



- The concept of the Pythagoras theorem
- Verification of the Pythagoras theorem
- Pythagorean Triplets
- Construction of triangles on the basis of different conditions.
- Median, centroid of triangle, altitude and ortho centre of the triangle.
- Perpendicular bisector and circum centre of the triangle.
- Angle bisectors and Incentre of the triangle.
- Properties of similar triangle
- Construction of similar triangle on the basis of given ratio of sides.

Introduction

You have studied that the triangle is a simple closed figure made of three line-segments. There are three vertices, three sides and three angles in it. The triangles have been classified on the basis of sides and angles. There are three types of triangle on the basis of sides equilateral triangle, isosceles triangle scalene triangle. There are three types of triangle on the basis of angles acute angled triangle, obtuse angled triangle and right angled triangle. You have also learned the construction of these triangles you have also learn the properties of a right angled triangle. In this unit we will learn about an important property of a right angled triangle known as Pythagoras Theorem.

Unani geometry expert Pythagoras 570–500 discovered very useful and important quality related to a right angled triangle which is known as the Pythagoras theorem. The Indian mathematician Boudhayan (800 BC), discussed this theorem of right angled triangle for a rectangle was almost 300 years ago before pythagoras. It is clear from this that in India this theorem had been in used much earlier.

But what we shall study today this theroem in its present format is known as, Pythagoras theorem.

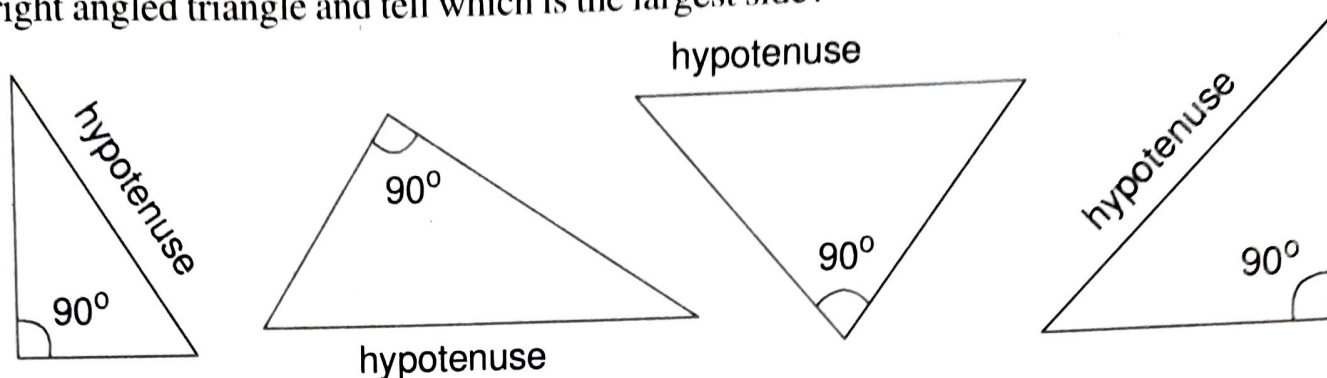
Here we will study the Pythagoras theroem in detail. Along with this, some special construction of triangle, altitude and orthocentre of triangle, median and center of gravity. perpendicular bisector of the length of the sides of the triangle and angle-bisector and incentre of a triangle. In addition, we will also study about similar triangle.

Do it :

Right angled triangle :

See the following triangles. What type of triangles are on the basis of angles ?

Measure the opposite sides of the right triangle and also measure the remaining two sides of the right angled triangle and tell which is the largest side?



We saw that the (opposite side) of right angled triangle is the largest. This side is known as hypotenuse—

The opposite side of the right angled triangle is called hypotenuse. It is larger than the two sides making right angle.

5.2 Concept of Pythagoras theorem :

Look at both the shaded right-angled triangles ABC and PQR , formed in fig. 5.1 (i) and (ii). In both the triangles the squares are made outward on all sides. These squares are covered by the same type of smaller triangles.

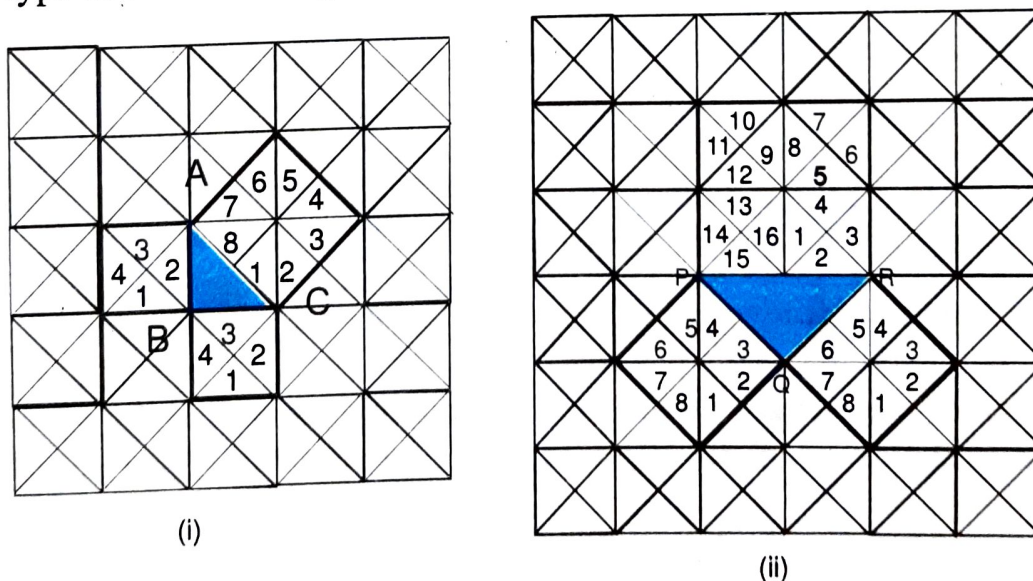


Fig. 5.1

What is the number of small triangles made on the square of hypotenuse AC of triangle ABC ? What is the sum of the number of triangles made on the squares of sides AB and AC of right angle?

We see that the number of small triangles made on the hypotenuse AC of right angle triangle ABC is 8 and the number of triangles made on squares on right angled sides AB and AC is $4 + 4 = 8$.

Likewise the square made on the hypotenuse PR of triangle PQR has 16 triangles and the square made on the sides QR and PR of right angled have $8 + 8$ triangles. Check it that their sum is equal to the 16 triangles made on the hypotenuse.

Experiment 1

Draw a right angled triangle ABC , such that angle BCA is right angle and $BC = 4$ unit, $CA = 3$ unit on measurement hypotenuse $AB = 5$ unit. Draw squares on AB , BC and CA outwards. Divide each squares in unit squares, as shown in the following :

We know that,

$$\text{Area of square} = (\text{side})^2$$

$$\therefore \text{Area of the square made on } AB = 5^2 \text{ square unit} \\ = 25 \text{ square unit}$$

$$\text{Area of the square made on } BC = 4^2 \text{ square unit} \\ = 16 \text{ square unit}$$

$$\text{Area of the square made on } CA = 3^2 \text{ square unit} \\ = 9 \text{ square unit}$$

$$\begin{aligned} \text{The sum of the area of the squares formed by } BC \text{ and } CA \\ &= (16 + 9) \text{ square unit} \\ &= 25 \text{ square unit} \end{aligned}$$

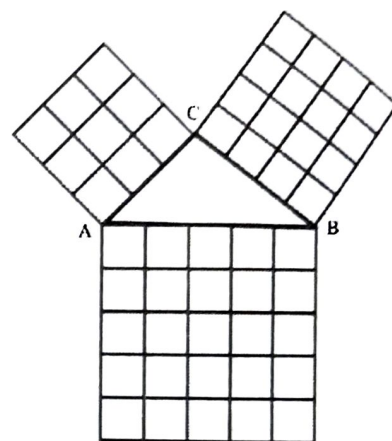


Fig. 5.2

We see that the area of the square formed on the hypotenuse AB is equal to 25 square unit two square made on $BC + CA$ is equal to 25 square unit.

On the basis of above results, we can say that

Square made on the hypotenuse of right angled triangle is equal to the sum of the squares made on the remaining two sides.

This statement is known as Pythagoras Theorem.

The above theorem can also be stated as :

The area of the square formed on the hypotenuse in a right angle triangle is equal to sum of the areas of two squares, made on remaining two sides.

Or

$a^2 + b^2 = c^2$, where c is the hypotenuse of right angled triangle and a and b are the measurement of the remaining sides.

Experiment - 1 : Verification of Pythagoras theorem (by cut and paste method) :

Make a right angle triangle ABC on a thick paper whose $\angle BCA$ right angle. Let's construct square on the outer side of the three sides. Now find the centre of square on BC (the point where its diagonals cuts each other). From this point draw α line parallel to AB and perpendicular lines passing through the same point. Now cut the squares into the direction of these straight lines and write them 1, 2, 3 and 4. Cut these four pieces and the square 5 on the side CA and put on the square made on AB according to the diagram so that the figure 5.3 is adjusted on the vertices of the square made on the hypotenuse of the remaining point will be covered by the square 5 right angle made on AC .

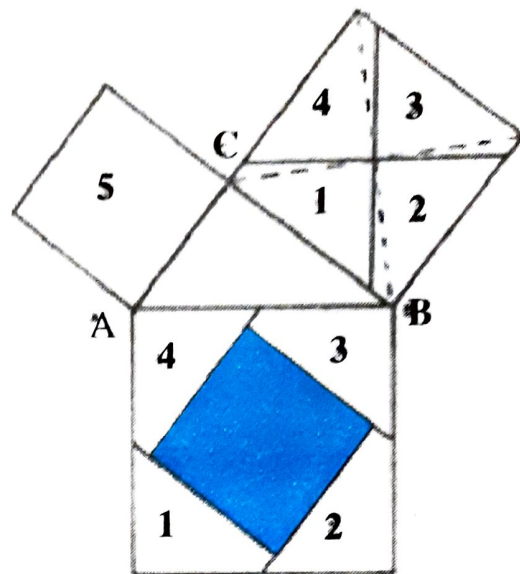


Fig. 5.3

Now it is completely clear from the picture that four pieces of square BC and the square made on the side CA , covered the whole of square made on the diagonal AB . This proves the authenticity of the Pythagoras theorem.

It is evident from the relation $c^2 = a^2 + b^2$ that $c^2 > a^2$ and $c^2 > b^2$

$$\therefore c > a \text{ and } c > b$$

In this way hypotenuse has the largest arm in a right angled triangle.

Experiment 2

Take a thick colored paper and make a right angled triangle of any measurement. Now let's make four models of it. For example, take a right angled triangle. Length of base is ' a ' unit, length of second side is b unit and length of hypotenuse is C unit. Make a square of the arm equal to the measurement of length C unit of the hypotenuse. On the other paper make a square on another coloured shape of the side $(a + b)$ unit.

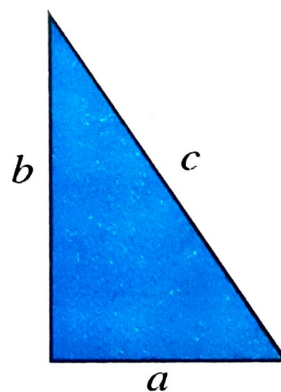


Fig. 5.4

Now arrange the 4 triangles you have made with the square of the side 4 in accordance with the figure 5.5

This $(a + b)$ becomes side of the square.
 Area of the square of side $(a + b)$
 $= 4 \times \text{Area of right angle (whose sides are } a \text{ and } b) + \text{square of the length of side } C.$

$$\begin{aligned}(a + b)^2 &= 4 \times \left(\frac{1}{2} a \times b \right) + c^2 \\(a + b)^2 &= 2ab + c^2 \\a^2 + 2ab + b^2 &= 2ab + c^2 \\a^2 + b^2 &= c^2\end{aligned}$$

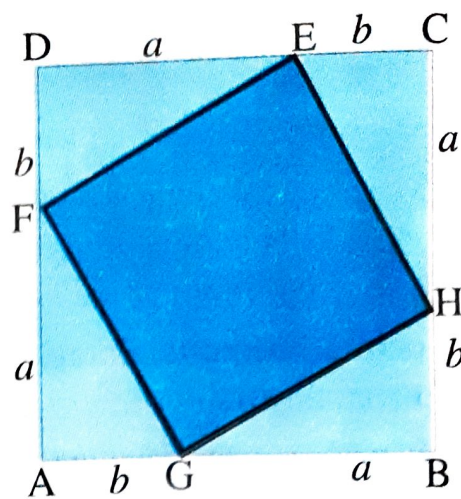


Fig. 5.5

On the basis of above both experiments, can be said :

Area of a square formed on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the remaining two square made on other two sides.

Boudhayan :-

It is not known about birth place of Boudhayan, parent family information. Historian beleve his time period as 800 BC.

First of all, he cited the Baudhayan Shulva Sutra, which is known tody as the Pythagoras Theorem.

This formula is as follows :

The square made on the ankya rope (Karna) (hypotenuse) of the (rectangle) is equal to the sum of the squares formed on Parshwamani (base) and Triyadmani (perpendicular).

5.2.2 Converse of Pythagoras Theorem :

Experiment-3

Cut three square paper of sides 3 cm, 4 cm and 5 cm. Place these three square on one paper, shaking the three heads of these three squares in such a way that a triangle is formed form their sides, accorrding to figure 5.6. Mark the triangle made on the paper. Measure the angles of this triangle. You will see that the only the angle infront of side 5 cm is right angle.

Pay attention to it :

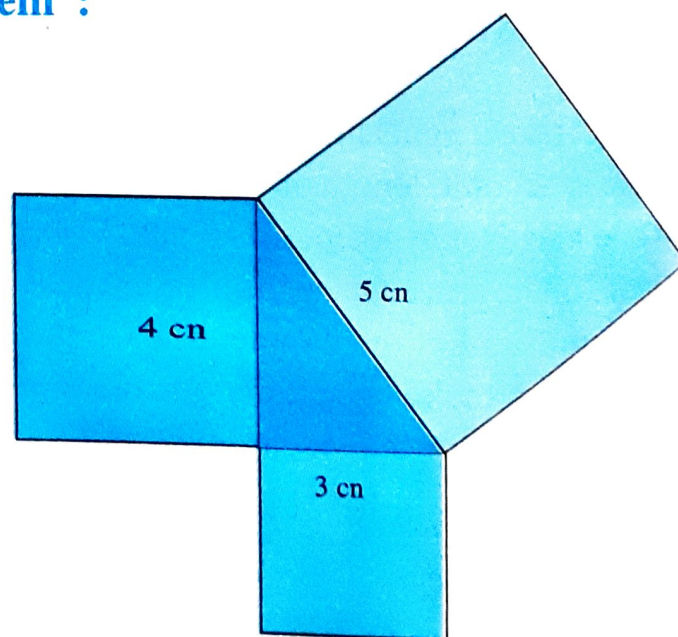


Fig. 5.6

$$3^2 + 4^2 = 5^2, 4^2 + 5^2 \neq 3^2 \text{ and}$$

$$3^2 + 5^2 \neq 4^2$$

Experiment 4

Repeat the above procedure by taking 4 cm, 5 cm and 7 cm as three sides of square. This time you will receive an obtuse angled triangle. Please also note that

$$4^2 + 5^2 \neq 7^2, \quad 7^2 + 5^2 \neq 4^2$$

$$\text{and } 4^2 + 7^2 \neq 5^2$$

$$\text{i.e. } 16 + 25 \neq 49, \quad 49 + 25 \neq 16$$

$$16 + 49 \neq 25$$

$$\text{or } 41 \neq 49, \quad 74 \neq 16$$

$$65 \neq 25$$

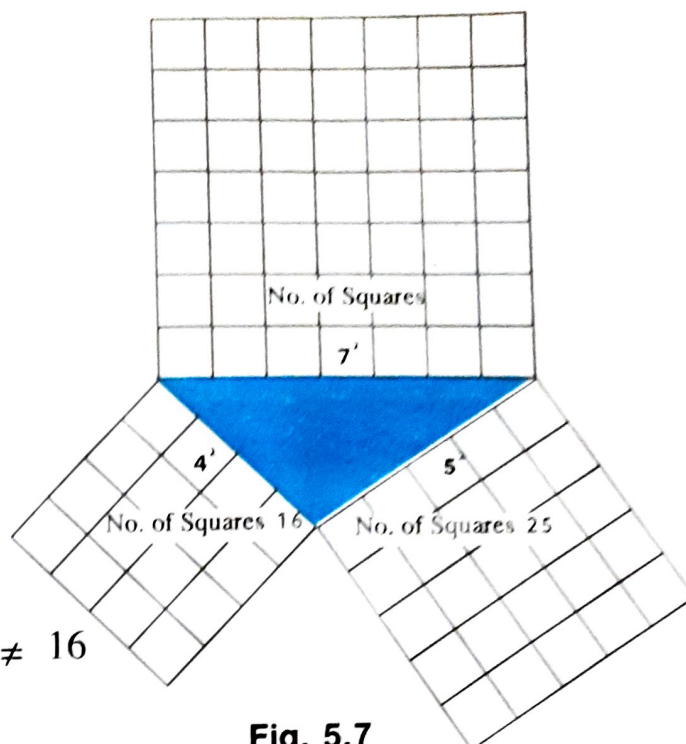


Fig. 5.7

This process shows that the Pythagoras theorem will be applicable only, if the triangle is a right angled triangle.

Similarly, repeat the above process by taking 5 cm, 12 cm and 13 cm as sides of three squares, by above process see whether the Pythagoras theorem is proved or not? (Check it)

Also do this :

- I. Draw a triangle ABC in which $AB = 5$ cm, $BC = 3$ cm and $CA = 4$ cm.

$$5^2 = 3^2 + 4^2$$

$$\text{or } 25 = 9 + 16$$

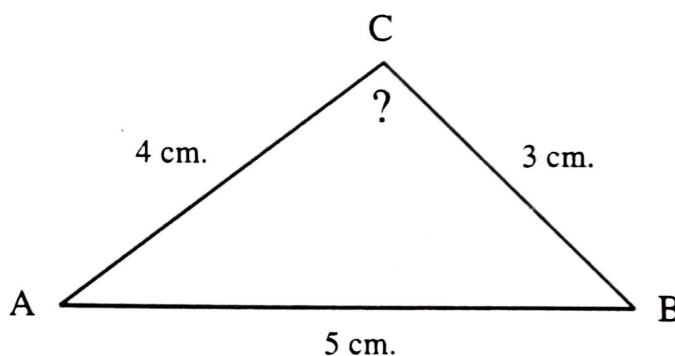


Fig. 5.8

Measure $\angle ACB$ and write its measurement in the empty space. (Fill in the blanks)

$\angle ACB = \dots$

(II). Construct ΔPQR , in which $PQ = 6$ cm,
 $QR = 2.5$ cm and $PR = 6.5$ cm

Here $(2.5)^2 + 6^2 = (6.5)^2$

Measure $\angle PQR$ and write its measurement
 in the empty space. (fill in the blanks)

$\angle PQR = \dots\dots\dots$

(III). Again, do the above experiment with triangle

EFG , in which $FG = 2.1$ cm,

$EG = 7.2$ cm and $FE = 7.5$ cm.

Measure $\angle EFG$ and write the measurement
 in the empty space. (fill in the blanks)

$\angle EGF = \dots\dots\dots$

We have seen that in each case, an angle of
 the triangle is 90.

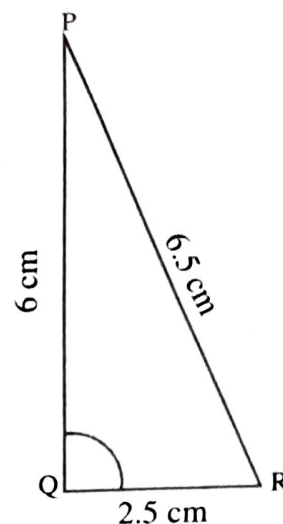


Fig. 5.9

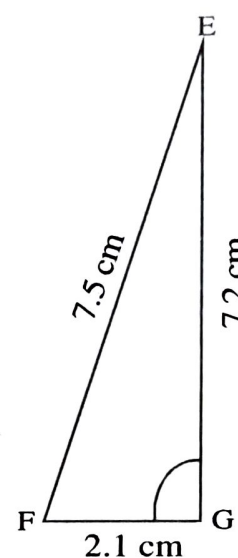


Fig. 5.10

Conclusions

If the square made on one side of the triangle is equal to the sum of the squares made on the remaining two sides, then the triangle formed is a right angle triangle. This is the inverse of Pythagoras theorem.

In any Right angled triangle, hypotenuse is the largest side among any of the other two sides.

Also do it :

- Take a point P outside the line l .

Draw a perpendicular from point P on line l .

From point P draw any other line-segments as PR , PQ , PM , PS and PT , other than this perpendicular.

Which of the line-segment is smallest among these line-segment?

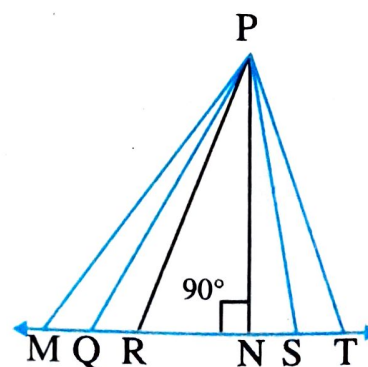


Fig. 5.11

We find out that PN is the smallest line-segment. In this way we can say that from any point outside the line.

Among all line segments drawn from P on the line l the perpendicular is the smallest.

5.3 Pythagorean triplets

If the lengths of the sides of the right angle of a right angled triangle are $2n$ and $n^2 - 1$, then the length of the hypotenuse is $n^2 + 1$. Because

$$(2n)^2 + (n^2 - 1)^2 = (n^2 + 1)^2 \text{ while } n > 1$$

Thus, $2n$, $n^2 - 1$ and $n^2 + 1$ are called Pythagorean Triplets.

Fill blanks in the table of contents :

Side n	BC $n^2 - 1$	AB $2n$	AC $n^2 + 1$	$AC^2 = BC^2 + AB^2$
2	3	4	5	
3	8	6	10	
4	15	8	17	
5	24	10	26	

Example : Show that 8, 9 and 12 are not Pythagorean triplet ?

Solution :

$$8^2 = 64$$

$$9^2 = 81$$

$$8^2 + 9^2 = 64 + 81$$

$$= 145 \quad \dots (i)$$

and $12^2 = 144 \quad \dots (ii)$

On comparing (i) and (ii) $8^2 + 9^2 \neq 12^2$.

Thus 8, 9 and 12 are not Pythagorean triplet.

Some method for finding Pythagorean Triplet :

Consider Pythagorean Triplets 3, 4 and 5.

Here $5^2 = 3^2 + 4^2$ or $25 = 9 + 16 = 25$

Multiply (3, 4, 5) by 2

$3 \times 2, 4 \times 2, 5 \times 2$ That is 6, 8, 10

∴ 6, 8 and 10 are Pythagorean triplet.

Check it :

Solution : $10^2 = 6^2 + 8^2$
 $100 = 36 + 64 = 100$

Thus,

If (a, b, c) are Pythagorean Triplet and k is a positive integer, then ak, bk and ck are also Pythagorean Triplet.

Pythagorash Theorem

Activity

Draw a right angle triangle whose sides perpendicular, base and hypotenuse are 3 cm, 4 cm and 5 cm respectively. Draw equilateral triangles on hypotenuse, perpendicular and base of same lengths of 5 cm, 3 cm and 4 cm. (Fig 5.3)

Area of the equilateral triangle ADC on the hypotenuse of $\triangle ABC$

$$\frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times (5)^2 = \frac{25\sqrt{3}}{4} \text{ cm}^2, \dots\dots\dots (i)$$

Similarly, Area of the equilateral triangles of AFB and BCE on perpendicular and base sides of $\triangle ABC$,

$$\text{Area of } \triangle AFB = \frac{\sqrt{3}}{4} \times (3)^2 = \frac{9\sqrt{3}}{4} \text{ cm}^2 \dots\dots\dots (ii)$$

$$\text{Area of } \triangle BCE = \frac{\sqrt{3}}{4} \times (4)^2 = \frac{16\sqrt{3}}{4} \text{ cm}^2 \dots\dots\dots (iii)$$

Sum of the area of the triangle formed on the base and perpendicular of $\triangle ABC$ is

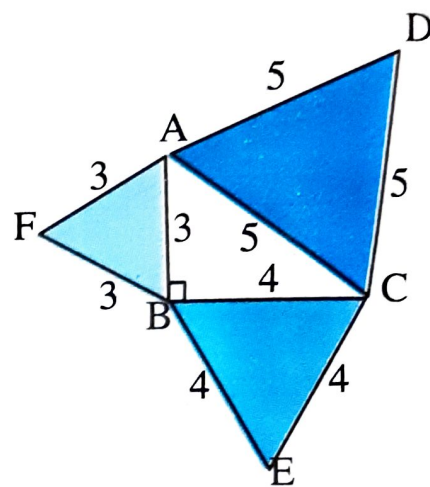
$$\left(\frac{16\sqrt{3}}{4} + \frac{9\sqrt{3}}{4} \right) \text{ cm}^2 = \frac{25\sqrt{3}}{4} \text{ cm}^2 \dots\dots\dots (iv)$$

From equation (i) and (iv) we see that

area of $\triangle ACD$ = area of $\triangle ABF$ + area of $\triangle BCE$.

Therefore, Area of the square on the hypotenuse of $\triangle ABC$ is equal to sum of the area of the square on the base and the area of the square on the perpendicular of $\triangle ABC$.

In similar manner the area of the equilateral triangle on the hypotenuse is equal to the sum of the area of the equilateral triangle on the perpendicular of the triangle $\triangle ABC$.



Conclusion:

Area of the polygon formed on the hypotenuse of $\triangle ABC$ equal to the sum of the area of the polygon formed on the base and the area of the polygon formed on the perpendicular of the right angle triangle ABC.

To find the hypotenuse of right angled triangle

Example 1 : Find the hypotenuse of the right angled triangle whose other two sides are 8 cm and 15 cm.

Solution : Suppose that the hypotenuse of the right angled triangle is 'a' cm.

From Pythagoras theorem,

$$a^2 = 8^2 + 15^2$$

or $a^2 = 64 + 225$

or $a^2 = 289$

or $a^2 = 17^2$

$\therefore a = 17$

$\therefore$ The length of hypotenuse is 17 cm.

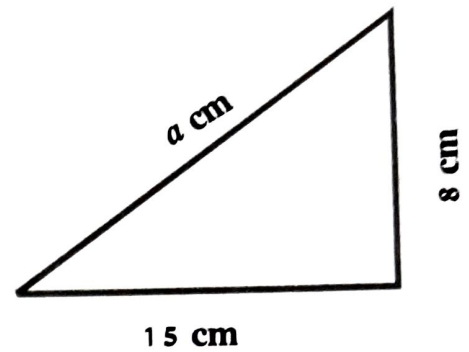


Fig. 5.12

- In the right angled triangle to find a side other than hypotenuse of the two sides.

Example 2 : Find the length of the unknown side y in figure 5.13.

Solution : By Pythagoras theorem,

$$y^2 + 12^2 = 13^2$$

or $y^2 + 144 = 169$

or $y^2 = 169 - 144$

or $y^2 = 25$

or $y = 5$

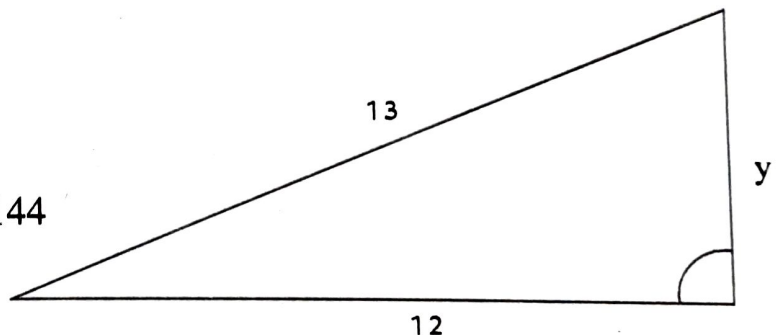


Fig. 5.13

Therefore, the length of unknown side will be 5 cm.

Example 3 : 10 metre long staircase is mounted on a wall at a distance 8 metre from the wall. How long the stair case will reach up the wall?

Solution : Suppose that the staircase reached the wall at a height h m.

By Pythagoras theorem

$$\begin{aligned}
 10^2 &= 8^2 + h^2 \\
 \therefore h^2 &= 10^2 - 8^2 \\
 &= 100 - 64 \\
 h^2 &= 36 \\
 \text{or } h^2 &= 6^2 \\
 h &= 6
 \end{aligned}$$

So, the staircase will reach the wall upto 6 metres high.

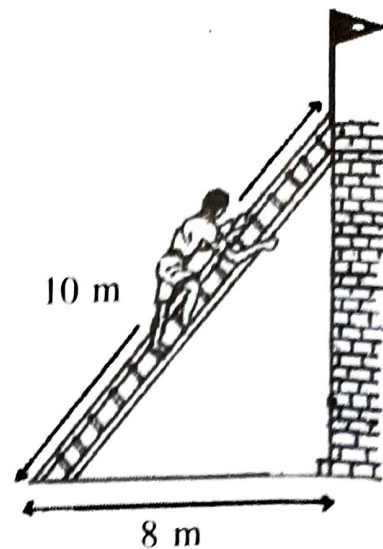


Fig. 5.14

Example 4 : Find out the length of the wire supporting the electric pillar with the help of figure 5.15

Solution : Suppose that the length of the wire is x metre. Since the right angle triangle will be formed in the picture. So from the Pythagoras theorem,

$$x^2 = 4^2 + 3^2$$

$$= 16 + 9$$

$$\text{or } x^2 = 25$$

$$\text{or } x = 5$$

$$\therefore x = 5$$

So the length of the wire is 5 metres.

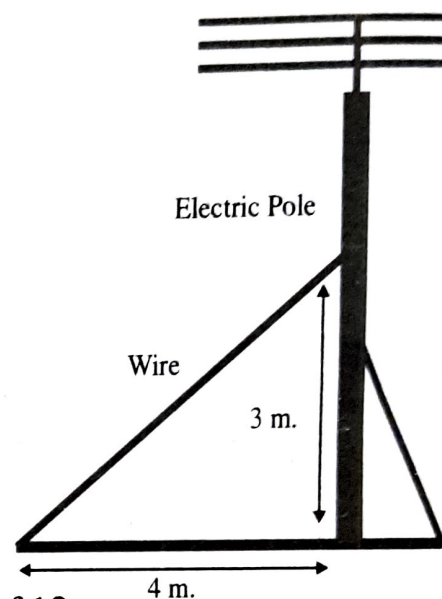


Fig. 5.15

Example 5 : The light pillar is broken by a tree at a height of 12 m from the ground, but two pieces did not separate. In the place where the top of the pillar touch the ground it was 5 metres from tree base. How high was the pillar before breaking?

Solution : Suppose that the part x of the pillar is broken. (let's say that the x -part of the column is broken)

After breaking up, 12 m were saved.

Its upper part is touching the ground 5m away from the pillar. We can display this from the figure ABC. $\angle C$ is a right angle.

$$\begin{aligned}
 \text{So } AB^2 &= AC^2 + BC^2 \\
 x^2 &= 12^2 + 5^2 \\
 &= 144 + 25
 \end{aligned}$$

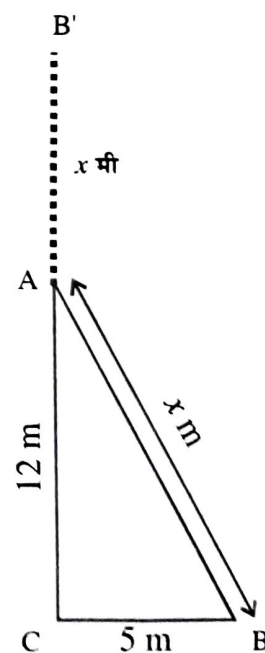


Fig. 5.16

$$x^2 = 169$$

$$x^2 = 13^2$$

$$x = 13$$

The part BA of the figure has got fallen down and in right angle $\triangle ABC$ $AB = x$ represents the hypotenuse.

Thus the total height of the light pillar before breaking

$$= 12 \text{ m} + x \text{ m} = 12 \text{ m} + 13 \text{ m} = 25 \text{ m}$$

Try it

1. Which one will be the right angled of the triangle?

(i) 5 cm, 4 cm, 3 cm

(ii) 6 cm, 8 cm, 7 cm

Exercise – 5(a)

1. In figure 5.17, $\angle B$ is a right angle triangle, the length of side CA will be—

(i) 5 cm

(ii) 10 cm

(iii) 8 cm

(iv) 16 cm

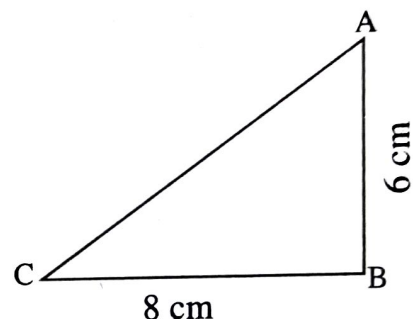


Fig. 5.17

2. In figure 5.18, $\angle A$ is a right angle, then the measure of AB will be :

(i) 25 cm

(ii) 13 cm

(iii) 5 cm

(iv) 12 cm

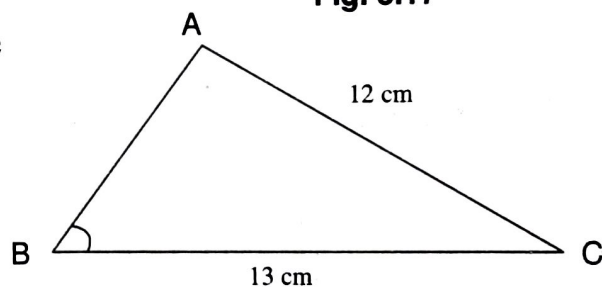


Fig. 5.18

3. See, right angled triangle and fill in the blanks in your exercise book.

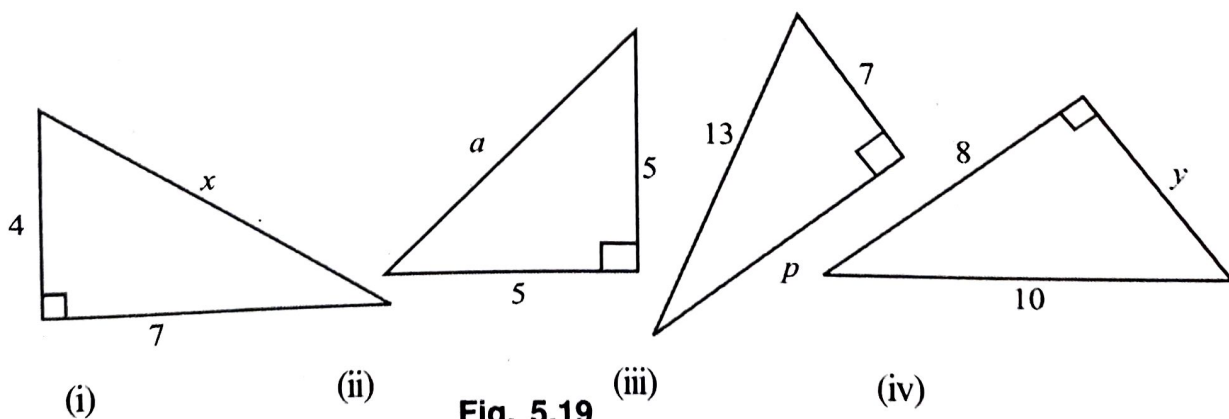


Fig. 5.19

- (i) $4^2 + 7^2 = \dots$ (ii) $5^2 + \dots = a^2$
 (iii) $13^2 - 7^2 = \dots$ (iv) $y^2 = 10^2 - \dots$

4. In $\triangle ABC$, $\angle ABC$ is right angle. If $AB = 7$ cm and $BC = 24$ cm then find the length of CA .

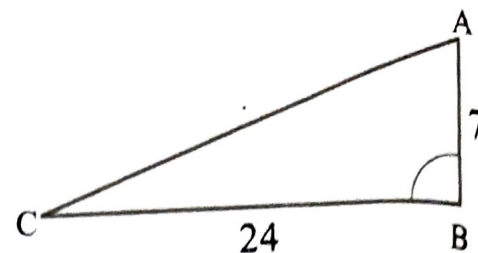


Fig. 5.20

5. In rectangle $ABCD$, diagonal $CA = 20$ cm and $AB = 16$ cm. Find the length of BC .

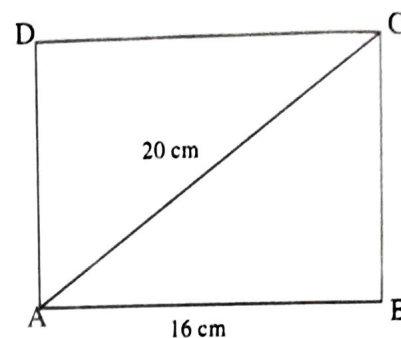


Fig. 5.21

6. A 65 decimetre long ladder has been placed at a distance of 25 decimeter from the wall. Find the height of the upper end of the ladder from the base.
7. There is a wire of 26 metres long. Its one end is tied to the top of the 24 metre high pillar and the other end is grounded in ground. Find the distance between the pillar on the ground and the lower end of the wire.
8. Followings are Pythagorean triplets make three pythagorean triplets from each of them.
- (i) 3, 4, 5 (ii) 5, 12, 13 (iii) 8, 15, 17
9. Sort out the Pythagorean triplets in the followings :
- (i) 5, 12, 13 (ii) 7, 8, 15 (iii) 6, 7, 8
 (iv) 8, 15, 17 (v) 3, 4, 5
10. Verify if n is any odd number, then n , $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ are Pythagorean triplets.

5.5.1 : Altitude of Triangle

In $\triangle ABC$, we can draw several line segments from the vertex A to BC .

Look at these line-segements in the figure 5.22 given below. Notice which of these lines represents the height at the triangle?

Line-segment AL is the height of the triangle.

The line-segment, which is vertical to BC from vertices A and is perpendicular to it, is the height of the triangle. It is also called the (vertical) perpendicular of the triangle.

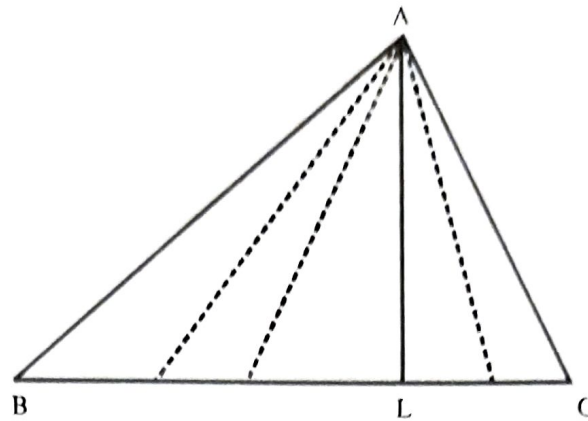


Fig. 5.22

Line segment AL is a altitude of the triangle, the terminal end of the altitude is vertex of the triangle and the other end point is on the opposite side, on which it is perpendicular is falling. Only one altitude can be drawn from each vertices.

Think, discuss and write :

- (i) How many vertices can there be in a triangle?
- (ii) Can you think of such a trinagle, whose two altitudes are its two sides.
- (iii) Does an altitude lie always inside the triangle?

Indication : For questions (ii) and (iii) draw each type of triangle to find it.

Think, Discuss and write :

1. In the ΔPQR , angle P is a right angle. Which is the longest side of the triangle?
2. Which is the longest side in the right angled triangle ?

See these :

In the following figure 5.23, look at (i), (ii) and (iii). In figure (i) of ΔABC , perpendicular AD from vertex A to the side BC . In fig. (ii) of ΔABC perpendicular BE from ventex B to the side AC and in fig. (iii) of ΔABC , perpendicular CF from top C to the side AB .

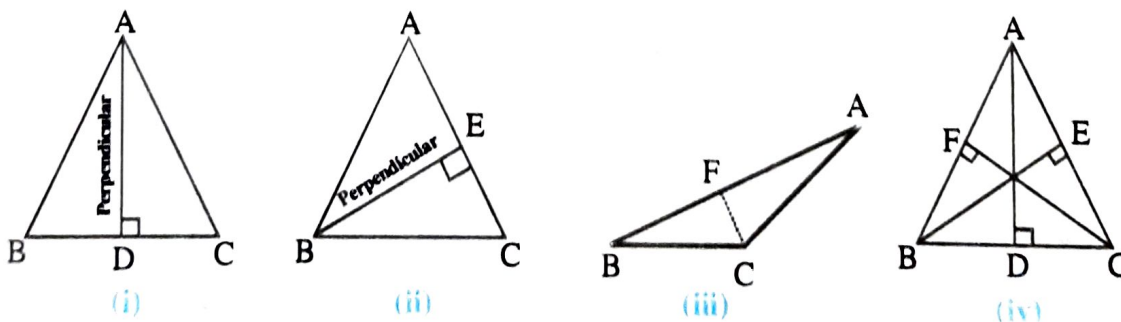


Fig. 5.23

Line-segment AD , BE and CF , are altitude of $\triangle ABC$.

In figure 5.23(iv), how many altitude are there in a triangle ABC ?

We see three altitudes in the triangle, which pass through one point.

The perpendicular line-segments which is drawn from the vertex, on opposite side a triangle, is called the altitude of the triangle. There are three altitudes in the triangle.

Experiment 1

Construct a triangle ABC , Whose sides are 4 cm, 5 cm and 6 cm. Draw the altitudes from the vertices A , B and C of this triangle.

The Steps of Constructions are as follows :

1. Make $\triangle ABC$ from the measurements of given sides.
2. Draw $AL \perp BC$ from vertex A .
3. Draw $BM \perp AC$ from vertex B .
4. Draw $CN \perp AB$ from the vertex C .

Are the three altitudes passing through the same point O ?

We see that all the three altitudes AL , BM and CN of $\triangle ABC$ are passing through the same point O .

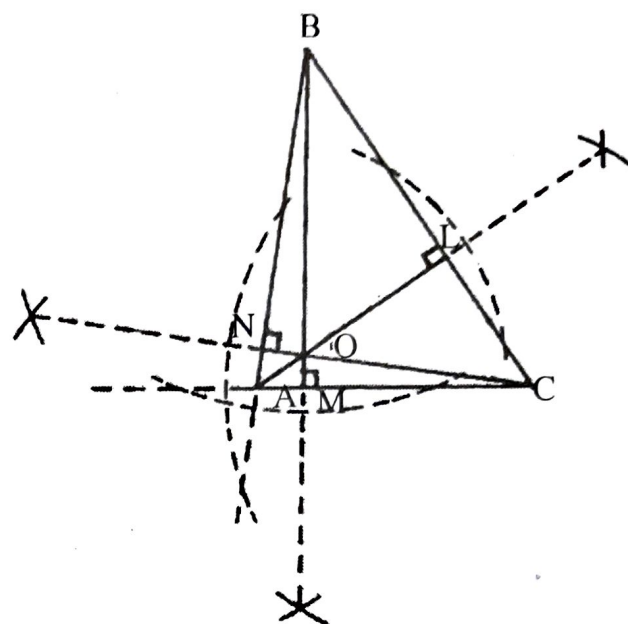


Fig. 5.24

Experiment 2

Draw triangle ABC , Whose sides are 7 cm, 6 cm and 5 cm from the vertices B and C draw altitudes.

The Steps of Constructions are as follows :

1. Draw $\triangle ABC$ according to the given sides.
2. Draw a perpendicular BM from the point B on AC .
3. Draw a perpendicular CN from the point C on AB .
4. Display the point at which both the perpendiculars BM and CN intersect with point O .
5. Join A to O and extend it which cuts BC at the point L .

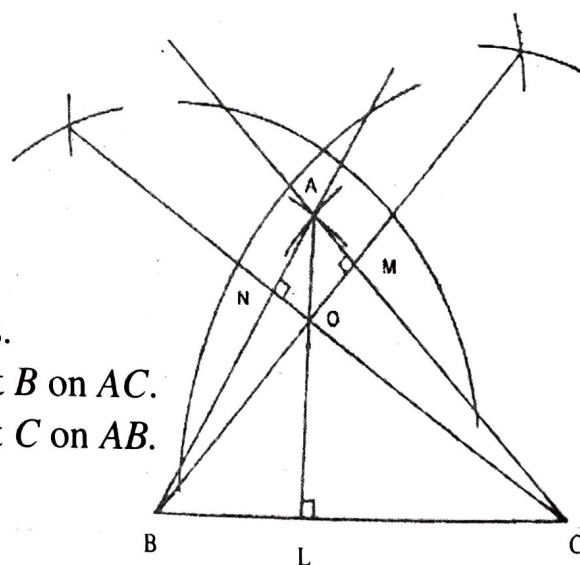


Fig. 5.25

Measure $\angle ALB$. We see that $\angle ALB = 90^\circ$

Thus $AL \perp BC$, which is the third altitude.

We have also seen in earlier that the intersection point of two altitudes is O .

We conclude from the above that :

Altitudes of a triangle are concurrent. The concurrent point of the altitudes of the triangle is called the orthocentre of the triangle.

Try it :

Now draw a triangle. Draw its two altitudes. Join its point of intersection O to third vertices and proceed it till it cut the third side. Now measure the angle formed. We see that the angle is 90° . Thus, these three altitudes pass through the point O .

Note : In order to find out the orthocentre of the triangle, it is sufficient to draw two altitudes, because the third altitude will also pass through the orthocentre.

Exercise – 5(b)

1. Draw altitudes of the triangle by given below sides and angle check it that each triangle has concurrent altitudes :
 - (i) 3.4 cm, 5.4 cm, 4.5 cm
 - (ii) Two sides 6 cm and 4.5 cm and an angle between them 120° .
 - (iii) Two sides 5 cm and 4 cm and an angle between them 90° .
 - (iv) Two angles 65° and 80° and side between them is 5 cm.
2. Draw an isosceles triangle ABC , in which $AB = AC$, draw an altitude AL from the vertex A . By measuring tell that the following statements are true or not?
 - (i) $BL = LC$
 - (ii) $\angle B = \angle C$
 - (iii) $\angle BAL = \angle CAL$
 - (iv) Altitude AL has divided the isosceles $\triangle ABC$ into two congruent triangles.

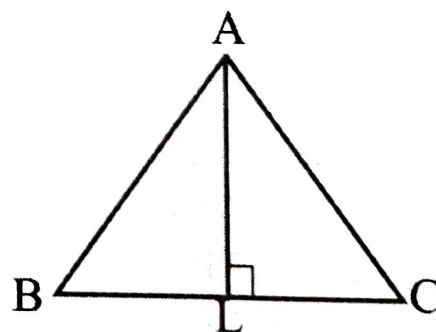


Fig. 5.26

3. In $\triangle ABC$, $\angle C$ is a right angle.
Find orthocentre of this triangle without drawing altitudes.

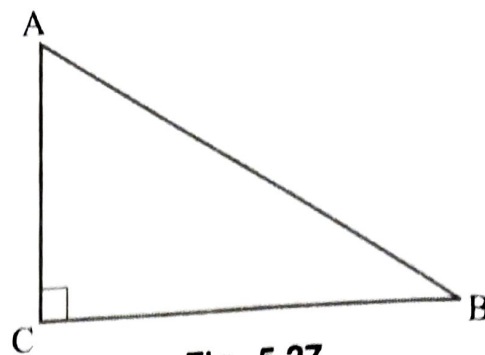


Fig. 5.27

5.5.2 Median of triangle :

Activity : Cut a triangle ABC from a piece of paper. Take a side as in fig. 5.28 BC . Join the point B and C of the triangle, find the perpendicular bisector of BC , by the process of folding paper. The folding of the paper, cuts the side BC at point D , which is its middle point. Join vertex A to D .

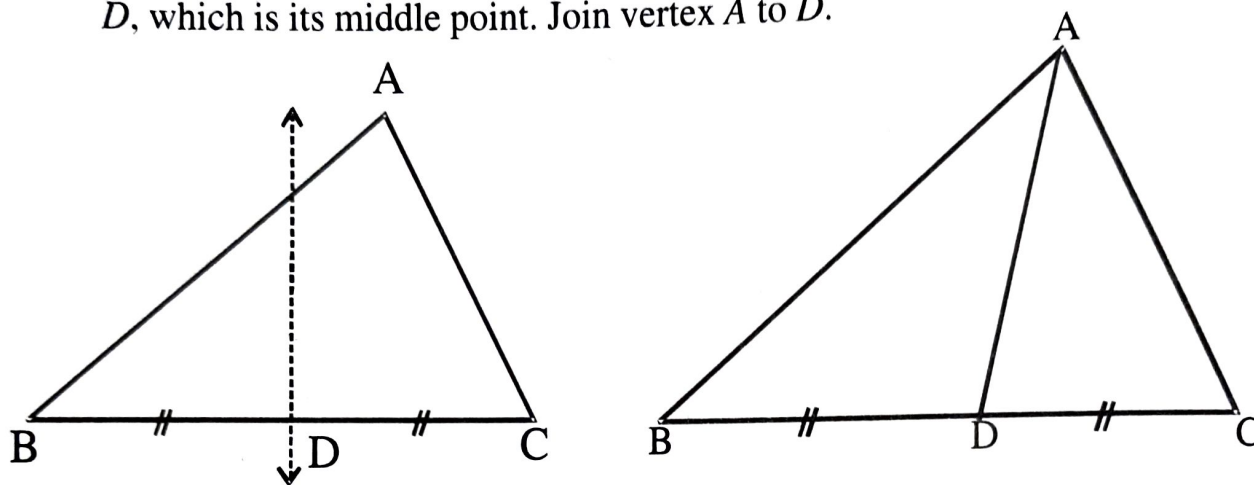


Fig. 5.28

Line-segment AD which join the mid-point D of BC to the vertex A , is a median of the triangle.

By taking sides AB and CA , draw two more medians of this triangle.

Median, joins one vertex of the triangle to the middle point of the opposite side.

Check it :

Look at the following figure 5.29

In figure (i) join vertex A of the triangle to the mid point D of the front side BC .

In figure (ii) join the vertex C to the mid point E of the front side AB .

In figure (iii) join the vertex B to the mid-point F of the front side AC .

In figure (iv) how many medians are in the $\triangle ABC$?

We see that there are three medians in the triangle, which are passing through only one point.

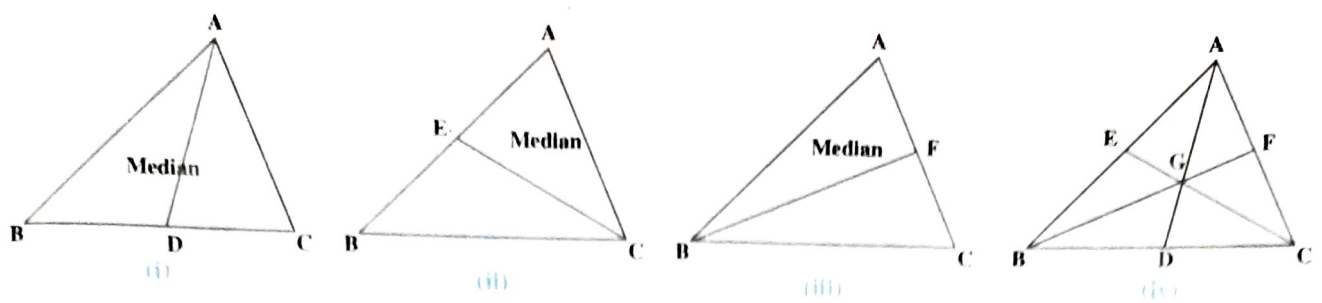


Fig. 5.29

Thus, line segment joining any vertex with the mid point of the (opposite) side of a triangle is called its median.

Experiment 1 :

Draw a ΔPQR whose sides are 5 cm, 6 cm and 7 cm. The mid points of PQ , QR and RP are S , N , M respectively join S , N , M with their front vertices. Tell whether the line segments PN , RS and QM of ΔPQR , are the medians or not? Do all the three medians pass through the same point.

Experiment 2 :

Draw a ΔPQR whose sides are 4 cm, 5 cm and 6 cm. Join the mid point M to PR to the vertex Q .

Join the mid-point S of PQ to R . QM and RS are two medians out of the three medians of the triangle, locate the intersection point of QM and RS as the point G .

Join P and G and extend it. Suppose it cuts (meets) QR at the point N .

Measure QN and NR .

Is $QN = NR$?

We find that $QN = NR$.

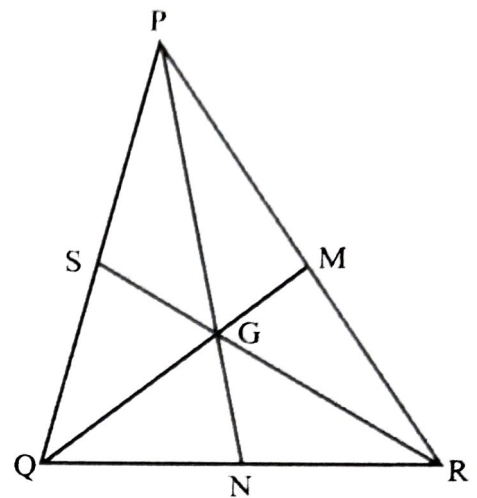


Fig. 5.30

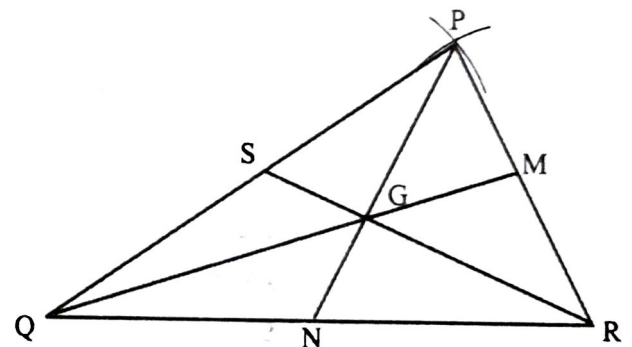


Fig. 5.31

Therefore, N is the mid-point of the side QR and thus PN is the third median of the triangle and all the three medians are concurrent at a point G .

Now make a $\triangle PQR$. Draw its two medians and name its point of intersection as G . Join the third vertex with G extend it and check whether it is the median or not.

We conclude that it is the third median of the triangle.

Therefore,

The medians of triangle are concurrent. The point of intersection of all the three medians of a triangle is called the Centroid of the Triangle.

Note : To locate the centroid of a triangle is sufficient to draw just two medians of the triangle. The point of their intersection is the centroid of the triangle.

Exercise – 5(c)

1. Write down the following statements in your notebook and fill in the blanks therein:
 - (i) A median of a triangle is that line-segment which join any vertex to the of the opposite side.
 - (ii) Medians of a triangle are
 - (iii) Intersection point of the medians of triangles is called
2. Draw an isosceles $\triangle ABC$, in which $AB = AC$. Draw a median AD . Tell by measurement whether the following statements are true or not ?
 - (i) AD is perpendicular to BC ,
 - (ii) AD bisects the $\angle A$.
 - (iii) AD is the perpendicular bisector of BC .
3. AD , BE and CF are the medians of any $\triangle ABC$ and G is its centroid. If $BE = CF$, then what is the type of $\triangle GBC$. Investigate through drawing the triangle.
 - (i) Scalene triangle
 - (ii) Isosceles triangle
 - (iii) Equilateral triangle

Perpendicular bisectors of the sides of a triangle

Do it

Experiment-1

Draw a triangle ABC whose sides are 6 cm, 7 cm and 8 cm respectively. Draw also the perpendicular bisectors of the sides of the triangle.

Construction :

- (i) Construct $\triangle ABC$ with the given measurements.

- (ii) Draw the perpendicular bisector l of the side AB .
- (iii) Draw the perpendicular bisector m of the side AC .
- (iv) Draw the perpendicular bisector n of the side BC .

Are all the three perpendicular bisectors are passing through the point O , as in above figure?

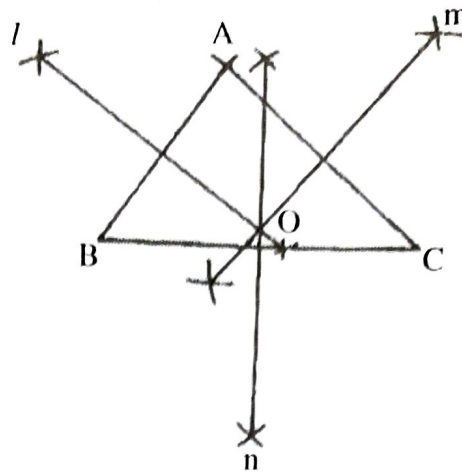


Fig. 5.32

Experiment 2 :

Draw a $\triangle ABC$ whose sides are 5 cm, 6 cm and 7 cm. Draw also its perpendicular bisectors of sides.

Constructions :

- (i) Construct $\triangle ABC$ with the given measurements.
- (ii) Draw the perpendicular bisector l of the side AB .
- (iii) Draw the perpendicular bisector m of the side AC .
- (iv) Name the point of intersection of the perpendicular bisectors l and m as O .
- (v) Draw $n \perp BC$ at O which cuts BC at D .
- (vi) Measure BD and CD and whether $BD = CD$?

We see that $BD = CD$.

Thus, n is the perpendicular bisector of BC .

Thus, Point O is the common point of the three perpendicular bisectors of the three sides of the triangle.

Now draw a triangle ABC . Draw perpendicular bisectors of sides AB and AC . Name the point of intersection of both perpendicular bisectors as O . Draw a perpendicular from O on BC and check whether it is the perpendicular bisectors of the BC or not.

We see that it is also a perpendicular bisector passing through O i.e. all the three perpendicular bisectors are concurrent.

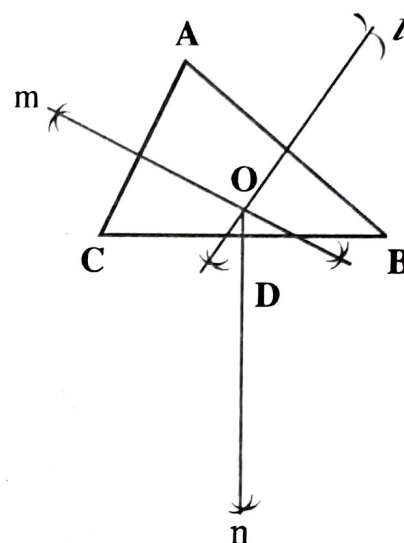


Fig. 5.33

Perpendicular bisectors of a triangle are concurrent. The point of intersection is known as the circum centre of the triangle.

Note : In order to find the circum centre of a triangle it is sufficient to draw just two perpendicular bisectors of the triangle.

Exercise – 5(d)

1. Draw a perpendicular bisector l of the line-segment AB . Take any point P on l and measure PA and PB .
2. Draw a right angled triangle ABC , in which $\angle C$ is a right angle. Find the mid point O of AB . Draw a circle of radius OA with centre O . Does it pass through C ? In context of ΔABC , what the point O is called for .
3. Draw perpendicular bisectors of sides of the triangle by drawing triangle with given sides and angle given below check it whether they are concurrent.
 - (i) 7 cm, 5 cm and 4 cm
 - (ii) 6 cm and angle made on side are 90° and 30° .
 - (iii) 4.5 cm, 2.5 cm, angle between them is 105° .

5.5.4 Angle bisectors of a triangle

Activity

Experiment -1 :

Draw a ΔABC with sides 6 cm, 7 cm and 8 cm. Draw the bisectors of the angles $\angle A$, $\angle B$ and $\angle C$ respectively.

Construction :

Construct a ΔABC with the given measurements. Draw the angle of bisectors of $\angle A$, $\angle B$ and $\angle C$ respectively. Are the three bisectors concurrent?

Experiment-2 :

Construct a ΔABC with sides 4 cm, 5 cm and 6 cm respectively. Draw the bisectors of $\angle B$ and $\angle C$. Name the point of their intersection as I . Join A and I and extend it further. Suppose it meets BC at R . Measure $\angle BAR$ and $\angle CAR$. We see that $\angle BAR = \angle CAR$.

Therefore, AR is the bisector of $\angle A$. Likewise, the third angle bisector does also pass through I .

Now construct a ΔABC . Draw bisectors $\angle B$ and $\angle C$. Name their point of intersection as I . Join points I and A , check whether AI is the bisector of $\angle A$ or not?

We see that AI is the bisectors of $\angle A$ and all the three angle bisectors pass through a point I .

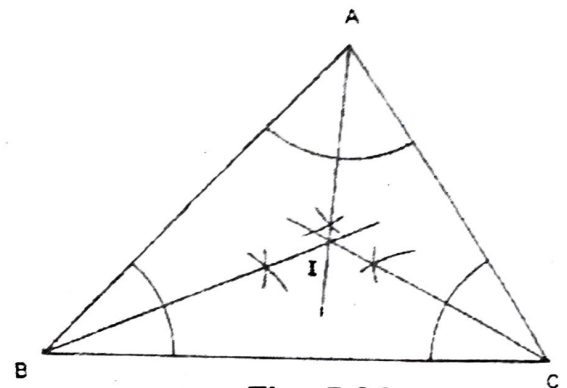


Fig. 5.34

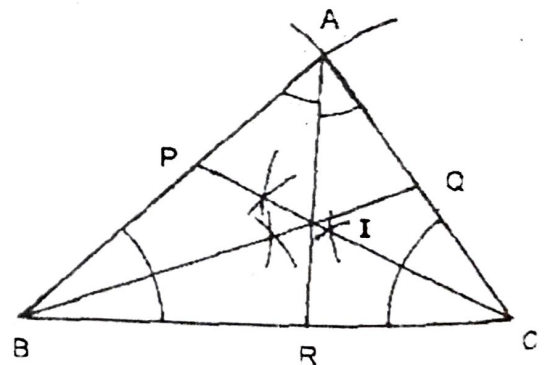


Fig. 5.35

Therefore,

The angle bisectors of triangle are concurrent. The point of concurrency of the angle bisectors of a triangle is called the incentre of a triangle.

Important : It is sufficient to draw only two angle bisectors of any triangle to determine the incentre. Their point of intersection is called the incentre.

Exercise – 5(e)

1. Make triangle from the sides and angles given below and draw their bisectors of angles and check it whether they are concurrent.
 - (i) 3.4 cm, 4 cm and 2.5 cm
 - (ii) 4.6 cm, 3.5 cm and angle between them is 120° .
 - (iii) 5 cm, 4 cm and 2.5°
2. Draw an equilateral triangle $\triangle ABC$ and find its incentre. Also find that circum centre, orthocentre and centroid lie on its incentre.
3. In figure $\triangle ABC$ is an isosceles triangle, in which $AB = AC$, AP is an angle bisector of $\angle A$. This line-segment AP is a perpendicular bisector of BC or not. Tell by drawing figure and measurement.

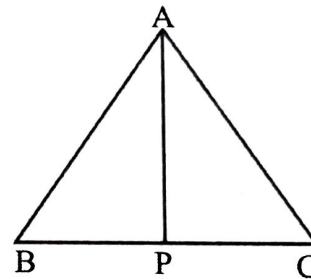


Fig. 5.36

5.7 The Concept of Similarity in triangles :

You know that the two congruent shapes have the same shape and the same size, there are also examples of some such figures in nature which are similar in shape, but not equal in measurements.

Attention : All flowers in a flower plant are not of the same size even though they are of the same shape. Such shapes are called similar shapes.

You can also see that two congruent shapes are identical, but it is not necessary that two identical shapes are congruent.

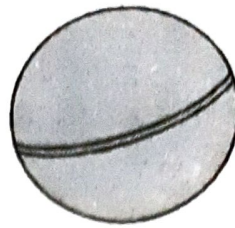
Concept of Similarity



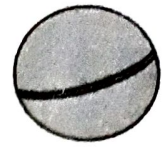
(i)



(ii)



(iii)



(iv)



(v)



(vi)



(vii)



(viii)

In the above pictures (i) and (ii) we see that both pictures are of the same child, but the size of both are different. It is also clear from seeing pictures (iii) and (iv) that both diagrams are of the same ball, but the sizes are different. Look at the pictures (v) and (vi) of these pictures, they also have two pictures of the same car, whose sizes are different.

Different bicycles are visible in this picture (vii) and (viii), in which the sizes of both bicycles are different, but they are similar.

So we conclude that the pairs of the above-mentioned pictures are similar in appearance but different in size.

Such types of figures which are similar in shape, but their sizes are different, are called similar figures.

By looking at the following tell the type of relationship between them :

1. Two different size photos of younger brother in your house.
2. Pictures of two different shapes of a hockey.
3. Two different size photos of globes.
4. Two different size photos of any tree.

We see that younger brother photos; photos of hockey; pictures of the globe and pictures of the tree are same in shape but the sizes are different.

Now you collect the leaves and flowers of the same shape but the sizes are different.

See also these :

See the following shapes :

In 5.41 four equilateral triangles of different measurements, in 5.42 three squares of three different measurements and 5.43 has three regular pentagon of different measurements. Are they of the same shape ? So these are not similar? The answer to all the above questions is clearly 'Yes'.

From this we concludes that, **“Regular Polyons having same numbers of sides are similar for example equilateral triangle, squares etc. are similar.”**

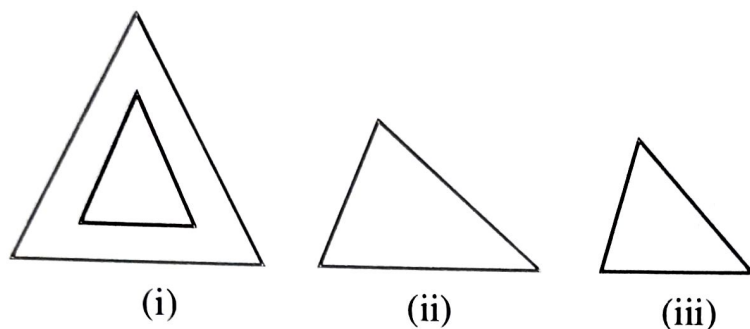
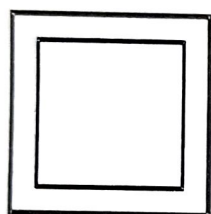
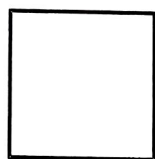


Fig. 5.41

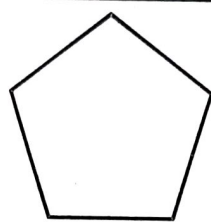


(i)

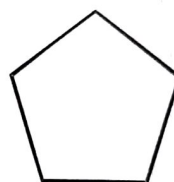


(ii)

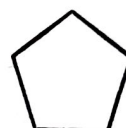
Fig. 5.42



(i)



(ii)



(iii)

Fig. 5.43

Similar Polygon :

In photographer's shop, you must have seen photos of any person or of a single object or of a single figure made from a single negative in different measurements. The photographer is usually takes photos of small size film 35 mm measurements. Then he makes the size on a 45 mm or 55 mm film. If we find the ratio of adjacent sides of the photo picture then it is $\frac{45}{35}$ or $\frac{55}{35}$.

In fact it means that the photographer raises each side of the small picture in equal proportion i.e. 35 : 45 or 35 : 55.

Again, two polygons whose numbers of sides are equal are similar, if

- (i) their adjacent angles are equal, and
- (ii) their adjacent arms are in equal proportion.

Thus, we can say that the following quadrilateral $ABCD$ and $A'B'C'D'$ are identical/
similar.

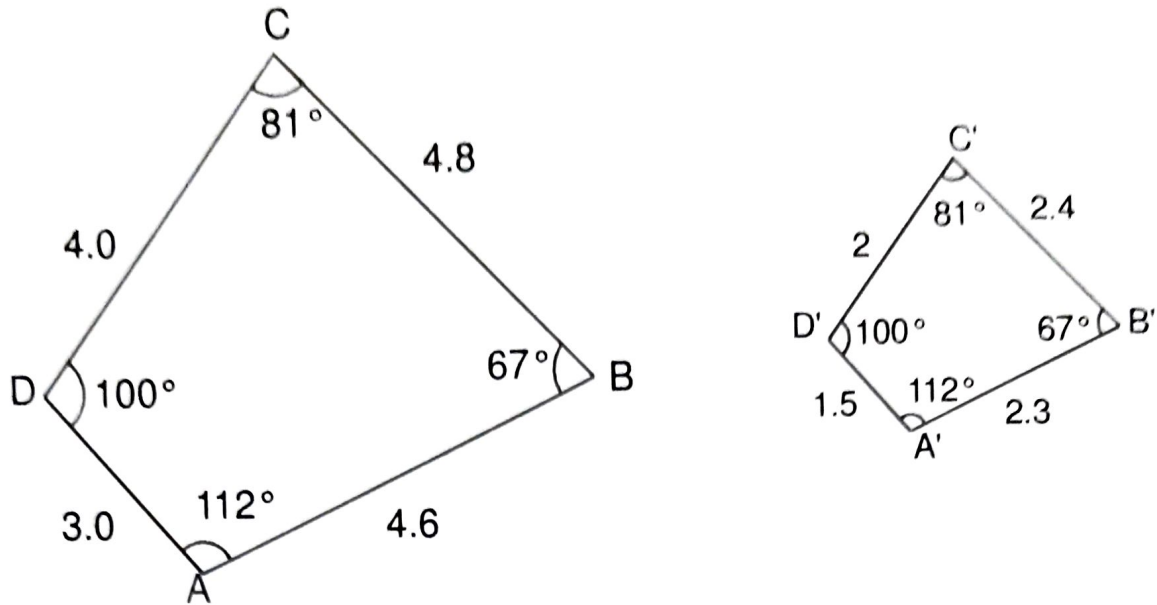


Fig. 5.44

Now pay attention on the following two shapes, one square and another rectangle. Their corresponding angles are equal but their corresponding arms are not in the same proportion. So these are not similar.

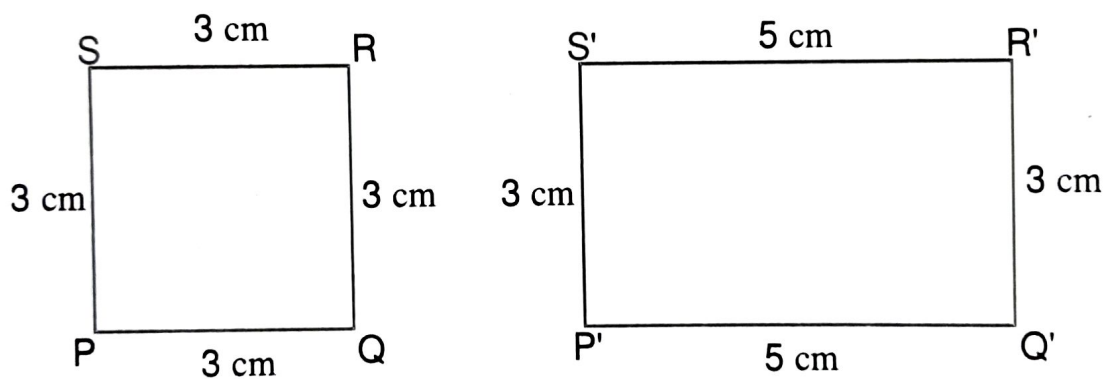


Fig. 5.45

Similarly, we can see that the corresponding sides of the square and rhombus are in the same ratio but their corresponding angles are not equal. So both shapes are not similar.

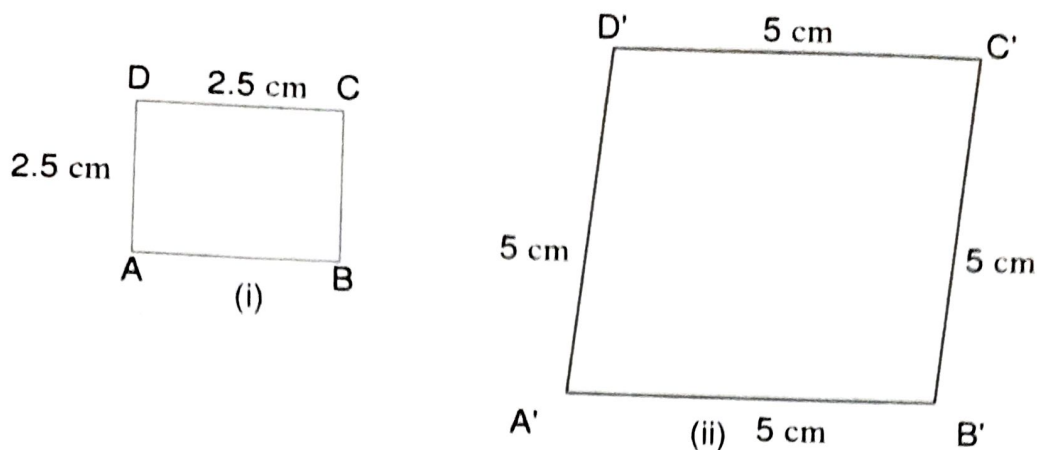


Fig. 5.46

So in the given figure 5.4.6 (i) and figure 5.46(ii) only one of the restriction is not enough for similarity of the polygon.

The Greek mathematician Thales (600 B.C.) has an important theorem in relation to similar angular triangles was meditated. That theorem was.

The proportion of any two corresponding sides (arms) of equal angle triangles is always equal. Irrespective of their measurements.

Recall that for the congruency of two triangles, only three pairs of the components of triangles were taken. Try the same way to establish the similarity of triangles using lesser number of components of the similarity we triangles are required. Let's do some activity for this.

Activity

Draw two line-segment BC and EF of 3 cm. and 5 cm. respectively. Construct $\angle YBC$ and $\angle QEF$ with angle of 60° at the point B and E and construct $\angle XCB$ and $\angle PFE$ with angle of 45° at point C and F. Sides BY and CX intersect each other at point A. and sides QE and PF intersect at D.

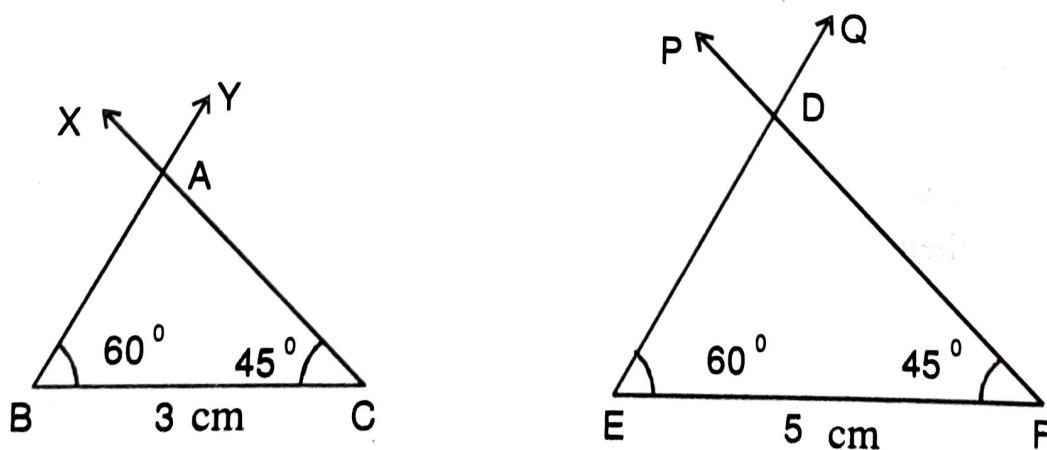


Fig. 5.47

In this way, we get two triangles ABC and DEF . Clearly angle $\angle A$ and $\angle D$ are of 75° . give attention that (notice). In triangles ABC and DEF $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$.

Which are the adjacent angles. So the corresponding angles are equal.

Now find the measurements of the sides of both the triangles AB , AC , DE , DF , and BC , EF find the ratio $\frac{AB}{DE}$, $\frac{AC}{DF}$ and $\frac{BC}{EF}$.

In the above figure, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} (= .6)$ that is why the triangle ABC and DEF are similar.

Please do this

Do this activity by taking different pairs of triangles. Whose corresponding angles are equal. we will get the following conclusion.

If the corresponding angles of two triangles are equal, their corresponding sides will be in equal proportion and in this way both the triangles are similar.

Activity 1 :

Try to make two similar triangles, for this see the figure 5.48 (i) and (ii) and draw two triangles according to the measurement on your exercise book. Find the ratio of their corresponding sides. We see that

$$\frac{AB}{DE} = \frac{4.5}{3} = \frac{3}{2}$$

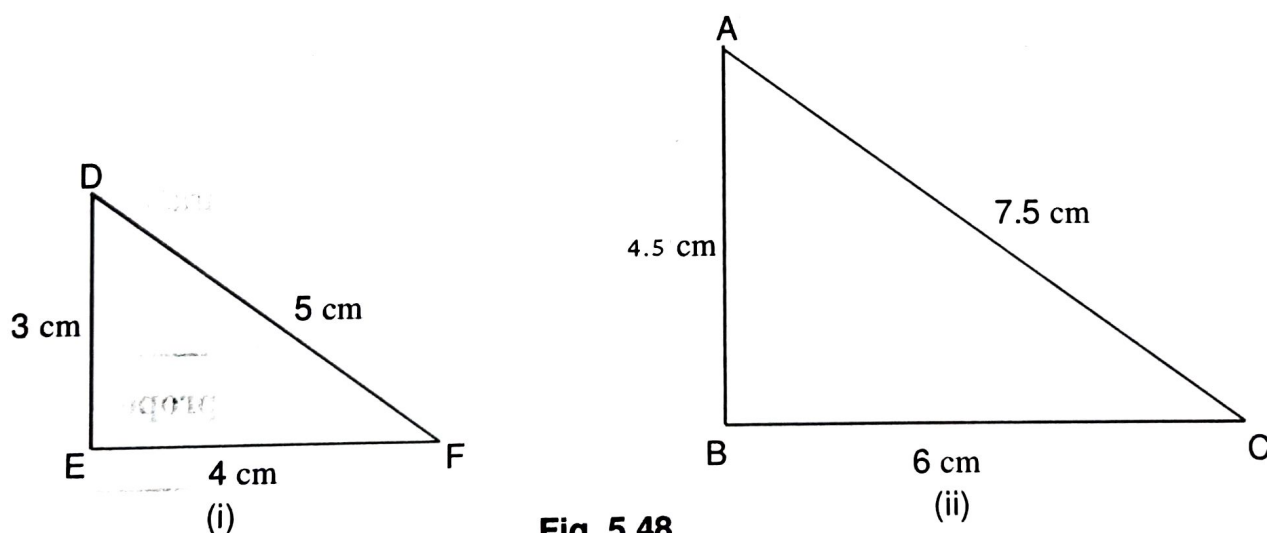


Fig. 5.48

$$\frac{BC}{EF} = \frac{6}{4} = \frac{3}{2} \text{ and}$$

$$\frac{CA}{FD} = \frac{7.5}{5} = \frac{3}{2}$$

The proportion of the corresponding sides of both the triangles are the same. Now measure their angles:

$$\angle A = \angle D, \quad \angle B = \angle E \quad \text{and} \quad \angle C = \angle F$$

If the proportions of the corresponding sides of two triangles are equal, then their corresponding angles will also equal/same.

Activity 2 : Also do this :

Draw two triangles on your exercise book whose measurements is like figure 5.49 (i) and (ii). Now measure the corresponding sides of the both triangles AB and DE , BC and EF and CA and FD and find their proportions (ratio). We see that

$$\frac{BC}{EF} = \frac{6}{3} = \frac{2}{1}$$

$$\frac{2}{1} = \frac{BC}{EF}$$

Thus,

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{2}{1}$$

That is, the proportions of the corresponding sides of the both the triangles are the same, from the figure it is clear that the corresponding angles are equal.

So,

If the two triangles have the same angle, then the proportions of their corresponding sides are equal.

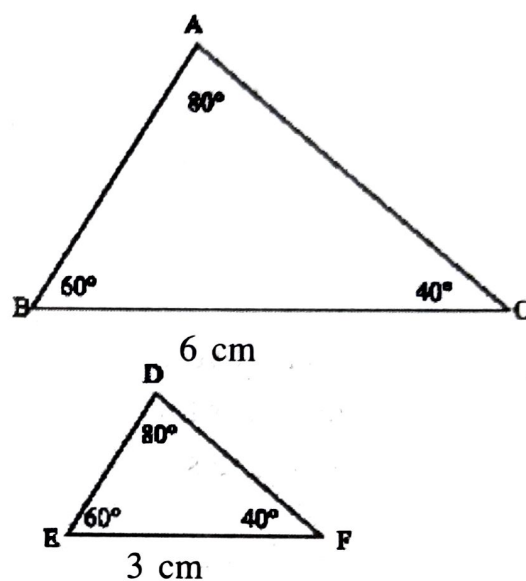


Fig. 5.49

Exercise 5(j)

1. In figure 5.50, two triangles ABC and PQR are similar. The lengths of their sides are given in cm. Find x and y .

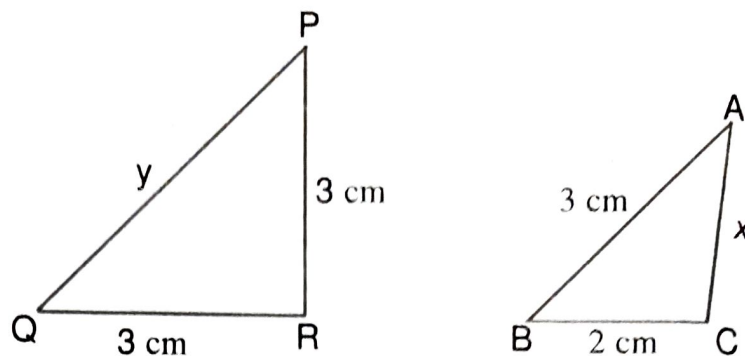


Fig. 5.50

2. If a triangle ABC and triangle PQR , their sides are respectively : $AB = 3$ cm, $BC = 5$ cm, $CA = 4$ cm and $PQ = 7$ cm, $PR = 6$ cm and $QR = 8$ cm, then triangles are similar or not?

3. In figure 5.51, triangle ABC and triangle PQR are similar. Find the remaining sides of the triangle PQR .

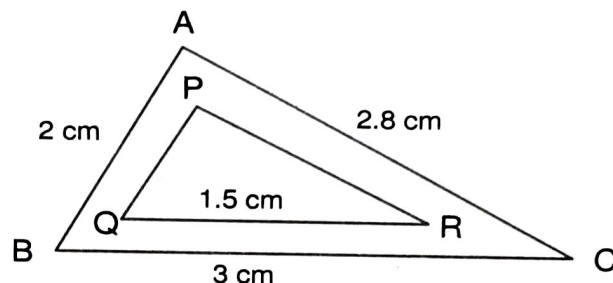


Fig. 5.51

4. In figure 5.52 triangle ABC , MN is parallel to BC . Find the measurements of the remaining angles of $\triangle ABC$. Are the triangle ABC and triangle AMN similar?

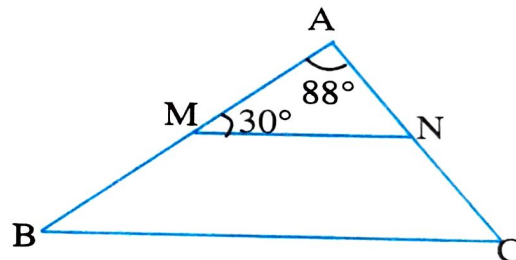


Fig. 5.52

5. In figure 5.53, if M and N are the mid-points of sides AB and AC respectively, find the ratio of the sides BC and MN .

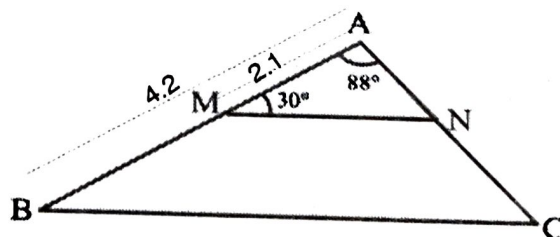


Fig. 5.53

6. Draw two parallel line-segments AB and CD . Join A to D and B to C . Let's assume that they cut each other at point O . Will triangle OAB and triangle ODC be similar?

7. In figure 5.54 are the triangle ABC and triangle PRQ are similar? If yes, then tell the reason.

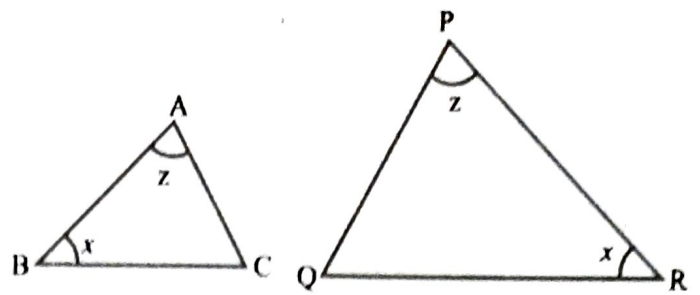


Fig. 5.54

8. Assuming $20\text{ m} = 1\text{ cm}$ size of a field shown in figure 5.5. Considering 40 metres = 1 cm, show the new size of the farm on your exercise book.

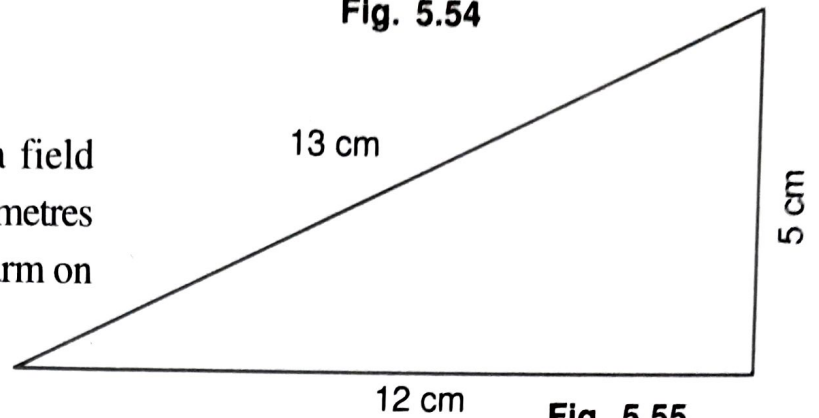


Fig. 5.55

Ability Exercise - 5

1. In a right angled triangle ABC , $\angle B = 90^\circ$, $AB = 6.0\text{ cm}$, $AC = 10\text{ cm}$ then the measure of BC will be :
- (i) 6 cm (ii) 10 cm
(iii) 8 cm (iv) None of these

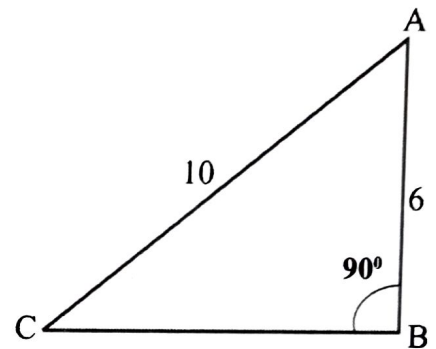


Fig. 5.57

2. The length of a rectangular field is 24 m, width is 10 m. Diagonal of this rectangular field will be :
- (i) 26 m (ii) 29 m
(iii) 20 m (iv) 24 m

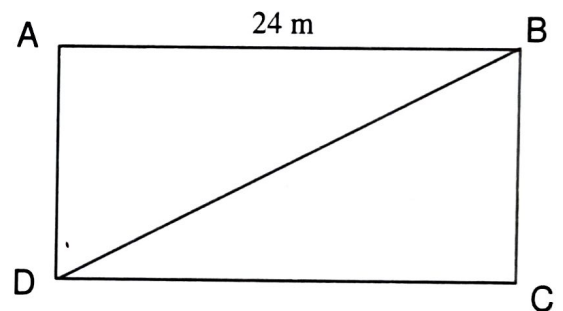


Fig. 5.58

3. A surveyor wants to know the distance between the point A and B , but he cannot straight from A to B , so he creates a right angled triangle as in picture, in which $AC = 21\text{ m}$, $BC = 35\text{ m}$, how long is AB ?

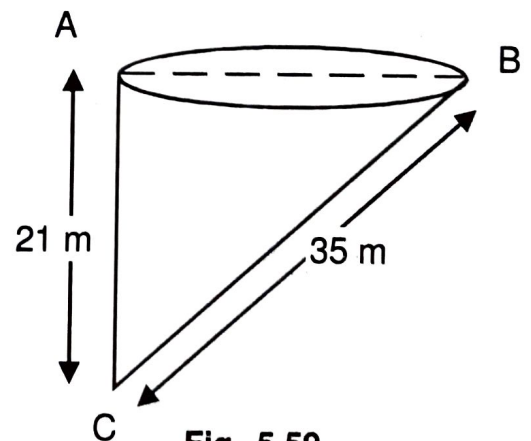


Fig. 5.59

4. AB is a hedge. To find the distance between A & B a surveyor creates a right angled triangle as shown in figure 5.60 $\triangle ADC$. Find the distance between point A to point B, while $BC = 12$ metres
(Hint—In a right triangle ADC)

$$\begin{aligned} AC^2 &= AD^2 + DC^2 \\ &= (30)^2 + 40^2 \\ &= 900 + 1600 \end{aligned}$$

$$AC^2 = 2500$$

$$AC = 50 \text{ m}$$

$$\therefore AB = AC - BC = 50 - 12 = 38 \text{ m}$$

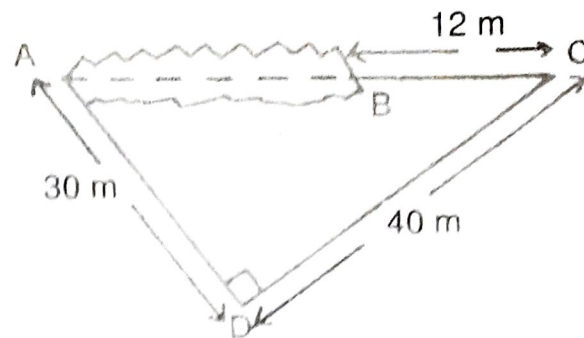


Fig. 5.60

5. Two houses on both sides of a road are face-to-face. One end of the 50 metre long staircase reaches a height of 48 metre on the wall of the house on one side of the road. And on the wall of the second house it reaches only 14 metres find the width of the road.

(Hint—According to the picture width of the road $BD = BC + CD$)

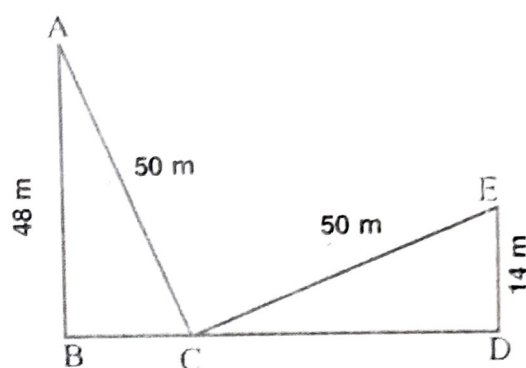
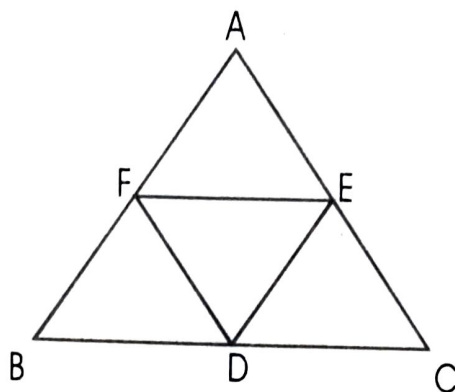


Fig. 5.61

6. Name the triangle whose circumcentre and incentre points are the same.
7. 3, 4, 5 are Pythagorean triplet. Write three triplet of this triplet. Show that 15, 20, 25 also Pythagorean triplet.
8. Construct a triangle ABC , whose base is 7 cm, $\angle B = 60^\circ$ and the sum of two sides is 9 cm.
9. Construct a triangle ABC , whose angle is 120° and the two sides are 5 cm and 6 cm respectively, then find its circumcentre of the triangle.
10. Construct an equilateral triangle, whose side is 6 cm, then find the centroid of the triangle and find the distance between the vertex and centroid.
11. The ratio of the sides of a triangle PQR is 4 : 5 : 6 construct a triangle ABC similar to PQR . Whose base is 8 cm.

M.S.E. Question

12. Point at the plane of a triangle, which is at equal distance from the sides of the triangle is :
- (i) incentre (ii) circumcentre
(iii) orthocentre (iv) centroid
13. In the enclosed figure the area of DEF will be :



D, E, F are the mid point of the sides.

- (i) $\frac{1}{2} \Delta ABC$ (ii) $\frac{1}{3} \Delta ABC$
(iii) $\frac{1}{4} \Delta ABC$ (iv) ΔABC

In this unit we learned that

1. In the right angled triangle, the side facing the right angle is called hypotenuse. It is larger than the sides making right angle.
2. The square made on the hypotenuse in a right angled triangle is equal to the sum of the squares made on the other two sides. This is called Pythagoras theorem.
3. The perpendicular is the shortest line that can be drawn on a line from a point outside the line.
4. If the square made on one side of a triangle equals the sum of the squares of each of the remaining two sides, then the triangle is called a right angled triangle. This is the converse of the Pythagoras Theorem.
5. If a, b, c is a Pythagorean Triplet and k is a positive integer, then ak, bk and ck is also a Pythagorean Triplet.

6. Perpendicular drawn from a vertex of a triangle to the opposite side is called the altitude. There are three vertices in triangle.
7. Altitudes of a triangle are concurrent. The point of intersection of the altitudes of a triangle is called the orthocentre of the triangle.
8. The line-segment joining any vertex of a triangle and the mid-point of the opposite side is called median of the triangle.
9. Medians of a triangle are concurrent. The point where all the three medians of the triangle meet is called the centroid of the triangle.
10. Perpendicular bisectors of the sides of the triangle are concurrent. The point where the perpendicular bisectors of the triangle meet is called the circumcentre of the triangle.
11. Angle bisectors of a triangle are concurrent. The point of concurrency is called the incentre of the triangle.
12. (i) Two polygons whose number of sides are same (equal) are similar, if :
 - (a) their corresponding angles are equal, and
 - (b) their corresponding sides are in equal proportion.
- (ii) If the proportion of the corresponding sides of the triangle are equal, then their corresponding angles are also equal.
- (iii) If the angles of two triangles are equal, then the ratio of corresponding sides are equal.

ANSWERS

Exercise – 5(a)

1. (ii) 10 cm, 2. (iii) 5 cm, 3. (i) x^2 , (ii) 5^2 (iii) p^2 , (iv) 8^2 , 4. 25 cm, 5. 12 cm, 6. 60 Decimeter, 7. 10 m, 9. (i) (5, 12, 13) (iv) (8, 15, 17), (v) (3, 4, 5)

Exercise – 5(b)

2. (i) true (ii) true (iii) true (iv) true; 3. the point of intersection of AC and BC is C.

Exercise – 5(c)

1. (i) Mid point, (ii) Concurrent, (iii) Centroid

2. (i) true, (ii) true, (iii) true, 3. Isosceles

Exercise – 5(d)

1. Yes, 2. Yes, circumcentre

Exercise – 5(e)

1. (i) Yes, (ii) Yes, (iii) Yes; 2. True; 3. Yes

Exercise – 5(f)

1. 2, 4, 5; 2. NO; 3. $PQ = 1$ cm. $PR = 1.4$ cm; 4. $\angle B = 30$, $\angle C = 62^\circ$, yes;
5. 2 : 1, 6. Yes, 7. Yes because $\angle C = \angle Q = 180^\circ - (x + z)$, 8. 2.5 cm, 6 cm, 6.2 cm

Ability Exercise - 5

1. (iii) 8 cm, 2. (i) 26 m, 3. $AB = 28$ m, 4. $AC = 38$ m, 5. 62 m, 6. Equilateral triangle,
7. (6, 8, 10), (12, 16, 20), (18, 24, 30), 10. 3 cm, 11. in centre, 12. $\frac{1}{4}\Delta ABC$.

