

UNIT-2

EXPONENTS



- Rules of Exponents
- Representation of rational numbers by Exponents.
- Positive and negative Exponents.
- To represent the big and small numbers in the form of Exponents and comparing of them.

2.1 Introduction

As we know, in the infinite group of natural numbers no number is largest. Consider the number like 10000000000000000000000000000. Now its not that easy to read, write or understand it just by a glance over it. Practically the mass of earth is 5,970,000,000,000,000,000,000,000 kg, mass of Uranus is 86,800,000,000,000,000,000,000 K, Similarly the distance between sun and saturn or distance between sun and Uranus also are related to of larger magnitude and are really difficult to read, write and understand well with only a flip of seconds time. Thus we find the idea of exponents was envolved and defined to know about pretty, smaller numbers and larger ones as well easily and comfortably.

2.2 Exponents

Let us observe the number 100000 as follows :

$$100000 = 10 \times 10 \times 10 \times 10 \times 10$$

$$10^4$$

power of 10 is 5

Here the number 10 in multiplication has been used five times in a row. It is abbreviated as ten to the power of five and written as 10^5 in short form. In this type of notational form of numbers, Number 10 is called base and number 5 is called power of index or exponent.

Similarly take the number 10000, then

$$10000 = 10 \times 10 \times 10 \times 10 = 10^4$$

multiplying 10, at a time, 3 times.

(1) Also $100 = 10 \times 10 = 10^2$ to multiplied twice at a time.

Decimal numbers and other fractional numbers can also be expressed as follows :

$$0.01 = \frac{1}{100} = \frac{1}{10 \times 10} = \frac{1}{10^2} = 10^{-2}$$

$$0.001 = \frac{1}{1000} = \frac{1}{10 \times 10 \times 10} = \frac{1}{10^3} = 10^{-3}$$

Such type of numbers having negative power represents several physical situations in our environment. It is again important to note that charge on electron particle is $1.6 \times 10^{-19} = 0.00000000000000000016$ coulomb.

In the same way for $2^3 = 8$ the base is 2 and exponents is 3. As a result minor small and pretty large number can be written, likely understood and fairly compared with the help of exponents very well.

Mathematical activity.

Square Table

1 2	2 2×2	3 $2 \times 2 \times 2$	4 $2 \times 2 \times 2 \times 2$	5 $2 \times 2 \dots 5$ times	6 $2 \times 2 \dots 6$ times	7 $2 \times 2 \dots 7$ times	8 $2 \times 2 \dots 8$ times
9 2×2 ... 9 times	10 2×2 .. 10 times	11 2×2 .. 11 times	12 2×2 .. 12 times	13 2×2 .. 13 times	14 2×2 .. 14 times	15 2×2 .. 15 times	16 2×2 .. 16 times
17 2×2 ... 17 times	18 2×2 .. 18 times	19 2×2 .. 19 times	20 2×2 .. 20 times	21 2×2 .. 21 times	22 2×2 .. 22 times	23 2×2 .. 23 times	24 2×2 .. 24 times
25 2×2 .. 25 times	26 2×2 .. 26 times	27 2×2 .. 27 times	28 2×2 .. 28 times	29 2×2 .. 29 times	30 2×2 .. 30 times	31 2×2 .. 31 times	32 2×2 .. 32 times
33 2×2 .. 33 times	34 2×2 .. 34 times	35 2×2 .. 35 times	36 2×2 .. 36 times	37 2×2 .. 37 times	38 2×2 .. 38 times	39 2×2 .. 39 times	40 2×2 .. 40 times
41 2×2 .. 41 times	42 2×2 .. 42 times	43 2×2 .. 43 times	44 2×2 .. 44 times	45 2×2 .. 45 times	46 2×2 .. 46 times	47 2×2 .. 47 times	48 2×2 .. 48 times
49 2×2 .. 49 times	50 2×2 .. 50 times	51 2×2 .. 51 times	52 2×2 .. 52 times	53 2×2 .. 53 times	54 2×2 .. 54 times	55 2×2 .. 55 times	56 2×2 .. 56 times
57 2×2 .. 57 times	58 2×2 .. 58 times	59 2×2 .. 59 times	60 2×2 .. 60 times	61 2×2 .. 61 times	62 2×2 .. 62 times	63 2×2 .. 63 times	64 2×2 .. 64 times

Let the teacher encourage and instruct to multiply 2 as many times as the serial allotted to each subsquare of the given square. For example we multiply 2, six times *i.e.* $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$ corresponding to subsquare number 6. Repeated by corresponding to subsquare 30 we get the number $2^{30} = 2 \times 2 \times \dots 30 \text{ times} = 107502824$. Ultimately the number 2^{64} corresponding to subsquare 64 can be realised and guessed to become a very large number of several digits.

Example 1. Express 64 as powers of 2.

Solution : $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$.

Here the base is 2 and exponent 6.

Example 2. Find exponential form of

1000000000000000000000

$= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

$= 10^{21}$

(simplified form of a large expressed)

Here, base = 10 and

exponent = 21

2.3 Base and exponents of a number in exponential form.

Observe the table

Exponential (power) Form	Factor	Value	Base	Exponent
2^3	$2 \times 2 \times 2$	8	2	3
3^2	3×3	9	3	2
5^6	$5 \times 5 \times 5 \times 5 \times 5 \times 5$	15625	5	6

Try these

- Find base and exponent for 6^5
- Express 125 in exponential form and find its base and exponent.
- Express 343 in exponential form and find its base and exponent.

It is to be remembered that some of the exponents (powers) are given specific name e.g.

For Example:

- (a) Consider 10^2 i.e. 10 to the power 2. It is also called 10 square.
- (b) Consider 10^3 i.e. 10 to the power 3. It is also called 10 cube.

Example 3. Find which one out of 3^4 and 4^3 is greater.

Solution : Now $3^4 = 3 \times 3 \times 3 \times 3 = 81$
also that $4^3 = 4 \times 4 \times 4 = 64$
but $81 > 64$
so $3^4 > 4^3$

Thus 3^4 is greater than 4^3 .

Example 4. Which number among 2^8 and 8^2 is greater?

Solution : $2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
 $= 256$
and $8^2 = 8 \times 8 = 64$
Since, $256 > 64$
Therefore, $2^8 > 8^2$

which implies that 2^8 is greater than 8^2

Observe to note that, just like integers we can express a rational number in exponential form if the number given be multiplied repeated by to itself.

e.g.

$$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^4$$

KNOW THIS ALSO

It is only a coincidence that great scholar and great Indian mathematician Srinivasa Ramanujan (1887–1920) had explained about the number 1729 the only smallest possible number that can be written as the sum of cubes of two numbers in two ways. When his scholar friend Dr. G.H. Hardy told him about his taxi number (1729) as an in auspicious 1729 is Ramanujan's number and $1729 = 12^3 + 1^3$; $1729 = 10^3 + 9^3$

This is an exponential form with base $\left(\frac{2}{3}\right)$ and exponent (power) 4.

Similarly,

$$\left(\frac{-5}{6}\right) \times \left(\frac{-5}{6}\right) \times \left(\frac{-5}{6}\right) \times \left(\frac{-5}{6}\right) \times \left(\frac{-5}{6}\right)$$

can be expressed by $\left(\frac{-5}{6}\right)^5$

which is in exponential form with base $(-5/6)$ and exponent (power) 5.

Numbers with negative base.

Let us try to find out $(-2)^3$ where we have to follow exponential in restricted to the numbers with negative base.

Thus,

$$(-2)^3 = (-2) \times (-2) \times (-2) = -8, (-2) \text{ is being multiplied 3 times repeatedly.}$$

also,

$$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$$

(-2) is being multiplied 4 time so itself.

Observe the following:

Consider a number having base a then

$$a \times a \times a = a^3$$

(called power 3 or a cube)

$$a \times a \times a \times a \times a = a^5$$

(called a to the power 5)

Example 5. Find the values of following :

$$(1)^5, (-1)^4, (-5)^4 \text{ and } (-10)^5$$

Solution : A : $(1)^5 = 1 \times 1 \times 1 \times 1 \times 1$
 $= 1$

B : $(-1)^4 = (-1) \times (-1) \times (-1) \times (-1)$
 $= 1 \times 1 = 1$

C : $(-5)^4 = (-5) \times (-5) \times (-5) \times (-5)$
 $= 25 \times 25$
 $= 625$

$$\begin{aligned}
 \text{E: } (-10)^5 &= (-10) \times (-10) \times (-10) \times (-10) \times (-10) \\
 &= 100 \times 100 \times (-10) \\
 &= -100000
 \end{aligned}$$

Example 5. Find the values of followings :

$$(1)^3, (-1)^4, (-5)^4, (-10)^5 \text{ and } (-10)^2$$

Solution :

$$\begin{aligned}
 \text{A : } (1)^3 &= 1 \times 1 \times 1 \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \text{B : } (-1)^4 &= (-1) \times (-1) \times (-1) \times (-1) \\
 &= 1 \times 1 = 1
 \end{aligned}$$

$$\begin{aligned}
 \text{C : } (-5)^4 &= (-5) \times (-5) \times (-5) \times (-5) \\
 &= 25 \times 25 \\
 &= 625
 \end{aligned}$$

$$\begin{aligned}
 \text{D : } (-10)^5 &= (-10) \times (-10) \times (-10) \times (-10) \times (-10) \\
 &= 100 \times 100 \times (-10) \\
 &= -100000
 \end{aligned}$$

$$\begin{aligned}
 \text{E : } (-10)^2 &= (-10) \times (-10) \\
 &= 100
 \end{aligned}$$

Note that :

The even power of a number is always a positive number while the odd power of a negative number is always a negative number.

Exponential Form	Expanded form of multiplication	Value	Base	Power
$(-3)^4$	$(-3) \times (-3) \times (-3) \times (-3)$	81	-3	4
$(-5)^3$	$(-5) \times (-5) \times (-5)$	-125	-5	3
$(-7)^4$	$(-7) \times (-7) \times (-7) \times (-7)$	2401	-7	4
5^6	$5 \times 5 \times 5 \times 5 \times 5 \times 5$	15625	5	6

Conclusions:

The above discussions leads to the fact that :

- $(\text{positive integer})^n = \text{positive integer}$, where " n " a natural number.
- $(\text{negative integer})^n = \text{positive integer}$ if " n " is an even positive integer.
- $(\text{negative integer})^n = \text{negative integer}$, if n is an odd natural number.
- $(-1)^n = 1$ if n is even natural number.
- $(-1)^n = -1$ if n is an odd natural number.

Attempt the following:

1. Which one 5^3 or 3^5 is greater?
2. What is the exponential form of $6 \times 6 \times 6 \times 6$ with respect to base 6.
3. Evaluate $(-1)^{17}$

Exercise-2(a)

1. The values of $\left(\frac{-2}{3}\right)^5$ is:
(i) $\frac{32}{243}$ (ii) $\frac{-32}{243}$ (iii) $\frac{10}{15}$ (iv) $\frac{-10}{15}$
2. The exponential form of 3125 is :
(i) 5^5 (ii) 5^2 (iii) 5^3 (iv) 5^4
3. The value of 2 to the power 7 is :
(i) 49 (ii) 14 (iii) 128 (iv) 32
4. Find, what is the area of a square figure with side 5 metre in exponential form.
5. Simplify the following :
(i) $2^4 \times 3^2$ (ii) $(-2)^3 \times (-10)^3$
6. Find the prime factors of 15625 and then express it in an exponential form.

Rules for Exponents

Rule 1. Multiplication of exponential form of numbers having same base.

Example :

Consider

$$\begin{aligned}2^3 \times 2^4 &= (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7\end{aligned}$$

i.e. $2^3 \times 2^4 = 2^{3+4} = 2^7$

Be sure that,

2^3 and 2^4 have same base and sum of their powers in $(3 + 4) = 7$

Example 6 :

(a) Simplify $4^3 \times 4^5$ to write its exponential form :

Clearly $4^3 = 4 \times 4 \times 4$ and $4^5 = 4 \times 4 \times 4 \times 4 \times 4$

Therefore,

$$\begin{aligned}4^3 \times 4^5 &= (4 \times 4 \times 4) \times (4 \times 4 \times 4 \times 4 \times 4) \\ &= 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \\ &= 4^8 = 4^{3+5}\end{aligned}$$

Note that 4^8 may be written as 2^{16} :

(b) Simplify $\left(\frac{3}{5}\right)^4 \times \left(\frac{3}{5}\right)^6$

Clearly $\left(\frac{3}{5}\right)^4 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

and $\left(\frac{3}{5}\right)^6 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

therefore,

$$\begin{aligned}\left(\frac{3}{5}\right)^4 \times \left(\frac{3}{5}\right)^6 &= \left(\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}\right) \times \left(\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}\right) \\ &= \frac{3}{5} \text{ is being multiplied } (4 + 6) \text{ times repeated} \\ &= \left(\frac{3}{5}\right)^{10}\end{aligned}$$

Example 7. Evaluate $(-3)^2 \times (-3)^5$ in exponential form.

Solution :

$$\begin{aligned}(-3)^2 \times (-3)^5 &= [(-3) \times (-3)] \times [(-3) \times (-3) \times (-3) \times (-3) \times (-3)] \\&= (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \\&= (-3)^7\end{aligned}$$

$$\therefore (-3)^2 \times (-3)^5 = (-3)^{2+5} = (-3)^7$$

Example 8. Find value of $a^3 \times a^4$ in exponential form.

Solution :

$$\begin{aligned}a^3 \times a^4 &= (a \times a \times a) \times (a \times a \times a \times a) \\&= a \times a \times a \times a \times a \times a \times a \\&= a^7\end{aligned}$$

Thus,

$$\begin{aligned}a^3 \times a^4 &= a^{3+4} \\&= a^7\end{aligned}$$

Special : Note that in each of examples base of multiplicands and products is same.

Try it :

1. Express $\left(\frac{5}{6}\right)^2 \times \left(\frac{5}{6}\right)^3$ to an exponential form with base $\frac{5}{6}$.

Fill in the blank boxes :

2. $(-5)^2 \times (-5)^6 = (-5)^{\square}$

3. $a^2 \times a^3 = a^{\square}$

Conclusion :

Now, we have that : $a^m \times a^n = a^{m+n}$

Where a is any non-zero positive rational number m , and n being any two non-zero positive integers.

Special : About the expression $2^3 \times 3^2$ or $3^4 \times 4^3$ be careful. Here, since bases of numbers in exponential form are not same therefore, we can't add the exponents (power) while simplify the given expression. If done so the result would lead to a wrong deduction.

Rule 2. Division of any two number in exponential form having same base.

Example 9. Find (i) $2^7 \div 2^3$

$$\text{Now } 2^7 \div 2^3 = \frac{2^7}{2^3} = \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times 2 \times 2}{\cancel{2} \times \cancel{2} \times \cancel{2}} \\ = 2^4$$

$$\therefore 2^7 \div 2^3 = 2^{7-3} = 2^4 \text{ holds true.}$$

Example 10. (i) Evaluate $5^5 \div 5^3$

Solution : Now,
$$5^5 \div 5^3 = \frac{5^5}{5^3} = \frac{\cancel{5} \times \cancel{5} \times \cancel{5} \times 5 \times 5}{\cancel{5} \times \cancel{5} \times \cancel{5}} \\ = 5 \times 5 = 5^2$$

$$\therefore 5^5 \div 5^3 = 5^{5-3} = 5^2 \text{ holds true.}$$

KNOW THIS ALSO

Long ago our Indian saints and scholars had developed, how to write and name the numbers upto (10^{17})
Paraardh

(ii) Evaluate $a^4 \div a^3$

Solution :
$$a^4 \div a^3 = \frac{a^4}{a^3} = \frac{\cancel{a} \times \cancel{a} \times \cancel{a} \times a}{\cancel{a} \times \cancel{a} \times \cancel{a}} \\ = a$$

$$\therefore a^4 \div a^3 = a^{4-3} = a^1 = a$$

(iii) Evaluate $7^8 \div 7^5$

Solution :
$$7^8 \div 7^5 = \frac{7^8}{7^5} = \frac{\cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times 7 \times 7 \times 7}{\cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7}} \\ = 7 \times 7 \times 7 \\ = 7^3$$

$$\therefore 7^8 \div 7^5 = 7^{8-5} = 7^3 \text{ holds true.}$$

(iv) Simplify $\left(\frac{5}{9}\right)^6 \div \left(\frac{5}{9}\right)^4$

Solution :

$$\begin{aligned}\left(\frac{5}{9}\right)^6 \div \left(\frac{5}{9}\right)^4 &= \frac{\left(\frac{5}{9}\right) \times \left(\frac{5}{9}\right) \times \left(\frac{5}{9}\right) \times \left(\frac{5}{9}\right) \times \left(\frac{5}{9}\right) \times \left(\frac{5}{9}\right)}{\left(\frac{5}{9}\right) \times \left(\frac{5}{9}\right) \times \left(\frac{5}{9}\right) \times \left(\frac{5}{9}\right)} \\ &= \frac{5}{9} \times \frac{5}{9} = \left(\frac{5}{9}\right)^2\end{aligned}$$

Thus clearly,

$$\left(\frac{5}{9}\right)^6 \div \left(\frac{5}{9}\right)^4 = \frac{\left(\frac{5}{9}\right)^6}{\left(\frac{5}{9}\right)^4} = \left(\frac{5}{9}\right)^{6-4} = \left(\frac{5}{9}\right)^2$$

Therefore,

$$\left(\frac{5}{9}\right)^6 \div \left(\frac{5}{9}\right)^4 = \left(\frac{5}{9}\right)^{6-4} = \left(\frac{5}{9}\right)^2 \text{ holds true.}$$

Try it :

Simplify the following and write in exponential form :

(i) $10^6 \div 10^2$

(ii) $2^9 \div 2^9$

(iii) $7^{13} \div 7^{10}$

So from discussed examples it leads to.

Conclusion :

For a being non-zero positive rational number $a^m \div a^n = a^{m-n}$, where $m > n$, m and n being any two positive integers.

Let us observe again that :

$$3^4 = 3 \times 3 \times 3 \times 3$$

and

$$3^7 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

Therefore,
$$3^4 \div 3^7 = \frac{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times 3 \times 3 \times 3} = \frac{1}{3 \times 3 \times 3}$$

Thus,

$$3^4 \div 3^7 = \frac{1}{3^{7-4}} = \frac{1}{3^3}$$

Similarly,

$$\left(\frac{2}{3}\right)^4 \div \left(\frac{2}{3}\right)^9 = \frac{\left(\frac{2}{3}\right)^4}{\left(\frac{2}{3}\right)^9}$$

$$= \frac{\frac{2}{\cancel{3}} \times \frac{2}{\cancel{3}} \times \frac{2}{\cancel{3}} \times \frac{2}{\cancel{3}}}{\frac{2}{\cancel{3}} \times \frac{2}{\cancel{3}} \times \frac{2}{\cancel{3}} \times \frac{2}{\cancel{3}} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}}$$

$$= \frac{1}{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}} = \frac{1}{\left(\frac{2}{3}\right)^5}$$

$$= \frac{1}{\left(\frac{2}{3}\right)^{9-4}} = \frac{1}{\left(\frac{2}{3}\right)^5}$$

$$\therefore \left(\frac{2}{3}\right)^4 \div \left(\frac{2}{3}\right)^9 = \frac{1}{\left(\frac{2}{3}\right)^{9-4}} = \frac{1}{\left(\frac{2}{3}\right)^5}$$

For a being a non-negative, non-zero rational number with m, n beign any two non-zero positive integers then

$$a^m \div a^n = \frac{1}{a^{n-m}} \quad (\text{where } m < n)$$

$$= 1 \quad (\text{if } m = n)$$

Try :

1. Show that if $4^3 \div 4^3 = 1$ implies that $4^0 = 1$

2. Show that if $\left(\frac{2}{3}\right)^8 \div \left(\frac{2}{3}\right)^8 = 1$ implies that $\left(\frac{2}{3}\right)^0 = 1$

Observe that :

$$a^m \div a^m = 1$$

implies that $a^{m-m} = 1$ $\left(\because a^m \div a^n = a^{m-n} = \frac{1}{a^{n-m}} = 1, \text{ where } n = m \right)$

$$\text{or } a^0 = 1$$

Thus we may conclude :

If a is any non-zero rational number then $a^0 = 1$.

Remark :

Here $a \neq 0$ as division by zero is not defined.

Rule 3 : To find a number in power form.

Example 11. (a) Simplify $[(5^3)]^4$

Solution :

$$\begin{aligned} [(5^3)]^4 &= (5^3) \times (5^3) \times (5^3) \times (5^3) \\ &= (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5) \\ &= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \\ &= 5^{12} \end{aligned}$$

$$\text{Thus } [(5^3)]^4 = 5^{3 \times 4} = 5^{12} \text{ holds true.}$$

Similarly,

(a) Find $\left[\left(\frac{4}{7}\right)^2\right]^3$

Consider,

$$\begin{aligned}\left[\left(\frac{4}{7}\right)^2\right]^3 &= \left(\frac{4}{7}\right)^2 \times \left(\frac{4}{7}\right)^2 \times \left(\frac{4}{7}\right)^2 \\ &= \left(\frac{4}{7} \times \frac{4}{7}\right) \times \left(\frac{4}{7} \times \frac{4}{7}\right) \times \left(\frac{4}{7} \times \frac{4}{7}\right) \\ &= \left(\frac{4}{7}\right)^6\end{aligned}$$

i.e. $\left[\left(\frac{4}{7}\right)^2\right]^3 = \left(\frac{4}{7}\right)^{2 \times 3} = \left(\frac{4}{7}\right)^6$ holds true.

Try :

$$(i) \quad \left[\left(\frac{2}{3}\right)^5\right]^2 = \left(\frac{2}{3}\right)^{5 \times 2} \quad (ii) \quad \left[\left(\frac{3}{5}\right)^4\right]^3 = \left(\frac{3}{5}\right)^{4 \times 3}$$

Now, as we have discussed, so far, we can conclude.

If a is any given rational number such that m and n are any two positive integers, then $(a^m)^n = a^{mn}$.

Note : Rule 3 also holds for the case when $a = 0$ as well $[(0^m)^n = 0^{mn}]$ holds good.

Rule 4. This rule is related to those type of numbers which are in exponent form having same exponent but different base as such.

Can we simplify a number $2^4 \times 3^4$; if we are asked to do so? The expression for simplification involves number that there same exponent but base happen to be different.

Consider,

$$\begin{aligned}(2)^4 \times (3)^4 &= (2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3) \\ &= (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)\end{aligned}$$

(applying the law of commutativity associating with respect to multiplied repeated by)

$$\begin{aligned}&= (6) \times (6) \times (6) \times (6) \\ &= 6^4\end{aligned}$$

i.e. $2^4 \times 3^4 = (2 \times 3)^4 = 6^4$

One the same lines again consider rational numbers having different base and same exponent to multiply as follows :

$$\begin{aligned}
 \text{e.g.} \quad \left(\frac{5}{7}\right)^3 \times \left(\frac{3}{4}\right)^3 &= \left(\frac{5}{7} \times \frac{5}{7} \times \frac{5}{7}\right) \times \left(\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}\right) \\
 &\left(\text{number with same exponent 3 and different base as } \frac{5}{7} \text{ and } \frac{3}{4}\right) \\
 &= \left(\frac{5}{7} \times \frac{3}{4}\right) \times \left(\frac{5}{7} \times \frac{3}{4}\right) \times \left(\frac{5}{7} \times \frac{3}{4}\right)
 \end{aligned}$$

(By repeated application of commutativity and with respect to multiplication)

$$\begin{aligned}
 &= \left(\frac{15}{28}\right) \times \left(\frac{15}{28}\right) \times \left(\frac{15}{28}\right) = \left(\frac{15}{28}\right)^3 \\
 &= \left(\frac{15}{28}\right)^3
 \end{aligned}$$

Therefore, in the same way we simplify the following the specific expression as :

$$\begin{aligned}
 \text{(i)} \quad \left(\frac{5}{6}\right)^5 \times \left(\frac{6}{7}\right)^5 &= \left(\frac{5}{\cancel{6}} \times \frac{\cancel{6}}{7}\right)^5 = \left(\frac{5}{7}\right)^5 \\
 \text{(ii)} \quad \left(\frac{8}{9}\right)^6 \times \left(\frac{9}{12}\right)^6 &= \left(\frac{8}{\cancel{9}} \times \frac{\cancel{9}}{12}\right)^6 = \left(\frac{8}{12}\right)^6 = \left(\frac{2}{3}\right)^6
 \end{aligned}$$

Conclusion :

$a^m \times b^m = (a \times b)^m$ if a, b are being any two rational numbers and m being a positive integer.

Rule 5. Discussion about division of two numbers which are in exponent form with different base but same exponent (power).

Let us observe that :

$$\begin{aligned}
 8^5 \div 9^5 &= \frac{8^5}{9^5} = \frac{8 \times 8 \times 8 \times 8 \times 8}{9 \times 9 \times 9 \times 9 \times 9} \\
 &= \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} = \left(\frac{8}{9}\right)^5
 \end{aligned}$$

$$\text{i.e. } 8^5 \div 9^5 = \frac{8^5}{9^5} = \left(\frac{8}{9}\right)^5$$

Again consider an expression of the type :

$$\left(\frac{3}{4}\right)^3 \div \left(\frac{2}{5}\right)^3$$

Try to simplify as follows :

$$\text{Now, } \left(\frac{3}{4}\right)^3 \div \left(\frac{2}{5}\right)^3 = \frac{\left(\frac{3}{4}\right)^3}{\left(\frac{2}{5}\right)^3} = \frac{\left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right)}{\left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right)}$$

$$\left(\frac{3}{4}\right)^3 \div \left(\frac{2}{5}\right)^3 = \left(\frac{\frac{3}{4}}{\frac{2}{5}}\right) \times \left(\frac{\frac{3}{4}}{\frac{2}{5}}\right) \times \left(\frac{\frac{3}{4}}{\frac{2}{5}}\right) = \frac{\left(\frac{3}{4}\right)^3}{\left(\frac{2}{5}\right)^3}$$

$$\text{Thus, } = \left(\frac{\frac{3}{4}}{\frac{2}{5}}\right)^3$$

Try it : Verify the following statements :

$$(i) \quad 5^3 \div 6^3 = \left(\frac{5}{6}\right)^3 \quad (ii) \quad \left(\frac{4}{5}\right)^4 \div \left(\frac{3}{8}\right)^4 = \left(\frac{\frac{4}{5}}{\frac{3}{8}}\right)^4$$

(iii) Fill the number in box :

$$(20)^4 \div (4)^4 = (\square)^4$$

Conclusion :

If a and b are any two non-zero rational numbers such that m is a positive integer

$$a^m \div b^m = \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m \quad \text{and} \quad b^m \div a^m = \frac{b^m}{a^m} = \left(\frac{b}{a}\right)^m$$

Note : The above relations also hence proved even if $a = 0$ in first and $b = 0$ in second relation respectively.

Example 12. Illustrate the method to find out the value of

$$\left(\frac{3}{4}\right)^2 \times \left\{ \left(\frac{3}{4}\right)^{10} \div \left(\frac{3}{4}\right)^8 \right\}$$

$$\begin{aligned} \text{Solution : } \left(\frac{3}{4}\right)^2 \times \left\{ \left(\frac{3}{4}\right)^{10} \div \left(\frac{3}{4}\right)^8 \right\} &= \left(\frac{3}{4}\right)^2 \times \frac{\left(\frac{3}{4}\right)^{10}}{\left(\frac{3}{4}\right)^8} \\ &= \left(\frac{3}{4}\right)^2 \times \left(\frac{3}{4}\right)^{10-8} = \left(\frac{3}{4}\right)^2 \times \left(\frac{3}{4}\right)^2 \\ &= \left(\frac{3}{4}\right)^{2+2} = \left(\frac{3}{4}\right)^4 \\ &= \frac{3 \times 3 \times 3 \times 3}{4 \times 4 \times 4 \times 4} = \frac{81}{256} \end{aligned}$$

Note : In general simplification ($\div$) division should be performed before ($\times$) multiplication operation.

Group Discussion

1. What could be $2^3 \times 2^6$ having base 2 in exponent form?
2. Find the value of $(15)^0$.
3. Find the value of $(1^3)^5$.
4. Simplify $(20)^4 \div (5)^4$ to express the result in exponential form with base 4.

Exercise-2(b)

1. Simplify :

(i) $3^7 \times 3^8$

(ii) $6^4 \times 6^2 \div 6^5$

(iii) $5^9 \times 5^4 \div 5^8$

(iv) $2 \times 5^2 + 5 \times 2^5$

(v) $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3$

(vi) $\left(\frac{4}{9}\right)^3 \times \left(\frac{4}{9}\right)^4 \div \left(\frac{4}{9}\right)^5$

2. Simplify :

(i) $15^8 \times 15^{12} \div 15^{20}$

(ii) $25^3 \times 25^7 \div 25^{10}$

(iii) $\left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^3 \div \left(\frac{1}{2}\right)^8$

(iv) $\left(\frac{3}{4}\right)^2 \times \left(\frac{3}{4}\right)^6 \div \left(\frac{3}{4}\right)^5$

3. Find the value of $(-1)^3 \times (-1)^2 \times (-1)^{15}$.

4. Find the value of $(-1)^{49} \div (-1)^{25}$.

5. Find the value of $3^{12} \times 3^7 \div 3^{25}$.

6. Fill in the blanks in your answer books :

(i) $4 \times 4 \times \dots\dots 20 \text{ times} = (\dots\dots)^{20}$ (ii) $8^6 \times 7^6 = (\dots\dots)^6$

(iii) $\frac{9^{12}}{5^{12}} = \left\{ \frac{9}{(\dots\dots)} \right\}^{12}$ (iv) $(3^8)^3 = (3)^{(\dots\dots)}$ (v) $(2^3)^5 = (2^5)^{(\dots\dots)}$

7. The correct option for $5^5 \times 8^5$ is :

(i) 40^5 (ii) 40^{10} (iii) 40^{25} (iv) 5^{40} .

8. The correct option for $(-3)^4 \div (-3)^2$ is :

(i) 81 (ii) -81 (iii) 9 (iv) -9

9. The correct option for $4 \times 5^2 + 5 \times 4^2$ is :

(i) 100 (ii) 80 (iii) 200 (iv) 180

2.5 To express rational numbers in terms of exponents.

We know that general form of rational numbers is $\frac{p}{q}$ where p, q are integers, $q \neq 0$, and every integer is a rational number.

Consider,

$$2 = (2)^1, 3 = 3^1, \frac{4}{7} = \left(\frac{4}{7}\right)^1$$

$$6 = (6)^1, 8 = 8^1, 8 = (2)^3, \frac{4}{9} = \left(\frac{4}{9}\right)^1$$

and $\frac{4}{9} = \frac{2 \times 2}{3 \times 3} = \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^2$

Again $\frac{16}{625} = \left(\frac{16}{625}\right)^1, \left(\frac{16}{625}\right) = \left(\frac{4}{25}\right)^2$ and $\frac{16}{625} = \left(\frac{2}{5}\right)^4,$

$$\left(\frac{-27}{343}\right) = \left(\frac{-27}{343}\right)^1, \frac{-27}{343} = \frac{(-3) \times (-3) \times (-3)}{7 \times 7 \times 7} = \left(\frac{-3}{7}\right)^3$$

Try :

1. Express $\frac{16}{81}$ in exponent form having base $\frac{4}{9}$.

2. Which fraction's exponent form is $\left(\frac{3}{5}\right)^2$?

Note : Number that can be expressed, in exponent form one way only has unique exponent form.

For example,

$$\frac{5}{12} = \left(\frac{5}{12}\right)^1, 6 = 6^1, 3 = 3^1 \text{ and } 17 = 17^1 \text{ etc.}$$

Contrary to the above result if a number has several representation with different base then the number attains different exponents over different base scales. For example consider 729.

$$\begin{aligned} \text{Then } 729 &= 3^6 \text{ base 3 raised to the power 6.} \\ &= 9^3 \text{ base 9 raised to the power 3.} \\ &= 27^2 \text{ base 27 raised to the power 2.} \end{aligned}$$

Thus these type of examples discussed above establish the generalisation of following facts :

1. Every rational number can be expressed as having exponent 1 with base number at the rational number itself for example $a = (a)^1$.
2. Every prime number has a unique exponential form of representation.
3. The exponential representation in corresponding to other numbers than that of prime one is sometime unique and some time it is not so.

Again, try to consider that :

$$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^5 \dots(i)$$

and
$$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3} = \frac{2^5}{3^5} \quad \dots(ii)$$

from (i) and (ii) we find that :

$$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5}$$

Similarly,
$$\frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} = \left(\frac{4}{7}\right)^6$$

also that
$$\frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} = \frac{4 \times 4 \times 4 \times 4 \times 4 \times 4}{7 \times 7 \times 7 \times 7 \times 7 \times 7} = \frac{4^6}{7^6}$$

thus we have,
$$\left(\frac{4}{7}\right)^6 = \frac{4^6}{7^6}$$

these examples help us to generalise that

$$\frac{p}{q} \times \frac{p}{q} \times \frac{p}{q} \times \dots \times m \text{ times} = \left(\frac{p}{q}\right)^m \quad \dots(i)$$

and
$$\frac{p}{q} \times \frac{p}{q} \times \frac{p}{q} \times \dots \times m \text{ times} = \frac{p \times p \times p \times \dots \times m \text{ times}}{q \times q \times q \times \dots \times m \text{ times}} = \frac{p^m}{q^m} \quad \dots(ii)$$

from (i) and (ii) we get
$$\left(\frac{p}{q}\right)^m = \frac{p^m}{q^m} \quad \dots(iii)$$

Using this fact we can write exponential form of any rational number into a rational number. While some other rational numbers can be expressed as exponent of some other rational number.

For example :
$$\left(\frac{6}{7}\right)^3 = \frac{6^3}{7^3} = \frac{216}{343} \quad \left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \right]$$

and that
$$\frac{216}{343} = \frac{6 \times 6 \times 6}{7 \times 7 \times 7} = \left(\frac{6}{7}\right)^3$$

Using formulae of different base and same exponent we can write any one of rational number in exponent form.

$$a^m \times b^m = (a \times b)^m = (ab)^m$$

A numerical example related to above type elaboration may be supplied as under :

$$(27 \times 343) = 3^3 \times 7^3 = (3 \times 7)^3 = (21)^3$$

Example 13. Express 64×729 in the form of exponent having base 6.

Solution : $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

$$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$$

$$\text{Thus, } 64 \times 729 = 2^6 \times 3^6 = (2 \times 3)^6 = 6^6 \quad [\because (a)^m \times b^m = (ab)^m]$$

Exercise-2(c)

- Simplify $12 \times 12 \times 12 \times 12 \times 12 \times 12$ in exponent form.
- Express 15625 in exponent form over the bases 5, 25 and 125 respectively.
- Express 0.0001 in exponent form having base 0.01.
- Express $\frac{-343}{512}$ in exponential form to the base $\frac{-7}{8}$.
- Express $\frac{1000}{1331}$ in exponential form with base $\frac{10}{11}$.
- Express $\frac{1024}{1081}$ as square of some number.
- Express $\frac{512}{729}$ as cube of some number.
- 0.000001 is the square of :
 (i) $(0.01)^2$ (ii) $(0.001)^2$ (iii) $(0.0001)^2$ (iv) $(0.00001)^2$
- The value $(0.05)^3$ is :
 (i) 0.125 (ii) 0.0125 (iii) 0.00125 (iv) 0.000125
- The base corresponding to number $\frac{64}{125}$ in cubic form is :
 (i) $\frac{2}{5}$ (ii) $\frac{4}{5}$ (iii) $\frac{8}{25}$ (iv) $\frac{4}{25}$

2.6 Positive and Negative Exponents :

Meaning of exponent (-1).

Consider $2 \times \frac{1}{2} = 1$
or $2 = \frac{1}{\left(\frac{1}{2}\right)}$ (i)

Similarly $7 \times \frac{1}{7} = 1$
or $7 = \frac{1}{\left(\frac{1}{7}\right)}$ (ii)

also that $\frac{3}{4} \times \frac{4}{3} = 1$
or $\frac{3}{4} = \frac{1}{\left(\frac{4}{3}\right)}$ (iii)

Thus in general for a non-zero rational number a , we have

$$a \times \frac{1}{a} = 1 \quad \text{or} \quad a = \frac{1}{\left(\frac{1}{a}\right)} \quad \text{....(iv)}$$

It could be very well supposed that if the multiplication of any two rational number turn out to be '1' then they are called inverse or reciprocal of each other. Thus in (i) the reciprocal or inverse of 2 is $\frac{1}{2}$ so on and so forth we can decide inverse in (ii), (iii) and (iv) respectively.

You know that

$$10^2 = 10 \times 10 = 100$$

$$10^1 = \frac{10 \times 10}{10} = \frac{100}{10}$$

Again $1 = 10^0 = 10^{1-1} = 10^1 \times 10^{-1}$... (i)

This implies

$$1 = 10^0 = \frac{10}{10}$$

Now (i) implies that : $\frac{1}{10} = 10^{-1}$

Similarly,

$$10^{-2} = \frac{1}{10 \times 10} = \frac{1}{100}$$

$$10^{-3} = \frac{1}{10 \times 10 \times 10} = \frac{1}{1000}$$

Attention when exponents decreases by 1, then value becomes $\frac{1}{10}$ or one tenth part of initial value. i.e., $10^{-1} = \frac{1}{10}$

Thus $10^{-2} = \frac{1}{10^2}$ or $10^2 = \frac{1}{10^{-2}}$

$$10^{-3} = \frac{1}{10^3} \text{ or } 10^3 = \frac{1}{10^{-3}}$$

Similarly, $3^3 = 3 \times 3 \times 3 = 27$

$$\frac{3^3}{3} = \frac{3 \times 3 \times 3}{3} = \frac{27}{3}$$

$$3^2 = 3 \times 3 = 9$$

$$\frac{3^2}{3} = \frac{3 \times 3}{3} = \frac{9}{3}$$

$$3^1 = 3$$

$$\frac{3^1}{3} = \frac{3}{3}$$

$$3^0 = 1$$

Continuing in same method we may say that :

$$3^{-1} = \frac{1}{3}$$

$$3^{-2} = \frac{1}{3^2}$$

$$3^{-3} = \frac{1}{3^3}$$

$a^{-m} = \frac{1}{a^m}$ where a is any non-zero rational number m being a positive rational number asserting that a^m is multiplicative inverse of a^{-m} .

Try :

- (i) Find the multiplicative inverse of 7.
- (ii) What is the corresponding inverse of $\frac{1}{5}$?
- (iii) What is the corresponding inverse of $\frac{4}{3}$?
- (iv) What is the multiplicative inverse of a ? ($a \neq 0$)

We know that every one is known to the fact that the multiplicative inverse of a ($a \neq 0$) is $\frac{1}{a}$ also denoted by a^{-1} and is pronounced as a to the power -1 . Similarly, the multiplicative inverse of 2 is 2^{-1} and that of 7 is 7^{-1} . The multiplicative inverse of $\left(\frac{3}{4}\right)$ is obviously $\left(\frac{3}{4}\right)^{-1}$.

Thus $\frac{1}{2} = 2^{-1}$, $\frac{1}{7} = 7^{-1}$, $\frac{4}{3} = \left(\frac{3}{4}\right)^{-1}$.

Similarly, $2 = \left(\frac{1}{2}\right)^{-1}$, $7 = \left(\frac{1}{7}\right)^{-1}$, $\frac{4}{3} = \left(\frac{4}{3}\right)^{-1}$,

Again look at, $8 \times \frac{1}{8} = 1$

$$\left[\begin{array}{l} \therefore (\text{Inverse of } 8) = \frac{1}{8} \\ \qquad \qquad \qquad = 8^{-1} \end{array} \right]$$

$$= (8)^{-1}$$

$$8 = \frac{1}{\cancel{1}_8} = \frac{1}{8^{-1}}$$

or $\frac{1}{2^3} = (2^3)^{-1} = 2^{-3}$

Similarly, $\frac{1}{25} = \text{inverse of } 25$

$$\frac{1}{25} = (25)^{-1}$$

So $\frac{1}{5^2} = (5^2)^{-1} = (5)^{-2}$

$$\frac{36}{49} = \left(\text{inverse of } \frac{49}{36} \right) = \left(\frac{49}{36} \right)^{-1}$$

so $\left(\frac{6}{7}\right)^2 = \left[\left(\frac{7}{6}\right)^2 \right]^{-1} = \left(\frac{7}{6}\right)^{-2}$

We can satisfied the rule (1) of exponent raised by the above statement :

$$a^{-3} \times a^3 = a^{-3+3} = a^0 = 1$$

so $a^{-3} = \frac{1}{a^3}$, and $a^3 = \frac{1}{a^{-3}}$

Let us again consider and try to understand as :

$$a^{-n} \times a^n = a^{-n+n} = a^0 = 1$$

$$\text{or } a^{-n} \times a^n = 1$$

$$\text{so } a^{-n} = \frac{1}{a^n}, a^n = \frac{1}{a^{-n}}$$

Conclusion:

1. The negative exponent (-1) of a non-zero rational number equals to its multiplicative inverse. In other words if $\frac{a}{b}$ is any non-zero rational number then its reciprocal $\left(\frac{b}{a}\right)$ is the multiplicative inverse of $\left(\frac{a}{b}\right)$ i.e. $\frac{b}{a} = \left(\frac{a}{b}\right)^{-1}$.
2. For a being non-zero rational number a^n is the multiplicative inverse of a^{-n} also.

Note :

In case, when exponents are negative we may have some relevant results related to exponents.

- (i) $a^m \times a^{-n} = a^{m-n}$
- (ii) $a^{-m} \times a^n = a^{-m+n}$
- (iii) $a^{-m} \times a^{-n} = a^{-m-n} = a^{-(m+n)}$
- (iv) $a^m \div a^n = a^{m-n} = a^m \cdot a^{-n}$
- (v) $a^m \div a^{-n} = a^m \times a^n = a^{m+n}$
- (vi) $a^{-m} \div a^{-n} = a^{-m} \times a^{+n} = a^{-m+n}$
- (vii) $(a^m)^{-n} = a^{-mn} = (a^{-m})^n$
- (viii) $(a^{-m})^{-n} = a^{(-m) \times (-n)} = a^{mn} = (a^m)^n$
- (ix) $(a^{-m}) \times (b^{-m}) = (ab)^{-m}$

Example 14. Simplify $(-5)^{-4} \times (-5)^{-3}$. and $(-7)^{-4} \times (-7)^2$

Solution : (A)

$$\begin{aligned}(-5)^{-4} \times (-5)^{-3} &= \frac{1}{(-5)^4} \times \frac{1}{(-5)^3} \\&= \frac{1}{(-5)^4 \times (-5)^3} = \frac{1}{(-5)^{4+3}} = \frac{1}{(-5)^7} \\&= (-5)^{-7}\end{aligned}$$

Ans. this can be directly simplified as :

(B) Simplify $(-7)^{-4} \times (-7)^2$

Consider

$$\begin{aligned}(-7)^{-4} \times (-7)^2 \\&= \frac{1}{(-7)^4} \times (-7)^2 \\&= \frac{1}{(-7)^4 \times (-7)^{-2}} = \frac{1}{(-7)^{4-2}} = \frac{1}{(-7)^2} = (-7)^{-2}\end{aligned}$$

$$\begin{aligned}(-7)^{-4} \times (-7)^2 \\&= (-7)^{-4+2} = (-7)^{-2}\end{aligned}$$

Example 15. Find the value of $(-25)^{-3}$.

Solution : Consider

$$(-25)^{-3} = \frac{+1}{(-25)^{+3}} = \frac{1}{(-1)^3 (25)^3} = \frac{-1}{(-1)^4 (25)^3} = \frac{-1}{15625}$$

Example 16. Simplify $(2^5 \div 2^8)^5 \times 2^{-5}$

Solution : $(2^5 \div 2^8)^5 \times 2^{-5}$

$$\begin{aligned}&= (2^5 \times 2^{-8})^5 \times 2^{-5} \\&= (2^{5+(-8)})^5 \times 2^{-5} \\&= (2^{-3})^5 \times 2^{-5} \\&= (2^{-15}) \times 2^{-5} \\&= 2^{-15+(-5)} = 2^{-20} = \frac{1}{2^{20}} \quad \text{Ans.}\end{aligned}$$

Example 17. Simplify $\frac{1}{8} \times (3)^{-3}$.

Solution : Consider

$$\begin{aligned}
 \frac{1}{8} \times (3)^{-3} &= \frac{1}{2 \times 2 \times 2} \times (3)^{-3} \\
 &= \frac{1}{2^3} \times (3)^{-3} \\
 &= 2^{-3} \times 3^{-3} = (2 \times 3)^{-3} \\
 &= \frac{1}{6^3}
 \end{aligned}$$

This can be also done as :

$$\frac{1}{8} \times (3)^{-3} = \frac{1}{2^3} \cdot \frac{1}{3^3} = \frac{1}{(2 \times 3)^3} = \frac{1}{6^3}$$

Example 18. Simplify $2^5 \times (16^{-2}) \div 2^{-3}$

Simple method :

This simplification could be done as follows :

Solution : Let us consider that

$$\begin{aligned}
 2^5 \times (16^{-2}) \div 2^{-3} \\
 &= 2^5 \times \frac{1}{(16)^2} \div \frac{1}{2^3} \\
 &= 2^5 \times \frac{1}{(2^4)^2} \div \frac{1}{2^3} \\
 &= 2^5 \times \frac{1}{2^8} \times \frac{2^3}{1} \\
 &= \frac{2^5 \times 1 \times 2^3}{2^8 \times 1} = 1
 \end{aligned}$$

$$\begin{aligned}
 2^5 \times (16)^{-2} \div (2)^{-3} \\
 &= 2^5 \times (2^4)^{-2} \times 2^3 \\
 &= 2^5 \times 2^{-8} \times 2^3 \\
 &= 2^{5-8+3} = 2^0 = 1
 \end{aligned}$$

Example 19. Simplify : $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{2}{3}\right)^{-4}$

Solution :

First method :

$$\begin{aligned}
 \left(\frac{3}{4}\right)^{-2} \div \left(\frac{2}{3}\right)^{-4} \\
 = \frac{1}{\left(\frac{3}{4}\right)^2} \div \frac{1}{\left(\frac{2}{3}\right)^4}
 \end{aligned}$$

Second method :

$$\begin{aligned}
 \left(\frac{3}{4}\right)^{-2} \div \left(\frac{2}{3}\right)^{-4} \\
 &= \left(\frac{1 \times 3}{2 \times 2}\right)^{-2} \times \left(\frac{2}{3}\right)^4 \\
 &= \left(\frac{1}{2}\right)^{-2} \times \left(\frac{3}{2}\right)^{-2} \times \left(\frac{2}{3}\right)^4
 \end{aligned}$$

$$= \frac{1}{3^2} \div \frac{1}{2^4}$$

$$= \frac{4^2}{3^2} \times \frac{2^4}{3^4} = \frac{16 \times 16}{9 \times 81}$$

$$= \frac{256}{729}$$

$$= \left(\frac{2}{1}\right)^2 \times \left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^4$$

$$= 4 \times \left(\frac{2}{3}\right)^6$$

$$= \frac{4 \times 64}{729} = \frac{256}{729}$$

Try :

1. Find the difference between 8^2 and $\left(\frac{1}{2}\right)^{-6}$.

2. Evaluate $\left(\frac{1}{3}\right)^{-3}$.

3. Evaluate $(4)^{-4}$.

4. Which one of these two is greater in each sub-part.

(i) 5^3 and 5^{-3} (ii) 2^2 and 2^{-2} (iii) 2^3 and $\frac{1}{2^{-3}}$

Exercise-2(d)

1. Find the values of $3^4 \times 3^5 \times 3^{-9}$.

2. Simplify $\left(\frac{2}{3}\right)^2 \times \left(\frac{4}{9}\right)^2 \times \left(\frac{2}{3}\right)^{-4}$.

3. The value of $\left(\frac{1}{2}\right)^{-2} \times 2^2$ is :

(i) 2

(ii) 4

(iii) 8

(iv) 16

4. The value of $3^{-2} \times 3^5$ is :

(i) 3

(ii) 9

(iii) $\frac{1}{27}$

(iv) 27

5. Attempt to simplify the following :

(i) $\left(\frac{1}{3}\right)^{-2} \times 3^2 \div 3^{-3}$ (ii) $7^4 \times \left(\frac{1}{7}\right)^3 \div 7$ (iii) $3^0 + 3^{-1} + 3^{-2}$ (iv) $(3^3)^3 \div (3)^{25} \times (3^4)^4$

6. Simplify $(2 \times 3)^6 \times 6^{-3}$

7. Simplify : $\left\{ (4 \times 5)^{-2} \div \left(\frac{1}{10} \right)^2 \right\} + \frac{3}{4}$

8. Simplify : $\left\{ \left(\frac{3}{5} \right)^0 + \left(\frac{3}{5} \right)^1 + \left(\frac{3}{5} \right)^2 \right\} \div \left(\frac{7}{5} \right)^2$

9. Simplify : $2^{-2} - \{-2^{-3} - (2^{-2} - 3^{-2})\}$

3.7. To express numbers of smaller and larger magnitude in the form of an exponent.

Consider the following :

$$54 = 5.4 \times 10 = 5.4 \times 10^1$$

$$540 = 5.4 \times 100 = 5.4 \times 10^2$$

$$5400 = 5.4 \times 1000 = 5.4 \times 10^3$$

$$54000 = 5.4 \times 10000 = 5.4 \times 10^4 \text{ and likewise}$$

We have tried to express 54, 540 and 5400 in standard form—

Similarly that

$$0.54 = 5.4 \times \frac{1}{10} = 5.4 \times 10^{-1}$$

$$0.054 = 5.4 \times \frac{1}{100} = 5.4 \times 10^{-2}$$

$$0.0054 = 5.4 \times \frac{1}{1000} = 5.4 \times 10^{-3}$$

$$0.00054 = 5.4 \times \frac{1}{10000} = 5.4 \times 10^{-4}$$

We have again tried to express smaller and smaller numbers in standard form. Now it is important to note the definition of a number in standard form.

Any number can be expressed as a product of a decimal number k , in between 1.0 and 10.0 (including 1 and excluding 10) and some power of 10. Such type of representation of a number is called standard form or scientific notation. In standard form or scientific notation is represented by $k \times 10^n$. Where $1 \leq K < 10$ and n is any integer.

Comparison between any two numbers that are either a number with greater magnitude or it may be of lesser magnitude, what so ever the case may.

Comparison between smaller and greater numbers.

The diameter of Sun is 1.4×10^9 m and the diameter of Earth is 1.2756×10^7 m. Now we compare these two, then

$$\begin{aligned} \frac{\text{Diameter of sun}}{\text{Diameter of earth}} &= \frac{1.4 \times 10^9}{1.2756 \times 10^7} = \frac{1.4 \times 10^{9-7}}{1.2756} \\ &= \frac{1.4 \times 10^2}{1.2756} \\ &\approx 100 \end{aligned}$$

Thus the diameter of sun is 100 times greater than the diameter of Earth.

To express a number of high magnitude in standard form.

We know that numbers with larger magnitude can be expressed in a simplified and significant way with the help of exponents. Let us try to write a number large in magnitude in standard form with the help of exponents.

We have just now learnt that in order to convert a number into standard form we use a number from 1 to a number less than 10.

For example let us try to find the standard form of the number 5985.

$$\begin{aligned} \text{We have } 5985 &= 5.985 \times 1000 \\ &= 5.985 \times 10^3 \end{aligned}$$

(decimal is shifted by 3 places to left side)

The mass of Earth :

$$\begin{aligned} &= 5976,000,000,000,000,000,000,000 \text{ kg} \\ &= 5.976 \times 10^{24} \text{ kg.} \end{aligned}$$

Now everybody will agree to the fact that numbers with larger magnitude or with very minute seized magnitude could be well understood written without much effort and found easy to compare of written instandard form approach with these facilities provided by scientific about expressing number could very well be written easily as follows :

Consider

$$150,000,000,000 = 1.5 \times 10^{11}$$

(Decimal is tried shifting 11 places to the left side)

also that,

$$0.000009 = \frac{9}{1000000} = \frac{9}{10^6} = 9 \times 10^{-6}$$

(The decimal point is shifted to right by six places)

It must required to mention that while adding any two number expressed in standard form, both the numbers, are made as equal powers of 10 is concerned.

Observe:

5415 can be expressed as follows :

$$5415 = 541.5 \times 10 \quad \text{or} \quad = 54.15 \times 100 \quad \text{or} \quad = 541.5 \times 10^1 \quad \text{or} \quad = 54.15 \times 10^2$$

All these representation for 5415 are of course not in standard form.

Example 20 : Express the following in standard form :

- (i) 63000 (ii) 100000 (iii) 0.000045 (iv) 0.0001

Solution :

(i) $63000 = 6.3 \times 10000 = 6.3 \times 10^4$

(clearly $k = 6.3$ and $n = 4$ for the starndard $k \times 10^n$)

(ii) $100000 = 1 \times 100000 = 1 \times 10^5 = 1.0 \times 10^5$

(Here $k = 1$ and $n = 5$)

(iii) $0.000045 = \frac{4.5}{100000} = \frac{4.5}{10^5} = 4.5 \times 10^{-5}$

(iv) $0.0001 = \frac{1}{10000} = 1 \times \frac{1}{10^4} = 1.0 \times 10^{-4}$

Example 21. A person consumes on an average 3000 Calorie energy every day to his daily meal. Find the total Calorie consumed by him in scientific standard form during one year.

Solution : $\therefore$ The energy consumed in one day = 3000 Calorie
 $\therefore$ The energy consumed in one year = 365×3000 Calorie
= 1095000 Calorie
= 1.095×10^6 Calorie

Example 22. An estimate say that Indian Railway makes 1 crore 30 lakh passengers travel from one place to other in one day. Now find out how many train passenger in standard form travel in 30 days likewise.

Solution : $\therefore$ Number of passenger travelled in one day = 1,30,00,000
 $\therefore$ Number of passenger travelled in 30 days = $1,30,00,000 \times 30$
= 39,00,00,000
= 3.9×10^8

Example 23. The population of India at any day was found to be 1,00,84,35,405 as per census. Express in scientific form.

Solution : The popular under consideration = 1,00,84,35,405
= $1.008435405 \times 1000000000$
= 1.008435405×10^9
 $\approx 1.008 \times 10^9$

Attempt to find

1. Express 425000 in standard form.
2. Express 0.000035 in standard form.
3. Express 3.54×10^5 in general form.
4. Express 7.54×10^{-4} in general form.

Exercise-2(e)

1. Express 62,00,00,000 in standard form.
2. Express 0.00008 in standard form.
3. Scientists presume that Moon came into existence back some 460 crore years. Estimate the age of Moon in standard form.

4. The distance of sun from the earth is about 15,00,00,000 km. Express it in standard scientific form.
5. The mass of earth is about 5.98×10^{24} kg. Find how many zeros are required to be placed after 598. While expressing in standard form.
6. Express the following in standard form :
 - (i) 425000
 - (ii) Two crore, twenty lakh
 - (iii) 0.000045
 - (iv) 0.005
7. Express the following in general form :
 - (i) 2.5×10^4
 - (ii) 1.75×10^6
 - (iii) 1.21×10^{-8}
 - (iv) 4.5×10^{-5}
 - (v) 2.3×10^{-7}
 - (vi) 2.5×10^{-4}
8. The mass of one electron is 9.107×10^{-28} gram. If it is changed into simple decimal fraction. How many zeroes will be between decimal point (.) and number (9).

Ability Exercise-2

1. Express $(-8)^7$ in factor form.
2. Find $(-1)^{999}$.
3. Express $3^8 \times 3^{12}$ in exponential form to using the base 3.
4. Express $9^7 \div 9^3$ in exponential form using base as 9.
5. Find $8^5 - 5$.
6. Express $(2^3)^8$ in exponential form using base as 2.
7. Express (-128) in exponential form using base as (-2) .
8. Simplify to find $\left(\frac{7}{9}\right)^2 \div \left(\frac{14}{3}\right)^2$.
9. Find the multiplicative inverse of $(-4)^4$.
10. The average distance of moon from earth surface is 384400000 metre. Express the distance in scientific standard form.
11. The distance travelled by light rays in one year is about 9460500000000000 metre.
12. Simplify to find number of digits there in $2^{38} \times 5^{32}$ [Hint $2^{38} \times 5^{32} = 2^6 \times 2^{32} \times 5^{32}$
 $= 2^6 \times (10)^{32}$
 $= 6.4 \times 10^{33}$]

13. There are 10,00,000 bacteria in 1 gram of curd. Express the population of bacteria in scientific conversion.
14. Every one in a village school had gone to practice washing hands for 2 minutes each on the eve of world clear day. If all 64 students in the class successively took their time turn by turn. Express the time consumed by all the students in the class in exponential form.
15. Most of the population in a town were exposed to breathing problem. A test conducted exposed that there were 2000 harmful bacteria present in the polluted environment air per cubic metre and that multiply 100 times in a week. Now express number of bacteria presenting in our vicinity in standard form of number. Express the number of bacteria after three week's in standard form.

We have learnt:

1. Number can be expressed in an exponential form. Use of exponent/power/indices makes it easy to read, write and compare pretty large and quite small numbers. This also makes when applied mathematical operations easy over to these numbers in exponent form.
2. The numbers in exponential form do follow some of the laws as follows. Suppose a and b are any two non-zero rational numbers and m and n being some whole numbers then:
 - (i) $a^m \times a^n = a^{m+n}$
 - (ii) $a^m \div a^n = a^{m-n}$
 - (iii) $(a^m)^n = a^{mn}$
 - (iv) $a^m \times b^m = (ab)^m$
 - (v) $a^m \div b^m = \left(\frac{a}{b}\right)^m$
 - (vi) $a^0 = 1$
 - (vii) $(-1)^{\text{even number}} = 1$
 - (viii) $(-1)^{\text{odd number}} = -1$
3. Expressing a number in scientific notation or standard form, to do this, the number is expressed as a decimal number between 1.0 and 10.0 (which contains 1.0 and does not include 10) and 10 is taken to be the product of any power.

Remember that

1. The measurement of a bacteria is 0.00000005 cm or 5.0×10^{-8} metre.
2. Distance of sun from earth surface is 1,49,60,00,00,000 metre or 1.49×10^{11} metre.
3. The distance of moon from earth surface is about 38,44,67,000 metre or 3.84467×10^8 metre.
4. On an average there are 1,00000000000 or 1.0×10^{11} stars in our galaxy.

5. There lies approximately $1,35,30,00,000 \text{ km}^3$ or $1.353 \times 10^9 \text{ km}^3$ sea water on the earth.
6. Biologists claim, in one gram of human waste there are 100 bacterial eggs, 1000 bacterial cells, 1000000 bacterias and 10000000 viruses.

ANSWERS

Exercise-2(a)

1. (ii) $\frac{-32}{243}$; 2. (i) 5^5 ; 3. (iii) 128; 4. 5^2 square metre; 5. (i) 144; (ii) 8000; 6. 5^6

Exercise-2(b)

1. (i) 3^{15} ; (ii) 6; (iii) 5^5 ; (iv) 210; (v) $\left(\frac{2}{3}\right)^5$; (vi) $\left(\frac{4}{9}\right)^2$; 2. (i) 1; (ii) 1; (iii) $\frac{1}{2}$; (iv) $\left(\frac{3}{4}\right)^3$; 3. 1; 4. 1; 5. $(3)^{-6}$; 6. (i) 4; (ii) 8×7 or 56; (iii) 5; (iv) 24; (v) 3; 7. (i) $(40)^5$; 8. (iii) 9 9. (iv) 180

Exercise-2(c)

1. $(12)^6$; 2. 5^6 , 25^3 , 125^2 ; 3. $(.01)^2$; 4. $\left(\frac{-7}{8}\right)^3$; 5. $\left(\frac{10}{11}\right)^3$; 6. $\left(\frac{32}{33}\right)^2$; 7. $\left(\frac{8}{9}\right)^3$; 8. (ii) $(0.001)^2$; 9. (iv) 0.000125; 10. (ii) $\frac{4}{5}$.

Exercise-2(d)

1. 1 2. $\left(\frac{4}{9}\right)$; 3. (iv) 16; 4. (iv) 27; 5. (i) $(3)^7$; (ii) 1; (iii) $\frac{13}{9}$; (iv) 1; 6. 216; 7. 1; 8. 1; 9. $\frac{37}{72}$

Exercise-2(e)

1. 6.2×10^8 ; 2. 8.0×10^{-5} ; 3. 4.6×10^9 years; 4. $1.5 \times 10^8 \text{ km}$; 5. 20; 6. (i) 4.25×10^6 ; (ii) 4.5×10^{-5} ; (iii) 2.2×10^7 (iv) 2.5×10^{-3} ; 7. (i) 25000; (ii) 1750000; (iii) 0.0000000121; (iv) 0.000045; (v) 0.00000023; (vi) 0.00025; 8. 27

Ability Exercise-2

1. $(-8) \times (-8) \times (-8) \times (-8) \times (-8) \times (-8) \times (-8)$; 2. (-1) ; 3. $(3)^{20}$; 4. 9^4 ; 5. 1; 6. 2^{24} ; 7. $(-2)^7$; 8. $1/36$; 9. 4^{-4} or $(-4)^{-4}$; 10. $3.844 \times 10^8 \text{ m}$; 11. $9.4605 \times 10^{15} \text{ m}$; 12. 6.4×10^{33} ; 13. 10^{37} ; 14. 2^7 minutes; 15. 2.0×10^9

