - y)$ rupees. find the value $(2x - y)$ pens of this type.
 - (ii) If $x = 6$, $y = 2$, then how much will be the value of pens.

8.5.3 Identity $(a + b)(a - b) = a^2 - b^2$

Row method

$$\begin{aligned}(a + b) \times (a - b) &= a(a - b) + b(a - b) \\ &= a^2 - ab + ba - b^2 \\ &= a^2 - ab + ba - b^2 \\ &= a^2 - b^2\end{aligned}$$

Column method

$$\begin{array}{r}a + b \\ \times a - b \\ \hline a^2 + ab \\ - ab - b^2 \\ \hline a^2 - b^2\end{array}$$

$$\therefore (a+b)(a-b) = a^2 - b^2$$

Numerical verification

Let $a = 4$, $b = 1$, so

$$\begin{aligned} L.H.S. &= (a+b)(a-b) \\ &= (4+1)(4-1) = 5 \times 3 = 15 \end{aligned}$$

$$\begin{aligned} R.H.S. &= a^2 - b^2 \\ &= 4^2 - 1^2 = 16 - 1 = 15 \end{aligned}$$

$$\therefore L.H.S. = R.H.S.$$

Now you verify $(a+b)(a-b) = a^2 - b^2$ for $a = 3$, $b = 2$.

We see that $(a+b)(a-b) = a^2 - b^2$ is true for each value of a and b .

So this is an identity.

Geometrical representation and verification :

Activity :

Let the side of $ABCD$ be a and the side of square $HMCI$ be b .

So if we remove the square $HMCI$ from the square $ABCD$ then figure $ABMHID$ is formed whose area is $a^2 - b^2$, of these rectangle $GHID$ those length $a - b$ and breadth b is cut and adding to the side of AG in the form of $EAGF$ rectangle $EBMF$ is formed.

Whose area is $(a+b)(a-b)$

$\therefore$ Area of rectangle $EMBF$ = Area of square $ABCD$ - Area of square $HMCI$

$$\therefore (a+b)(a-b) = a^2 - b^2$$

From above it is observed that:

The product of the sum of terms and their difference is equal to the difference between the squares of those terms.

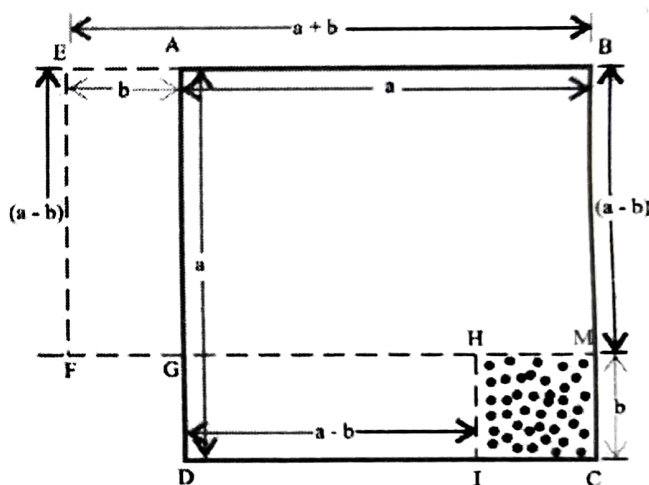


Figure 8.9

Note that:

$$\begin{aligned}(x+a)(x+b) &= x(x+b) + a(x+b) \\ &= x^2 + x.b + ax + b^2 = x^2 + x(b+a) + b^2 \\ &= x^2 + x(a+b) + b^2 \quad (\because b+a = a+b)\end{aligned}$$

Example 1. Using identity $(a+b)(a-b) = a^2 - b^2$ Find the product of $(x+5)(x-5)$.

Solution : We know that $(a+b)(a-b) = a^2 - b^2$

Substituting $a = x$ and $b = 5$

$$\begin{aligned}(x+5)(x-5) &= x^2 - 5^2 \\ &= x^2 - 25\end{aligned}$$

Example 2. Use identity $(a+b)(a-b) = a^2 - b^2$ to find the product of $(3x+2y) \times (3x-2y)$.

Solution : We know that $(a+b)(a-b) = a^2 - b^2$

Substituting $a = 3x$ and $b = 2y$

$$\begin{aligned}(3x+2y)(3x-2y) &= (3x)^2 - (2y)^2 \\ &= 9x^2 - 4y^2\end{aligned}$$

Exercise – 8(g)

Using identity $(a+b)(a-b) = a^2 - b^2$, find the product of:

- $(5+x)(5-x)$
- $(y+4)(y-4)$
- $(2x+3)(2x-3)$
- $(3x+4y)(3x-4y)$
- Using the identity, find the value of $(4x+2y)(4x-2y)$ and check the answer, if $x = 2$, $y = 1$.
- The length of a rectangular field is $(2x+1)$ m and the breadth is $(2x-1)$ m. Find the area of the rectangle.
- (i) The value of a book is $(3x+1)$ rupees. Similarly, find the value of $(3x-1)$ books.
(ii) If $x = 3$, then what is the number of books?
(iii) If $x = 4$, then how much money does it cost to buy books?

Using identity find the value of the following :

8. $153^2 - 147^2$

9. $(12.1)^2 - (7.9)^2$

10. $(983)^2 - (17)^2$

8.6 Application of identities

The use of identity provides also simple alternative method to multiply a two term expression and a number. Using identities it become easy to find the product of some big number.

It becomes easy to find the product of some number by using identities.

Example 1. Using $(a + b)^2 = a^2 + 2ab + b^2$ find the value of 101^2 .

$$\begin{aligned}\text{Solution : } (101)^2 &= (100 + 1)^2 \\ &= 100^2 + 2 \times 100 \times 1 + 1^2 \\ &= 10000 + 200 + 1 \\ &= 10201\end{aligned}$$

Example 2. Using $(a - b)^2 = a^2 - 2ab + b^2$. Find the value of 99^2 .

$$\begin{aligned}\text{Solution : } 99^2 &= (100 - 1)^2 \\ &= 100^2 - 2 \times 100 + 1^2 \\ &= 1000 - 200 + 1 \\ &= 9800 + 1 \\ &= 9801\end{aligned}$$

Example 3. Using $(a + b)(a - b) = a^2 - b^2$. Find the produce of 303 and 297.

$$\begin{aligned}\text{Solution : } 303 \times 297 &= (300 + 3) \times (300 - 3) \\ &= 300^2 - 3^2 \\ &= 90000 - 9 \\ &= 89991\end{aligned}$$

Example 4. If $a + \frac{1}{a} = 2$ then find the value of $a^2 + \frac{1}{a^2}$.

Solution : $\because (a + b)^2 = a^2 + 2ab + b^2$

$$\begin{aligned}\therefore \left(a + \frac{1}{a}\right)^2 &= a^2 + 2 \times a \times \frac{1}{a} + \left(\frac{1}{a}\right)^2 \\ &= a^2 + 2 + \frac{1}{a^2}\end{aligned}$$

$$\begin{aligned}a^2 + \frac{1}{a^2} + 2 &= \left(a + \frac{1}{a}\right)^2 \\ \therefore & \\ &= 2^2 = 4\end{aligned}$$

$$\therefore a^2 + \frac{1}{a^2} = 4 - 2 = 2$$

Exercise – 8(h)

Using identities find the value of following :

1. 51^2
2. 105^2
3. 201^2
4. 302^2
5. 1001^2
6. 49^2
7. 98^2
8. 95^2
9. 997^2
10. $(10.2)^2$
11. $(9.8)^2$
12. 103×97
13. 52×48
14. 10.5×9.5

15. If $a + \frac{1}{a} = \frac{5}{2}$ so find the value of $a^2 + \frac{1}{a^2}$.

16. If $a - \frac{1}{a} = 0$ so, find the value of $a^2 + \frac{1}{a^2}$.

Using identities find the value of the following :

17. 78×82
18. 95×103
19. 501×502

Ability Exercise – 8

Find the product of the following :

1. $(2x^2y) \times (-3xy^2)$
2. $(11y^2z^2), (5xyz)$
3. $x^6 \times 5x^3 \times 2y^2$
4. $7 \times (x + 3)$
5. $-6 \times (-m - 3)$
6. $ab(3a - 5b)$
7. $(2x + 3)(x + 5)$
8. $(x - 7)(x + 9)$
9. $(5x - 6y)(4x - 3y)$
10. $(x + a)(x + b)$

11. Check the following for identity :

(i) $(x + y)^2 = x^2 + 2xy + y^2$

(ii) $(x - y)^2 = x^2 - 2xy + y^2$

Using identities find the values of the following :

12. $(2x + 5)^2$

13. $(2x + 3y)^2$

14. $(x - 7)^2$

15. $(3m - 4n)^2$

16. $(mx - ny)^2$

17. $(7 + 3x)(7 - 3x)$

18. $(4a + 3b)(4a - 3b)$

19. Simplify the following :

(i) $a(b - c) + b(c - a) + c(a - b)$

(ii) $x(2y - 3z) + y(3z - 2x) + 3z(x - y)$

20. If $x + \frac{1}{x} = \frac{10}{3}$ then find the value of $x^2 + \frac{1}{x^2}$.

21. If $x - \frac{1}{x} = \frac{3}{2}$ then find the value of $x^2 + \frac{1}{x^2}$.

22. Show that :

(a) $\left(\frac{4}{3}m + \frac{3}{4}n\right)^2 - 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$

(b) $(4mn + 3n)^2 - (4mn - 3n)^2 = 48mn^2$

(c) $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

We have learned the following in this unit :

1. Expressions are formed with the use of variables and constants.
2. To multiply a number by a single variable algebraic expression, multiply the number with the coefficient of algebraic expression.
3. The product of the algebraic expressions with same is equal to the sum of their exponents of the same basis.
4. The coefficient of product of algebraic expressions is equal to the product of coefficient of expressions and variable parts of the product of algebraic expression it equal to the product the variable part of the expression.
5. To multiply an expressions with many terms from one term expression, we multiply each term of the expression.
6. In order to multiply two polynomial expressions, each of the first polynomials is mulplied by each term of the second polynomial.
7. The multiplication of expression can be under stood geometrically.
8. Difference in equation and identity :
 - * Such expression on statements which are satisfied for certain value of variable x are called equations and the fixed value of variable is the solution of the equation.
 - * The expression on statement which are true for each value for the variable is called identity.

9. Identity $(a + b)^2 = a^2 + 2ab + b^2$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

Verification of the above identities and verification by geometric representation.

10. The identity provides as simple alternation methods to find the product of numbers.
11. Generally an expresion which one or more term in which each term has non-zero coefficient and power of the expression is non-negative is called polynomial.
12. Similar terms are formed by similar variable and power of these variables are same.
13. The coefficient of the equal terms need be not necessarily equal.
14. We always get monomial when multiply monomial to a monomial.

ANSWERS

Exercise – 8(a)

1. (i) $-28x^2$, (ii) $-30x^3$, (iii) $21x^3y^3$; 2. (i) $12x^7$, (ii) $15y^6$, (iii) $-15x^2y$, (iv) $4a^2bx^2y$, (v) $15x^3y^3$, (vi) $6x^4y^2z^3$, (vii) $24p^2q^2$; 3. (i) $-6x^{11}$, -6 (ii) $-60x^4y^3$, -480 ; 4. (i) $2x^2y^3$, (ii) 144

Exercise – 8(b)

1. (i) $ax + bx$, (ii) $2y^3 + 10y^2$, (iii) $15ay - 21by + 3cy$, (iv) $3x^3 - 15x^2 + 12x$; 2. (i) $2x^3y - 2x^2y^2 + 2x^2yz$, (ii) $6x^2y^2 - 9x^2y^3 + 21xy^3$, (iii) $-y^3 + 8y^2$, (iv) $10a^2b - 14ab^2 + 2abc$; 3. $6x^2 + 15y^2$, (ii) $-xy + zy$, (iii) 0, (iv) 0; 4. (i) $2x^4 + 4x^2 + 4x$, (ii) 210; 5. (i) $6x^3 + 3x^2 + 12x$ km (ii) 885 km; 6. $2x^3 + 13x^2 + 27x + 18$; 7. $25x^2 - 9$; 8. $x^3 + 125$

Exercise – 8(c)

1. $x^2 + 7x + 10$; 2. $x^2 + 3x - 28$; 3. $x^2 - 11x + 24$; 4. $x^4 - 2x^2 - 35$; 5. $12x^2 + 11x - 56$; 6. $15x^2 + 11xy - 12y^2$; 7. $x^3 + x^2y - xy^2 - y^3$; 8. $10x^3 + 23x^2 - 23x - 28$; 9. $x^3 + y^3$; 10. $x^4 + x^2y^2 + y^4$; 11. (i) Rs. $(6x^2 + 11x - 7)$, (ii) Rs. 198

Exercise – 8(d)

1. (i) Equation, (ii) Identity, (iii) Identity, (iv) Equation

Exercise – 8(e)

1. $x^2 + 6x + 9$; 2. $4x^2 + 4x + 1$; 3. $9x^2 + 12x + 4$; 4. $4x^2 + 4xy + y^2$; 5. $4y^2 + 4yz + z^2$; 6. $4 + 4x + x^2$; 7. (i) $x^2 + 4xy + 4y^2$, (ii) 49; 8. $(9x^2 + 6xy + y^2)$ sq. m

Exercise – 8(f)

1. $x^2 - 10x + 25$; 2. $25x^2 - 70x + 49$; 3. $4x^2 - 4xy + y^2$; 4. $9x^2 - 12xy + 4y^2$; 5. $9 - 6x + x^2$; 6. $4y^2 - 4yz + z^2$; 7. $9x^2 - 6xy + y^2$; 8. (i) $x^2 - 4xy + 4y^2$, (ii) 64; 9. (i) $(4x^2 - 4xy + y^2)$ Rupees, (ii) 100 Rupees

Exercise – 8(g)

1. $25 - x^2$; 2. $y^3 - 16$; 3. $4x^2 - 9$; 4. $9x^2 - 16y^2$; 5. $16x^2 - 4y^2$; 6. $(4x^2 - 1)$ square meter; 7. (i) $(9x^2 - 1)$ Rupees, (ii) 8, (iii) 143 Rupees; 8. 1800; 9. 84. 10. 966000

Exercise – 8(h)

1. 2601; 2. 11025; 3. 40401; 4. 91204; 5. 1002001; 6. 2401; 7. 9605; 8. 9025; 9. 994009; 10. 104.04; 11. 96.04; 12. 9991; 13. 2496; 14. 99.75; 15. $\frac{17}{4}$; 16. 2; 17. 6369; 18. 9785; 19. 251502

Ability Exercise – 8

1. $-6x^3y^3$; 2. $55xy^3z^3$; 3. $10x^9y^2$; 4. $7x + 21$; 5. $6m + 18$; 6. $3a^2b - 5ab^2$; 7. $2x^2 + 13x + 15$; 8. $x^2 + 2x - 63$; 9. $20x^2 - 39xy + 18y^2$; 10. $x^2 + ax + bx + ab$; 12. $4x^2 + 20x + 25$; 13. $4x^2 + 12xy + 9y^2$; 14. $x^2 - 14x + 49$; 15. $9m^2 - 24mn + 16n^2$; 16. $m^2x^2 - 2mnxy + n^2y^2$; 17. $49 - 9x^2$; 18. $16a^2 - 9b^2$; 19. (i) 0, (ii) 0; 20. $\frac{82}{9}$; 21. $\frac{17}{4}$

