



- Operations and properties of addition, subtraction and multiplication on rational numbers.
- Distribution law of multiplication over addition of rational numbers.
- 1 and 0 in the form of rational number and their properties.
- Additive inverse and multiplicative inverse of a rational number.
- Division of a rational number by a rational number.
- Absolute value of a rational number.
- Rational numbers between two rational numbers.
- Mutual conversion of rational number and decimal number.

1.1. Introduction

In the previous class we saw that why rational numbers are necessary, for this we have obtained the concept of rational numbers by the expansion of set of integers. We have defined rational number in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$. We have studied in all respects the equivalency, their simplest (standard) form, their representation on the number line, their comparison to each other, in detail. Now in this unit we will study the main operations and their properties on rational numbers. Together with this we will learn absolute value of rational number, insertion of rational numbers between two rational numbers, to express the rational number in the form of decimal number and to express decimal number in the form of rational number.

1.2. Operations on rational numbers

We know how the operations of addition, subtraction, multiplication and division are done on integers and fractions. Now, we shall study the above operations in reference to rational numbers.

1.2.1. Addition of rational numbers

Let us add two rational numbers having equal denominators.

Example 1: Suppose we have to add $\frac{3}{5}$ and $\frac{-13}{5}$.

Solution :

$$\begin{aligned} & \frac{3}{5} + \left(\frac{-13}{5} \right) \\ &= \frac{3-13}{5} \\ &= \frac{-10}{5} = \frac{-2}{1} = -2 \end{aligned}$$

This answer is obtained by representation of these rational numbers on number line.

Solution by number line

We have learn to represent the rational numbers on the number line in the lower class.

$$\begin{aligned} &= \left(\frac{-2}{1} \right) \text{ simplest form} \\ &= -2 \end{aligned}$$

Clearly, we obtained same answer also from the number line.

Do these:

Find the value of $\frac{4}{7} + \frac{8}{7}$

Find the value of $\frac{5}{9} + \left(\frac{-4}{9} \right)$

Example 2 : Find the value of $\frac{5}{-9} + \frac{13}{9}$

Solution : $\frac{5}{-9} = \frac{5 \times (-1)}{-9 \times (-1)} = \frac{-5}{9}$ (Multiplying (-1) in numerator and denominator for making the negative denominator to positive)

$$\therefore \left(\frac{5}{-9} \right) + \frac{13}{9} = \frac{-5}{9} + \frac{13}{9}$$

$$\begin{aligned}
 &= \frac{(-5)+13}{9} \\
 &= \frac{-5+13}{9} \\
 &= \frac{8}{9}
 \end{aligned}$$

Try :

Simplify the following :

$$(i) = \frac{4}{7} + \left(\frac{-8}{7}\right) \quad (ii) = \frac{-4}{9} + \left(\frac{-2}{-9}\right)$$

Find the answer by both methods and verify that if both have same answers?

Thus, we see that:

1. Sum of rational number with same denominators

$$= \frac{\text{Sum of a numerators of rational numbers}}{\text{Same denominator}}$$

i.e.,

$$\text{Sum of two rational numbers } \frac{p}{q} \text{ and } \frac{r}{q} = \frac{p}{q} + \frac{r}{q} = \frac{p+r}{q}$$

2. The rational number so obtained is written in the simplest form.

Sum of rational numbers when their denominators are not same :

Example 1. Find the sum of $\frac{1}{4}$ and $\frac{-5}{6}$

First method : By number line

$$\text{Solution : } \frac{1}{4} + \left(\frac{-5}{6}\right) = \frac{1 \times 3}{4 \times 3} + \left(\frac{-5 \times 2}{6 \times 2}\right) = \frac{3}{12} + \frac{-10}{12} \text{ (same denominator)}$$

Second method : To make denominators equal

$$\begin{aligned}
 \text{Solution : } \frac{1}{4} + \left(\frac{-5}{6}\right) &= \frac{3}{12} + \left(\frac{-10}{12}\right) \\
 &= \frac{3+(-10)}{12}
 \end{aligned}$$

$$= \frac{3-10}{12}$$

$$= \left(\frac{-7}{12} \right)$$

Example 2 : Find the value of $\frac{2}{15} + \left(\frac{9}{-10} \right)$

Solution : $\frac{2}{15} + \left(\frac{9}{-10} \right)$

$$= \frac{2}{15} + \left(\frac{-9}{10} \right)$$

$$= \frac{2 \times 2}{15 \times 2} + \left(\frac{-9 \times 3}{10 \times 3} \right)$$

$$= \frac{4}{30} + \left(\frac{-27}{30} \right)$$

$$= \frac{4 + (-27)}{30}$$

$$= \frac{-23}{30}$$

Try :

Simplify the following:

(i) $\left(\frac{-5}{12} \right) + \frac{5}{18}$

(ii) $\frac{7}{12} + \left(\frac{-8}{15} \right)$

(iii) $\left(\frac{-9}{-10} \right) + \left(\frac{2}{-15} \right)$

We add two rational numbers having different denominators like fractions. First we find the L.C.M. of its denominators, then find such equivalent rational numbers whose denominators are equal to the L.C.M. obtained and obtain the answer by adding the numerators of both rational numbers.

Thus, we see that :

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers in standard form, then before adding, express them in the form of $\frac{p}{q}$ and $\frac{r}{q}$ respectively, where q is the L.C.M. of b and d and p and r are proportional quantities of a and c .

If we want to use the method of adding the fractions in the adding of rational numbers $\frac{-7}{-8}$ and $\frac{5}{-6}$, then we have to find the L.C.M. of (-8) and (-6) . But we know that the L.C.M. of only positive integers is defined. Thus we will have to make positive the denominators without changing the value of $\frac{-7}{-8}$ and $\frac{5}{-6}$.

How ?

Each rational number will have to be changed to such equivalent rational number whose denominator is positive. Thus-

$$\frac{-7}{-8} = \frac{-7 \times (-1)}{-8 \times (-1)} = \frac{7}{8}$$

and $\frac{5}{-6} = \frac{5 \times (-1)}{-6 \times (-1)} = \frac{-5}{6}$

Now we can obtain the solution as example 2.

Thus, we see that :

If the denominator of a rational number is negative, then the denominator is made positive by multiplying (-1) in both numerator and denominator.

i.e.

If $\frac{p}{-q}$ and $\frac{-p}{-q}$ are two rational numbers having negative denominators, then

$$\frac{p}{-q} = \frac{-p}{q} \text{ and } \frac{-p}{-q} = \frac{p}{q}$$

Try :

Express the denominators of the following rational numbers in the positive form :

(i) $\frac{4}{-15}$ (ii) $\frac{-9}{-13}$

Exercise 1 (a)

1. Simplify :

(i) $\frac{4}{5} + \left(\frac{-2}{5}\right)$

(ii) $\left(\frac{-4}{-5}\right) + \left(\frac{2}{-5}\right)$

2. Simplify with the help of number line :

(i) $\frac{3}{4} + \left(\frac{-3}{2}\right)$

(ii) $\left(\frac{-3}{-4}\right) + \left(\frac{5}{-6}\right)$

3. Simplify by taking L.C.M. of denominators :

(i) $\frac{5}{12} + \left(\frac{-7}{16}\right)$

(ii) $\left(\frac{-3}{-11}\right) + \left(\frac{-4}{33}\right)$

4. Find the value of :

(i) $2 + \left(\frac{-11}{9}\right)$

(ii) $(-3) + \left(\frac{-2}{-3}\right)$

Properties of addition of rational numbers :

1. Closure property :

Example : Add rational numbers $\left(\frac{2}{-15}\right)$ and $\left(\frac{-9}{10}\right)$.

Is the obtained sum a rational number?

Solution : $\left(\frac{2}{-15}\right) + \left(\frac{-9}{10}\right)$

$$= \frac{(-1) \times 2}{(-1)(-15)} + \left(\frac{-9}{10}\right) \quad (\text{To make positive the denominator } (-1) \text{ is multiplied to both numerator and denominator})$$

$$= \left(\frac{-2}{15}\right) + \left(\frac{-9}{10}\right)$$

$$= \frac{-2 \times 2 + (-9) \times 3}{30} \quad (\text{L.C.M. of } 15, 10 = 30)$$

$$= \frac{-4 - 27}{30}$$

$$= \frac{-31}{30}, \text{ which is a rational number}$$

$\therefore$ Obtained sum is a rational number.

Try :

Add following pairs of rational numbers and verify that sum is a rational number.

$$(i) = \left(\frac{-3}{-11}\right) + \left(\frac{-4}{-33}\right)$$

$$(ii) = \left(\frac{-7}{24}\right) + \left(\frac{2}{-9}\right)$$

Thus, we see that the sum of rational numbers is also a rational number, i.e.

If $\frac{p}{q}$ and $\frac{r}{s}$ are rational number, then their sum $\left(\frac{p}{q} + \frac{r}{s}\right)$ is also a rational number.

Therefore, 'process of addition' in rational numbers satisfies the closure property.

2. Commutative Property :

Example 1. Express each of the following rational numbers in the form of a rational number.

$$(i) \left(\frac{-5}{6}\right) + \frac{2}{7}$$

$$(ii) \frac{2}{7} + \left(\frac{-5}{6}\right)$$

Are these both rational numbers equal?

Solution :

$$(i) \left(\frac{-5}{6}\right) + \frac{2}{7}$$

$$(ii) \frac{2}{7} + \left(\frac{-5}{6}\right)$$

$$= \frac{(-35) + 12}{42}$$

$$= \frac{12 + (-35)}{42}$$

$$= \frac{-35 + 12}{42}$$

$$= \frac{12 - 35}{42}$$

$$= \frac{-23}{42}$$

$$= \frac{-23}{42}$$

In both cases, rational numbers obtained from sum are equal.

Try :

Simplify :

$$(i) \left(\frac{-5}{-8} \right) + \left(\frac{-7}{12} \right)$$

$$(ii) \left(\frac{-7}{12} \right) + \left(\frac{-5}{-8} \right)$$

and verify that rational numbers obtained from both simplifications are equal.

Thus, we see that on adding two rational numbers in any order, the sum is same.

i.e.

$$\text{If } \frac{p}{q} \text{ and } \frac{r}{s} \text{ are two rational numbers, then } \frac{p}{q} + \frac{r}{s} = \frac{r}{s} + \frac{p}{q}$$

Therefore, in rational numbers 'operation of addition' satisfies the commutative property.

3. Associative property

Example 1. Express the sum of the following rational numbers in the form of a rational number.

$$(i) \left[\left(\frac{-2}{3} \right) + \frac{3}{4} \right] + \left(\frac{1}{-5} \right)$$

$$(ii) \left(\frac{-2}{3} \right) + \left[\frac{3}{4} + \left(\frac{1}{-5} \right) \right]$$

What is the relation between the rational numbers thus obtained?

Solution :

$$(i) \left[\left(\frac{-2}{3} \right) + \frac{3}{4} \right] + \left(\frac{1}{-5} \right)$$

$$= \left[\frac{(-8)+9}{12} \right] + \left(\frac{-1}{5} \right)$$

$$= \left(\frac{-8+9}{12} \right) + \left(\frac{-1}{5} \right)$$

$$= \frac{1}{12} + \left(\frac{-1}{5} \right)$$

$$= \frac{5+(-12)}{60}$$

$$= \frac{5-12}{60}$$

$$(ii) \left(\frac{-2}{3} \right) + \left[\frac{3}{4} + \left(\frac{1}{-5} \right) \right]$$

$$= \left(\frac{-2}{3} \right) + \left[\frac{15+(-4)}{20} \right]$$

$$= \left(\frac{-2}{3} \right) + \left(\frac{15-4}{20} \right)$$

$$= \left(\frac{-2}{3} \right) + \frac{11}{20}$$

$$= \frac{(-40)+33}{60}$$

$$= \frac{-40+33}{60}$$

$$= \frac{-7}{60}$$

$$= \frac{-7}{60}$$

You see that in both cases rational numbers obtained from sum are equal.

Try :

Express the sum of following rational numbers in the form of a rational numbers and verify that the both rational numbers obtained from sum are equal.

$$(i) = \left[\left(\frac{-4}{7} \right) + \frac{1}{4} \right] + \left(\frac{1}{-2} \right) \qquad (ii) = \left(\frac{-4}{7} \right) + \left[\frac{1}{4} + \left(\frac{1}{-2} \right) \right]$$

Thus, you see that :

To add three rational numbers, add third number to the sum of first two rational numbers or add first number to the sum of last two numbers, in both conditions, the sum is same.

Conclusion :

If $\frac{p}{q}, \frac{r}{s}$ and $\frac{t}{u}$ are any three numbers, then $\left(\frac{p}{q} + \frac{r}{s} \right) + \frac{t}{u} = \frac{p}{q} + \left(\frac{r}{s} + \frac{t}{u} \right)$

Therefore, in rational numbers, 'operation of addition' satisfies the associative property.

We can add three or more rational numbers without using bracket due to this property of sum of rational numbers.

Example 2. Find the value of $\left(\frac{-5}{6} \right) + \frac{17}{10} + \left(\frac{7}{-12} \right)$

Solution : $\left(\frac{-5}{6} \right) + \frac{17}{10} + \left(\frac{7}{-12} \right) = \left(\frac{-5}{6} \right) + \frac{17}{10} + \left(\frac{-7}{12} \right)$

$$= \frac{(-50) + 102 + (-35)}{60} \qquad (\text{L.C.M. of 6, 10, 12} = 60)$$

$$= \frac{-50 + 102 - 35}{60} = \frac{17}{60}$$

Exercise 1 (b)

1. Tell true/false in the following statements :

(i) $\left(\frac{-4}{5}\right) + \left(\frac{-4}{5}\right) = \left(\frac{-8}{5}\right)$

(ii) $\left(\frac{-6}{7} + 6\right)$ is not rational number.

(iii) The sum of two rational numbers remains the same by changing the order of addition.

(iv) The sum of three rational numbers is not a rational number.

2. Tell by adding following rational numbers that the sum is rational number or not :

(i) $\left(\frac{-3}{4}\right)$ and 3

(ii) (-1) and $\left(\frac{-2}{3}\right)$

3. By solving both sides, verify that,

(i) $\left(\frac{-5}{8}\right) + \left(\frac{-9}{13}\right) = \left(\frac{-9}{13}\right) + \left(\frac{-5}{8}\right)$

(ii) $3 + \left(\frac{-7}{12}\right) = \left(\frac{-7}{12}\right) + 3$

(iii) $\left[\left(\frac{-4}{7}\right) + \frac{1}{4}\right] + \left(\frac{7}{-16}\right) = \left(\frac{-4}{7}\right) + \left[\frac{1}{4} + \left(\frac{7}{-16}\right)\right]$

4. If A and B are two rational numbers and $A + B = \frac{-15}{11}$, then find B + A.

5. Fill up the following blanks in your exercise book.

(i) $(...) + \left(\frac{-12}{7}\right) = \left(\frac{-12}{7}\right) + \left(\frac{5}{-11}\right)$

(ii) $[(-10) + (...)] + \left(\frac{-7}{12}\right) = (-10) + \left[\frac{5}{6} + \left(\frac{-7}{12}\right)\right]$

6. Simplify :

(i) $\frac{3}{4} + \left(\frac{-8}{9}\right) + \frac{5}{8}$

(ii) $\left(\frac{-9}{10}\right) + \frac{22}{15} + \left(\frac{13}{-20}\right)$

7. Solve the following by using the properties of addition :

$$(i) \frac{3}{7} + \left(\frac{-5}{14}\right) + \left(\frac{-11}{14}\right)$$

$$(ii) \frac{2}{5} + \left(\frac{-8}{-3}\right) + \left(\frac{4}{-5}\right) + \left(\frac{-2}{3}\right)$$

1.2.2 Subtraction of rational numbers

Subtraction of rational numbers when their denominators are same :

Example 1. Subtract $\left(\frac{-12}{7}\right)$ from $\left(\frac{-4}{7}\right)$

First Method : Solution by Number line

Displacement from one point to other point on number line is $\frac{1}{7}$.

Second method :

$$\begin{aligned}\text{Solution : } \left(\frac{-4}{7}\right) - \left(\frac{-12}{7}\right) &= \frac{(-4) - (-12)}{7} \\ &= \frac{-4 + 12}{7} \\ &= \frac{8}{7}\end{aligned}$$

Example 2. Find the value of $\left(\frac{3}{-4}\right) - \left(\frac{-5}{-4}\right)$.

$$\begin{aligned}\text{Solution : } \left(\frac{3}{-4}\right) - \left(\frac{-5}{-4}\right) &= \frac{(-1) \times 3}{(-1) \times (-4)} - \frac{(-1) \times (-5)}{(-1) \times (-4)} \\ &= \left(\frac{-3}{4}\right) - \frac{5}{4} \\ &= \frac{(-3) - 5}{4} \\ &= \frac{-3 - 5}{4} = \frac{-8}{4} \\ &= \frac{-2 \times 4}{4} = -2 \text{ (simplest form)}\end{aligned}$$

Try :

Simplify the following :

(i) $\left(\frac{7}{-11}\right) - \left(\frac{-3}{11}\right)$

(ii) $\left(\frac{-9}{-13}\right) - \left(\frac{-12}{13}\right)$

Thus, we see that :

Difference of rational numbers having same denominators =

Difference of numbers of rational numbers

Equal denominators

It means

If $\frac{p}{q}$ and $\frac{r}{q}$ be two rational numbers, then $\frac{p}{q} - \frac{r}{q} = \frac{p-r}{q}$

Subtraction of rational number when their denominators are not equal :

Example 1. Subtract $\left(\frac{5}{-4}\right)$ from $\left(\frac{-5}{6}\right)$

First method : By number line

Solution : $\frac{5}{-6} = \frac{5 \times (-1)}{-6 \times (-1)}$

$$= \frac{-5}{6}$$

$$\frac{-5}{4} = \frac{-5 \times 3}{4 \times 3} \text{ and}$$

$$\frac{-5}{6} = \frac{-5 \times 2}{6 \times 2}$$

$$= \frac{-15}{12}$$

$$= \frac{-10}{12} \text{ (same denominator)}$$

$$\therefore \left(\frac{-5}{4}\right) - \left(\frac{5}{-6}\right) = \left(\frac{-5}{4}\right) - \left(\frac{-5}{6}\right)$$

$$= \left(\frac{-15}{12}\right) - \left(\frac{-10}{12}\right) \quad (\text{L.C.M. of 4 and 6} = 12)$$

$$= \frac{-15}{12} + \frac{10}{12}$$

$\therefore \frac{-5}{4} - \left(\frac{5}{-6}\right) = 15$ displacement from 0 in negative direction on number line

and then from there 10 displacement in positive direction.

$= 5$ displacement from 0 in negative direction on number line.

$$= \left(\frac{-5}{12}\right)$$

Second method : To make denominators equal

Solution : $\left(\frac{-5}{4}\right) - \left(\frac{5}{-6}\right) = \left(\frac{-5}{4}\right) - \left(\frac{-5}{6}\right)$

$$= \frac{-5}{4} + \frac{5}{6}$$

$$= \frac{-15}{12} + \frac{10}{12} \quad (\text{L.C.M. of 4 and 6} = 12)$$

$$= \frac{-15+10}{12}$$

$$= \frac{-5}{12}$$

Try :

Simplify :

(i) $\left(\frac{-5}{12}\right) - \left(\frac{7}{-18}\right)$ (ii) $\left(\frac{-5}{16}\right) - \left(\frac{-11}{-12}\right)$ (iii) $\frac{4}{15} - \left(\frac{-9}{10}\right)$

Thus, we see that

If $\frac{a}{b}$ and $\frac{c}{d}$ are in the standard forms of two rational numbers, then before subtraction, express them respectively in the form of $\frac{p}{q}$ and $\frac{r}{q}$, where q is L.C.M. of b and d .

Example 2. The sum of two rational numbers is $\left(\frac{-1}{2}\right)$. If one of these numbers be $\left(\frac{-8}{19}\right)$, find second number.

Solution : Second number will be obtained by subtracting first number $\left(\frac{-8}{19}\right)$ from the sum $\left(\frac{-1}{2}\right)$.

$$\text{Therefore, second number} = \left(\frac{-1}{2}\right) - \left(\frac{-8}{19}\right)$$

$$= \left(\frac{-1}{2}\right) + \frac{8}{19}$$

$$= \frac{-19}{38} + \frac{16}{38} \quad (\text{L.C.M. of denominators 2 and 19} = 38)$$

$$= \frac{-19+16}{38}$$

$$= \frac{-3}{38}$$

Example 3. Simplify :

$$\left(\frac{-2}{3}\right) + \frac{5}{9} - \left(\frac{-7}{6}\right)$$

Solution : $\left(\frac{-2}{3}\right) + \frac{5}{9} - \left(\frac{-7}{6}\right)$

$$= \frac{-2}{3} + \frac{5}{9} + \frac{7}{6}$$

$$= \frac{-12}{18} + \frac{10}{18} + \frac{21}{18} \quad (\text{L.C.M. of denominators 3, 6, 9} = 18)$$

$$= \frac{-12+10+21}{18}$$

$$= \frac{19}{18} = 1\frac{1}{18}$$

Properties of operation of subtraction of rational numbers :

1. Closure property

Example 1 : Subtract $\frac{3}{-7}$ from $\frac{-5}{8}$. is obtained answer is a rational number?

Solution :

$$\begin{aligned} & \left(\frac{-5}{8}\right) - \left(\frac{3}{-7}\right) \\ &= \left(\frac{-5}{8}\right) - \left(\frac{-3}{7}\right) \\ &= \frac{-5}{8} + \frac{3}{7} \\ &= \frac{-35}{56} + \frac{24}{56} \\ &= \frac{-35 + 24}{56} \\ &= \frac{-11}{56} \end{aligned}$$

Obtained answer is a rational number.

Try :

Simplify the following and verify that obtained answer is a rational number :

(i) $\left(\frac{-5}{27}\right) - \left(\frac{10}{-9}\right)$

(ii) $\frac{7}{10} - \left(\frac{-5}{8}\right)$

Thus, we see that :

If $\frac{p}{q}$ and $\frac{r}{s}$ are rational numbers, then their difference $\left(\frac{p}{q} - \frac{r}{s}\right)$ is a rational number.

Therefore, in rational numbers operation of subtraction satisfies the closure property.

2. Commutative property :

Example 1. Express each of the following in the form of a rational number.

$$(i) \quad \frac{11}{24} - \left(\frac{-8}{9} \right)$$

$$= \frac{11}{24} + \frac{8}{9}$$

$$= \frac{33}{72} + \frac{64}{72}$$

$$= \frac{33+64}{72}$$

$$= \frac{97}{72}$$

$$(ii) \quad \frac{-8}{9} - \frac{11}{24}$$

$$= \frac{-64}{72} - \frac{33}{72}$$

$$= \frac{-64-33}{72}$$

$$= \frac{-97}{72}$$

$$= \frac{-97}{72}$$

In the two cases, the obtained rational numbers obtained by subtracting are not equal.

$$\text{It means } \frac{11}{24} - \left(\frac{-8}{9} \right) \neq \left(\frac{-8}{9} \right) - \frac{11}{24}$$

Try :

Express each of the following in the form of a rational number and verify that both rational numbers are not equal.

$$(i) \quad \left(\frac{-6}{13} \right) - \left(\frac{5}{-11} \right)$$

$$(ii) \quad \left(\frac{5}{-11} \right) - \left(\frac{-6}{13} \right)$$

Thus, we see that :

On subtracting two rational numbers from each other in any order, the difference does not remain same.

It means

$$\text{If } \frac{p}{q} \text{ and } \frac{r}{s} \text{ are any two rational numbers, then } \frac{p}{q} - \frac{r}{s} \neq \frac{r}{s} - \frac{p}{q}$$

Therefore, commutative property does not hold in the subtraction of rational numbers.

3. Associative property :

Example 1. Simplify :

$$(i) \left[\frac{3}{4} - \left(\frac{1}{-5} \right) \right] - \left(\frac{-2}{3} \right) \quad (ii) \frac{3}{4} - \left[\frac{1}{-5} - \left(\frac{-2}{3} \right) \right]$$

Are these obtained both rational numbers equal?

Solution :

$$\begin{aligned} (i) \quad & \left[\frac{3}{4} - \left(\frac{1}{-5} \right) \right] - \left(\frac{-2}{3} \right) \\ &= \left[\frac{3}{4} + \frac{1}{5} \right] + \frac{2}{3} \\ &= \left(\frac{15+4}{20} \right) + \frac{2}{3} \\ &= \frac{19}{20} + \frac{2}{3} \\ &= \frac{57+40}{60} \\ &= \frac{97}{60} \end{aligned} \quad \begin{aligned} (ii) \quad & \frac{3}{4} - \left[\frac{1}{-5} - \left(\frac{-2}{3} \right) \right] \\ &= \frac{3}{4} - \left[\frac{-1}{5} + \frac{2}{3} \right] \\ &= \frac{3}{4} - \left(\frac{-3+10}{15} \right) \\ &= \frac{3}{4} - \frac{7}{15} \\ &= \frac{45-28}{60} \\ &= \frac{17}{60} \end{aligned}$$

In both situations, rational numbers obtained are not equal.

It means $\left[\frac{3}{4} - \left(\frac{1}{-5} \right) \right] - \left(\frac{-2}{3} \right) \neq \frac{3}{4} - \left[\frac{1}{-5} - \left(\frac{-2}{3} \right) \right]$

Try :

Express each of the following in the form of a rational number and verify that obtained rational numbers are not equal:

$$(i) \left[\left(\frac{-2}{3} \right) - \left(\frac{-3}{2} \right) \right] - \frac{1}{6} \quad (ii) \left(\frac{-2}{3} \right) - \left[\left(\frac{-3}{2} \right) - \frac{1}{6} \right]$$

Thus, we see that :

For subtracting the rational numbers, subtract third number from the difference of first two numbers or subtract difference of last two numbers from first number, then rational numbers obtained from the two ways are not equal.

It means

If $\frac{p}{q}, \frac{r}{s}$ and $\frac{t}{u}$ are any three rational numbers, then $\left(\frac{p}{q} - \frac{r}{s}\right) - \frac{t}{u} \neq \frac{p}{q} - \left(\frac{r}{s} - \frac{t}{u}\right)$

Therefore, associative property does not hold in the subtraction of rational numbers.

Exercise 1(c)

1. Solve :

(i) $\frac{2}{3} - \frac{5}{3}$

(ii) $\frac{2}{3} - \left(\frac{-5}{3}\right)$

(iii) $\frac{-2}{3} - \left(\frac{-5}{3}\right)$

(iv) $\left(\frac{-2}{3}\right) - \frac{5}{3}$

2. Obtain rational numbers by simplifying the following with the help of number line :

(i) $\frac{3}{7} - \frac{8}{7}$

(ii) $\left(\frac{-4}{6}\right) - \left(\frac{3}{4}\right)$

3. Fill in the blanks by writing these in your exercise book.

(i) $\left(\frac{-15}{13}\right) - \left(\frac{-4}{26}\right) = \dots$

(ii) $\left(\frac{-7}{9}\right) - \dots = 3$

(iii) $2 - \dots = \left(\frac{-3}{4}\right)$

(iv) $\dots - \left(\frac{-2}{3}\right) = \left(\frac{5}{-6}\right)$

4. (i) Which number should be added to $\frac{6}{7}$ get sum $\left(\frac{-2}{9}\right)$?

(ii) What should be added to $\frac{-7}{12}$ get sum $\frac{25}{36}$?

5. Sum of two rational numbers is -5 . If one of these be $\frac{-16}{7}$, then find other number.

6. Simplify :

(i) $\frac{1}{6} + \left(\frac{-2}{5}\right) - \left(\frac{-2}{15}\right)$

(ii) $\frac{3}{8} - \left(\frac{-2}{3}\right) + \frac{-5}{36}$

7. Which statements are true in the following?

(i) Remainder is zero on subtracting $\left(\frac{-2}{5}\right)$ from $\frac{2}{5}$.

(ii) $\left(\frac{-2}{7}\right) - \frac{3}{7} = \frac{-3}{7} - \left(\frac{-2}{7}\right)$

(iii) $\left(\frac{-3}{4}\right) - \left(\frac{-7}{4}\right) > \frac{1}{4}$

(iv) $-5 < \left[\left(\frac{-12}{7}\right) - \frac{8}{7}\right]$

1.2.3. Multiplication of rational numbers :

Understand the following examples :

Example 1. Multiply $\left(\frac{-3}{5}\right)$ by $\frac{4}{7}$

Solution :
$$\begin{aligned}\left(\frac{-3}{5}\right) \times \frac{4}{7} &= \frac{(-3) \times 4}{5 \times 7} = \frac{-12}{35} \\ &= \frac{-12}{35}\end{aligned}$$

Example 2. Simplify $\left(\frac{-14}{9}\right) \times (-27)$

Solution :
$$\begin{aligned}\left(\frac{-14}{9}\right) \times (-27) &= \left(\frac{-14}{9}\right) \times \left(\frac{-27}{1}\right) \\ &= \frac{(-14) \times (-27)}{9 \times 1} \\ &= \frac{14 \times 27}{9} \\ &= \frac{14 \times 3 \times 9}{9}\end{aligned}$$

$$= \frac{14 \times 3}{1} = \frac{42}{1} = 42$$

Try :

Simplify the following :

(i) $\left(\frac{5}{-7}\right) \times \left(\frac{-13}{6}\right)$

(ii) $\left(\frac{-2}{3}\right) \times \left(\frac{-51}{-4}\right)$

(iii) $\frac{49}{39} \times \left(\frac{-26}{14}\right)$

Thus, we see that:

If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers, then $\frac{p}{q} \times \frac{r}{s} = \frac{p \times r}{q \times s}$

Exercise 1(d)

1. Solve :

(i) $\left(\frac{-9}{8}\right) \times \left(\frac{-32}{-3}\right)$

(ii) $(-32) \times \left(\frac{-7}{36}\right)$

(iii) $\left(\frac{-36}{5}\right) \times \left(\frac{20}{-3}\right)$

(iv) $\frac{13}{15} \times \left(\frac{-25}{26}\right)$

2. Multiply :

(i) $\frac{7}{6}$ by (-24)

(ii) $\left(\frac{-9}{8}\right)$ by $\left(\frac{-16}{-3}\right)$

(iii) $\left(\frac{16}{-21}\right)$ by $\left(\frac{-14}{5}\right)$

(iv) $\left(\frac{-13}{5}\right)$ by $\left(\frac{-11}{39}\right)$

Properties of operation of multiplication of rational number :

1. Closure property

Example 1. Multiply rational number $\left(\frac{-8}{7}\right)$ and $\left(\frac{-14}{-15}\right)$. Is the product rational?

Solution : $\left(\frac{-8}{7}\right) \times \left(\frac{-14}{-15}\right) = \frac{-8 \times (-14)}{7 \times (-15)}$

$$= \frac{112}{-105}$$

$$= \frac{16}{-15} = -\frac{16}{15}$$

Obtained product is a rational number.

Try :

Solve the following and verify that obtained product is a rational number :

$$(i) \left(\frac{15}{-2} \right) \times \left(\frac{-4}{11} \right) \quad (ii) \left(\frac{-3}{-8} \right) \times \left(\frac{20}{-9} \right)$$

Thus, we see that

The product of rational numbers is also a rational number i.e.

If $\frac{p}{q}$ and $\frac{r}{s}$ be any two rational numbers, then their product $\left(\frac{p}{q} \times \frac{r}{s} \right)$ also a rational number.

Therefore, the process of multiplication in rational numbers satisfies the closure property.

2. Commutative property :

Example 1. Express each of the following in the form of a rational number :

$$\left(\frac{-3}{4} \right) \times \left(\frac{7}{-11} \right) \text{ and } \left(\frac{7}{-11} \right) \times \left(\frac{-3}{4} \right)$$

What is the relation between so obtained rational numbers?

Solution :

$$(i) \left(\frac{-3}{4} \right) \times \left(\frac{7}{-11} \right) = \frac{(-3) \times 7}{4 \times (-11)} = \frac{-21}{-44}$$

$$(ii) \left(\frac{7}{-11} \right) \times \left(\frac{-3}{4} \right) = \frac{7 \times (-3)}{(-11) \times 4} = \frac{-21}{-44}$$

$$= \frac{21}{44}$$

$$= \frac{21}{44}$$

Rational numbers, obtained from product in both cases are equal.

Try :

Solve the following and show that in both conditions, the obtained products are equal.

1. $\left(\frac{-2}{-5}\right) \times \left(\frac{7}{-4}\right)$ and $\left(\frac{7}{-4}\right) \times \left(\frac{-2}{-5}\right)$

2. $\left(\frac{-3}{2}\right) \times \left(\frac{5}{-6}\right)$ and $\left(\frac{5}{-6}\right) \times \left(\frac{-3}{2}\right)$

Thus we see that:

In multiplication of two rational numbers on changing order of their places, the product remains same.

i.e.

If $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers then $\frac{p}{q} \times \frac{r}{s} = \frac{r}{s} \times \frac{p}{q}$

Therefore, in rational numbers, the process of multiplication satisfies the commutative property.

3. Associative property

Example 1. Obtain rational number from the product of following rational number

(i) $\left[\left(\frac{-4}{5}\right) \times \left(\frac{3}{-7}\right)\right] \times \frac{2}{9}$

(ii) $\left(\frac{-4}{5}\right) \left[\left(\frac{3}{-7}\right) \times \frac{2}{9}\right]$

What is the relation between so obtained rational numbers?

Solution :

(i) $\left[\left(\frac{-4}{5}\right) \times \left(\frac{3}{-7}\right)\right] \times \frac{2}{9}$

$$= \left[\frac{(-4) \times 3}{5 \times (-7)}\right] \times \frac{2}{9}$$

(ii) $\left(\frac{-4}{5}\right) \left[\left(\frac{3}{-7}\right) \times \frac{2}{9}\right]$

$$= \left(\frac{-4}{5}\right) \times \frac{3 \times 2}{(-7) \times 9}$$

$$= \left(\frac{-12}{-35} \right) \times \frac{2}{9}$$

$$= \frac{-24}{-315}$$

$$= \frac{(-3) \times 8}{(-3) \times 105}$$

$$= \frac{8}{105}$$

$$= \left(\frac{-4}{5} \right) \times \left(\frac{6}{-63} \right)$$

$$= \frac{(-4) \times 6}{5 \times (-63)}$$

$$= \frac{-24}{-315}$$

$$= \frac{(-3) \times 8}{(-3) \times 105}$$

$$= \frac{8}{105}$$

In both conditions, rational numbers obtained from multiplication are equal.

Try :

Obtain rational numbers by solving the following and show that the rational numbers obtained in both cases are equal.

1. $\left[\left(\frac{-7}{9} \right) \times \left(\frac{3}{-2} \right) \right] \times \left(\frac{-4}{-5} \right)$ and $\left(\frac{-7}{9} \right) \times \left[\left(\frac{3}{-2} \right) \times \left(\frac{-4}{-5} \right) \right]$

2. $\left[\left(\frac{2}{-3} \right) \times \left(\frac{-3}{-4} \right) \right] \times \left(\frac{-1}{2} \right)$ and $\left(\frac{2}{-3} \right) \times \left[\left(\frac{-3}{-4} \right) \times \left(\frac{-1}{2} \right) \right]$

Thus we see that :

If $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ are any three rational numbers, then $\left(\frac{p}{q} \times \frac{r}{s} \right) \times \frac{t}{u} = \frac{p}{q} \times \left(\frac{r}{s} \times \frac{t}{u} \right)$

Therefore, in rational numbers, operation of multiplication satisfies associative property.

According to this property of multiplication of rational numbers, we can multiply three or more than three rational numbers without using bracket.

Example : Simplify :

$$\left(\frac{-3}{5} \right) \times \left(\frac{-10}{-9} \right) \times \left(\frac{21}{-4} \right) \times (-6)$$

Solution :

$$\begin{aligned} & \left(\frac{-3}{5}\right) \times \left(\frac{-10}{-9}\right) \times \left(\frac{21}{-4}\right) \times (-6) \\ &= \frac{(-3) \times (-10) \times 21 \times (-6)}{5 \times (-9) \times (-4) \times 1} \\ &= \frac{-3 \times 10 \times 21 \times 6}{5 \times 9 \times 4 \times 1} \\ &= -21 \end{aligned}$$

1.2.4. Distributive property of multiplication over sum of rational numbers

We have read in integers that :

$$4 \times (5 + 6) = 4 \times 5 + 4 \times 6$$

$$(-5) \times (6 + 7) = (-5) \times 6 + (-5) \times 7$$

$$6 \times [(-7) + (-8)] = 6 \times (-7) + 6 \times (-8)$$

Is this property also true for rational numbers?

Example : Solve the following :

(i) $\left(\frac{-4}{3}\right) \times \left[\frac{1}{2} + \left(\frac{-7}{5}\right)\right]$

(ii) $\left(\frac{-4}{3}\right) \times \frac{1}{2} + \left(\frac{-4}{3}\right) \times \left(\frac{-7}{5}\right)$

Solution :

(i) $\left(\frac{-4}{3}\right) \times \left[\frac{1}{2} + \left(\frac{-7}{5}\right)\right]$

$$\begin{aligned} &= \left(\frac{-4}{3}\right) \times \left[\frac{5}{10} + \left(\frac{-14}{10}\right)\right] \\ &= \left(\frac{-4}{3}\right) \times \left[\frac{5 + (-14)}{10}\right] \\ &= \left(\frac{-4}{3}\right) \times \left[\frac{5 - 14}{10}\right] \\ &= \left(\frac{-4}{3}\right) \times \left(\frac{-9}{10}\right) \\ &= \frac{36}{30} \end{aligned}$$

(ii) $\left(\frac{-4}{3}\right) \times \frac{1}{2} + \left(\frac{-4}{3}\right) \times \left(\frac{-7}{5}\right)$

$$\begin{aligned} &= \frac{(-4) \times 1}{3 \times 2} + \frac{(-4) \times (-7)}{3 \times 5} \\ &= \frac{-4}{6} + \frac{28}{15} \\ &= \frac{-20}{30} + \frac{56}{30} \\ &= \frac{(-20) + 56}{30} \\ &= \frac{36}{30} \end{aligned}$$

$$= \frac{6}{5}$$

$$= \frac{6}{5}$$

We see that rational number obtained in both cases are equal.

$$\therefore \left(\frac{-4}{3}\right) \times \left[\frac{1}{2} + \left(\frac{-7}{5}\right)\right] = \left(\frac{-4}{3}\right) \times \frac{1}{2} + \left(\frac{-4}{3}\right) \times \left(\frac{-7}{5}\right)$$

Try :

Solving the following verify that in both cases obtained rational numbers are equal :

$$\left(\frac{-2}{3}\right) \times \left[\left(\frac{-4}{5}\right) + \left(\frac{-6}{-7}\right)\right] \text{ and } \left(\frac{-2}{3}\right) \times \left(\frac{-4}{5}\right) + \left(\frac{-2}{3}\right) \times \left(\frac{-6}{-7}\right)$$

Thus, we see that :

The product obtained by multiplication of a rational number and the sum of two rational numbers is equal to the sum of products obtained by the multiplication of this rational number to both the rational numbers separately.

That is,

$$\text{If } \frac{p}{q}, \frac{r}{s} \text{ and } \frac{t}{u} \text{ be three rational numbers, then } \frac{p}{q} \times \left(\frac{r}{s} + \frac{t}{u}\right) = \frac{p}{q} \times \frac{r}{s} + \frac{p}{q} \times \frac{t}{u}$$

Therefore, distributive property of multiplication is valid over addition of rational numbers.

Exercise 1 (e)

1. Choose true statements from the following :

(i) $\left(\frac{2}{-5}\right) \times \left(\frac{-3}{4}\right) = \left(\frac{-3}{4}\right) \times \left(\frac{2}{-5}\right)$

(ii) $\left[\left(\frac{-2}{-3}\right) \times \frac{1}{5}\right] \times \left(\frac{-2}{7}\right) = \left(\frac{-2}{-3}\right) \times \left[\frac{1}{5} \times \left(\frac{-2}{7}\right)\right]$

(iii) The product of $\frac{3}{4}$ and $\left(\frac{-5}{7}\right)$ is not rational number.

(iv) $\left[\left(\frac{7}{-8}\right) + \left(\frac{-5}{6}\right)\right] \times \frac{3}{4} = \left(\frac{7}{-8}\right) \times \frac{3}{4} + \left(\frac{-5}{6}\right) \times \frac{3}{4}$

2. Verify the following statements by doing the process of multiplication :

$$(i) \quad \frac{2}{7} \times \left(\frac{-11}{8} \right) = \left(\frac{-11}{8} \right) \times \frac{2}{7}$$

$$(ii) \quad \left(\frac{-8}{9} \right) \times \frac{15}{17} = \frac{15}{17} \times \left(\frac{-8}{9} \right)$$

$$(iii) \quad \left(\frac{-8}{7} \right) \times \left[\frac{1}{2} \times \left(\frac{-4}{3} \right) \right] = \left[\left(\frac{-8}{7} \right) \times \frac{1}{2} \right] \times \left(\frac{-4}{3} \right)$$

3. Fill in the blanks in the following on your exercise book and also write the name of appropriate property against each statement :

$$(i) \quad \frac{2}{11} \times \left(\frac{-3}{7} \right) = \left(\frac{-3}{7} \right) \times \dots$$

$$(ii) \quad \dots \times \left[\frac{2}{3} \times \left(\frac{3}{-2} \right) \right] = \left[\left(\frac{-11}{3} \right) \times \dots \right] \times \left(\frac{3}{-2} \right)$$

$$(iii) \quad \left(\frac{-5}{7} \right) \times \left[\dots + \left(\frac{-4}{7} \right) \right] = \left(\frac{-5}{7} \right) \times \frac{2}{5} + \dots \times \left(\frac{-4}{7} \right)$$

4. Simplify :

$$(i) \quad (-3) \times \left(\frac{-10}{9} \right) \times \left(\frac{18}{-5} \right) \times \left(\frac{-1}{-6} \right) \qquad (ii) \quad \left(\frac{-1}{-9} \right) \times \frac{3}{20} \times (-4) \times \left(\frac{6}{-5} \right)$$

1.2.5. 'Zero' and 'one' in the form of rational number and their properties

'Zero' in the form of rational number

We know that each integer is a rational number. 0 can be represented in the forms of

$$\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \dots, \frac{0}{-1}, \frac{0}{-2}, \frac{0}{-3}, \dots$$

Try :

$$1. \text{ Simplify : } \frac{0}{5}, \frac{0}{-1}, \frac{0}{10}, \frac{0}{-3}$$

$$2. \text{ Express 0 in the form of rational number } \frac{p}{q}, q \neq 0$$

Thus we see that :

$$0 = \frac{p}{q} \text{ where } p = 0, q \text{ is integer and } q \neq 0$$

Properties of zero

1. Additive identity element

We know that on adding 0 to any integer, the sum remains integer as, $(-9) + 0 = (-9)$

Is this property also true for rational numbers?

Example : Solve

(i) $\left(\frac{-2}{3}\right) + 0$

(ii) $0 + \left(\frac{-2}{3}\right)$

Solution : (i) $\left(\frac{-2}{3}\right) + 0 = \left(\frac{-2}{3}\right) + \frac{0}{3} = \frac{(-2) + 0}{3} = \frac{-2}{3}$

(ii) $0 + \left(\frac{-2}{3}\right) = \frac{0}{3} + \left(\frac{-2}{3}\right) = \frac{0 + (-2)}{3} = \frac{-2}{3}$

$$\therefore \left(\frac{-2}{3}\right) + 0 = \left(\frac{-2}{3}\right) = 0 + \left(\frac{-2}{3}\right)$$

Try :

Verify the following by solving :

$$\left(\frac{-3}{5}\right) + 0 = \left(\frac{-3}{5}\right) = 0 + \left(\frac{-3}{5}\right)$$

Thus, we see that :

If zero is added to any rational number or the same rational number is added to zero, then the sum remains same.

That is

$$\text{If } \frac{p}{q} \text{ be any rational number, then } \frac{p}{q} + 0 = \frac{p}{q} = 0 + \frac{p}{q}$$

Due to this property, we say identity element of addition is zero.

2. Multiplication of any rational number with zero.

We know that on multiplying integer by zero, the product is zero as $(-4) \times 0 = 0$.

Is this property also true for rational number?

Example : Simplify :

$$(i) \left(\frac{-6}{5}\right) \times 0$$

$$(ii) \left(\frac{7}{-11}\right) \times 0$$

Solution : (i) $\left(\frac{-6}{5}\right) \times 0 = \left(\frac{-6}{5}\right) \times \frac{0}{1} = \frac{(-6) \times 0}{5 \times 1} = \frac{0}{5} = 0$

$$(ii) \left(\frac{7}{-11}\right) \times 0 = \left(\frac{7}{-11}\right) \times \frac{0}{1} = \frac{7 \times 0}{(-11) \times 1} = \frac{0}{-11} = 0$$

Try :

Find the value of the following :

$$(i) (-5) \times 0 \qquad (ii) \left(\frac{2}{-15}\right) \times 0$$

$$(iii) \left(\frac{-8}{9}\right) \times 0 \qquad (iv) 0 \times \left(\frac{-3}{-4}\right)$$

Thus we see that :

The product of any rational number and zero remains always zero. That is

$$\text{If } \frac{p}{q} \text{ is any rational number, then } \frac{p}{q} \times 0 = 0 \times \frac{p}{q} = 0, q \neq 0$$

1 (one) in the form of rational number

We know that each integer is a rational number. We can write 1 in the form of

$$\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \dots, \frac{10}{10}, \dots, \frac{101}{101}, \frac{-1}{-1}, \frac{-5}{-5}, \frac{-20}{-20}, \dots$$

Try :

1. Simplify : $\frac{-100}{-100}, \frac{501}{501}, \frac{-36}{-36}, \frac{-2537}{-2537}$

2. Express 1 in the form of rational number $\frac{p}{q}, q \neq 0$.

Thus, we see that :

$$1 = \frac{p}{q}, \text{ where } p = q = \text{Integer and } p = q \neq 0.$$

Properties of 1 (one)

Identity element of multiplication

We know that on multiplying any integer by 1, the product remains same as $(-9) \times 1 = -9 = 1 \times (-9)$.

Example Find the rational number by solving the following :

(i) $\left(\frac{5}{-7}\right) \times 1$ (ii) $\left(\frac{-6}{-11}\right) \times 1$ (iii) $1 \times (-5)$ (iv) 1×0

Solution : (i) $\left(\frac{5}{-7}\right) \times 1 = \left(\frac{5}{-7}\right) \times \frac{1}{1} = \frac{5 \times 1}{(-7) \times 1} = \left(\frac{5}{-7}\right)$

(ii) $\left(\frac{-6}{-11}\right) \times 1 = \left(\frac{-6}{-11}\right) \times \frac{1}{1} = \frac{(-6) \times 1}{(-11) \times 1} = \left(\frac{-6}{-11}\right)$

(iii) $1 \times (-5) = \frac{1}{1} \times \left(\frac{-5}{1}\right) = \frac{1 \times (-5)}{1 \times 1} = \frac{-5}{1} = (-5)$

(iv) $1 \times 0 = \frac{1}{1} \times \frac{0}{1} = \frac{1 \times 0}{1 \times 1} = \frac{0}{1} = 0$

Try :

Find rational number to solve the following :

(i) $\left(\frac{-3}{4}\right) \times 1$

(ii) $1 \times \left(\frac{-8}{-9}\right)$

(iii) $(-6) \times 1$

(iv) $\left(\frac{11}{-12}\right) \times 1$

Thus we see that :

Any rational number multiplied by 1 or 1 multiplied by the rational number, the product so obtained is the same rational number. That is,

If $\frac{p}{q}$ be any rational number, then $\frac{p}{q} \times 1 = \frac{p}{q} = 1 \times \frac{p}{q}$

Due to this property 1 is called the identity element of multiplication of rational numbers.

Inverse of rational number :

1. Additive inverse of any rational number

We know that for each integer, there is an integer which on adding to the given integer, gives the sum as zero.

as, 12 is such integer for (-12) that $(-12) + 12 = 0$

Is this property of integers also true for rational numbers?

Example : Add the following :

(i) $\left(\frac{-3}{5}\right)$ to $\frac{3}{5}$

(ii) $\frac{4}{7}$ to $\left(\frac{-4}{7}\right)$

Solution :

(i) $\frac{3}{5} + \left(\frac{-3}{5}\right) = \frac{3+(-3)}{5} = \frac{3-3}{5} = \frac{0}{5} = 0$

(ii) $\left(\frac{-4}{7}\right) + \frac{4}{7} = \frac{(-4)+4}{7} = \frac{-4+4}{7} = \frac{0}{7} = 0$

Try :

Simplify the following :

(i) $\left(\frac{-6}{11}\right) + \left(\frac{6}{11}\right)$

(ii) $\frac{8}{15} + \left(\frac{-8}{15}\right)$

Additive inverse of $\frac{3}{5} = \left(\frac{-3}{5}\right)$ because $\frac{3}{5} + \left(\frac{-3}{5}\right) = 0$ and additive inverse of $\left(\frac{-4}{7}\right) = -\left(\frac{-4}{7}\right) = \frac{4}{7}$ because $\left(\frac{-4}{7}\right) + \frac{4}{7} = 0$

Try :

Tell additive inverse of the following rational numbers :

(i) $\left(\frac{-6}{11}\right)$

(ii) $\frac{8}{15}$

Additive inverse of rational number $x = -x$ because $x + (-x) = 0$, that is

If $\frac{p}{q}$ is any rational number, then additive inverse of $\frac{p}{q} = \frac{-p}{q}$ and additive inverse of $\left(\frac{-p}{q}\right) = -\left(\frac{-p}{q}\right) = \frac{p}{q}$

Additive inverse of rational number is also called negative or opposite of rational number.

2. Multiplicative inverse of rational number :

If the product of two rational numbers is 1, then each of them is called the multiplicative inverse of other.

Example : Simplify :

(i) $-5 \times \left(\frac{1}{-5}\right)$

(ii) $\left(\frac{-5}{8}\right) \times \left(\frac{8}{-5}\right)$

Solution : (i) $(-5) \times \left(\frac{1}{-5}\right) = \left(\frac{-5}{1}\right) \times \left(\frac{1}{-5}\right) = \frac{(-5) \times 1}{1 \times (-5)} = \left(\frac{-5}{-5}\right) = 1$

$$(ii) \left(\frac{-5}{8}\right) \times \left(\frac{8}{-5}\right) = \frac{(-5) \times 8}{8 \times (-5)} = \left(\frac{-40}{-40}\right) = 1$$

Try :

Simplify the following :

$$(i) \left(\frac{-4}{9}\right) \times \left(\frac{9}{-4}\right) \quad (ii) \frac{2}{3} \times \frac{3}{2} \quad (iii) \left(\frac{-11}{-15}\right) \times \left(\frac{-15}{-11}\right)$$

Multiplicative inverse of $(-5) = \left(\frac{1}{-5}\right)$, because $(-5) \times \left(\frac{1}{-5}\right) = 1$

Multiplicative inverse of $\left(\frac{-5}{8}\right) = \left(\frac{8}{-5}\right)$, because $\left(\frac{-5}{8}\right) \times \left(\frac{8}{-5}\right) = 1$

Multiplicative inverse of $\left(\frac{-12}{-7}\right) = \left(\frac{-7}{-12}\right)$, because $\left(\frac{-12}{-7}\right) \times \left(\frac{-7}{-12}\right) = 1$

Try :

Tell multiplicative inverse of the following rational numbers :

$$(i) \left(\frac{-4}{9}\right) \quad (ii) \frac{2}{3} \quad (iii) \left(\frac{-11}{-15}\right)$$

Thus, we see that,

Multiplicative inverse of rational number x is $\frac{1}{x}$, where $x \neq 0$, because $x \times \frac{1}{x} = 1$,

If $\frac{p}{q}$ is any rational number, then multiplicative inverse of $\frac{p}{q}$ is $\frac{q}{p}$, because

$$\frac{p}{q} \times \frac{q}{p} = 1, \text{ where } p \neq 0, q \neq 0$$

Note that there is no multiplicative inverse of zero '0' because there is no rational number x for which $0 \times x = 1$.

Multiplicative inverse of rational number is also called reciprocal of rational number.

Multiplicative inverse $\frac{1}{x}$ of rational number x is also written x^{-1} .

Therefore multiplicative inverse of 5 = $5^{-1} = \frac{1}{5}$

Multiplicative inverse of $\frac{12}{7} = \left(\frac{12}{7}\right)^{-1} = \frac{7}{12}$

and multiplicative inverse of $\left(\frac{-5}{8}\right) = \left(\frac{-5}{8}\right)^{-1} = \left(\frac{8}{-5}\right) = \left(\frac{-8}{5}\right)$

Try :

Tell multiplicative inverse of the following :

- (i) (-4) (ii) $\frac{6}{7}$ (iii) $\left(\frac{8}{-9}\right)$

Note that :

Multiplicative inverse of 1 is 1 because $1 \times 1 = 1$ and multiplicative inverse of (-1) is (-1) because $(-1) \times (-1) = 1$

Thus,

Only 1 and -1 are such rational numbers which are multiplicative inverse of themselves.

Exercise 1(f)

1. Write true or false, against the following statements:

- (i) $\left[\left(\frac{-6}{7}\right) + \frac{6}{7}\right]$ is not rational number.
- (ii) Identity element of addition in rational numbers is zero.
- (iii) There is no additive inverse of rational number zero '0'.
- (iv) Additive inverse of any negative rational number is a positive rational number.
- (v) Identity element of subtraction in rational numbers is zero.
- (vi) Identity element of multiplication in rational number is 1.
- (vii) Multiplicative inverse of a negative rational number is a negative rational number.
- (viii) If additive inverse of rational x is x , then $x = 0$.

2. Fill in the blanks :

(i) $\left(\frac{-5}{12}\right) + \dots = \left(\frac{-5}{12}\right)$

(ii) $\left(\frac{-4}{5}\right) - \dots = 0$

(iii) $\left(\frac{1}{-2}\right)^{-1} = \dots$

(iv) $\left[\left(\frac{-2}{3}\right) \times \frac{3}{4}\right]^{-1} = \dots$

3. Find the value of x in each situation :

(i) $\frac{2}{3} \times x = 1$

(ii) $\left(\frac{-3}{4}\right) + x = 0$

(iii) $x \times \left(\frac{-5}{4}\right) = 1$

(iv) $x \times \left(\frac{-6}{-7}\right) = 1$

(v) $\left(\frac{-5}{4}\right) \times \left(\frac{4}{-5}\right) = x$

(iv) $\left(\frac{-7}{6}\right) + \frac{7}{6} = x$

4. Tell the additive inverse of the following rational numbers :

(i) $\frac{1}{2}$

(ii) $\left(\frac{-3}{8}\right)$

(iii) 0

(iv) $\left(\frac{-4}{-7}\right)$

5. Find the multiplicative inverse of the following rational numbers :

(i) $\frac{8}{13}$

(ii) $\left(\frac{-6}{19}\right)$

(iii) $\left(\frac{17}{-16}\right)$

(iv) -9

(v) 17

(iv) $\frac{2}{5} \times \frac{9}{4}$

6. Simplify :

$$(-32) \times \left(\frac{5}{-14}\right) \times \left(\frac{-3}{-20}\right) \times 0 \times \left(\frac{-13}{5}\right)$$

1.2.6. Division of rational numbers

Understand the following examples :

Example 1. Divide $\frac{3}{5}$ by $\left(\frac{-2}{7}\right)$

Solution : $\frac{3}{5} \div \left(\frac{-2}{7}\right) = \frac{3}{5} \times \left(\frac{7}{-2}\right)$ [Multiplicative inverse of divisor $\left(\frac{-2}{7}\right) = \left(\frac{7}{-2}\right)$]

$$= \frac{3 \times 7}{5 \times (-2)}$$

$$= \frac{21}{-10}$$

Example 2. Simplify :

$$(i) \left(\frac{-4}{9} \right) \div \left(\frac{-4}{9} \right)$$

$$(ii) \left(\frac{1}{-6} \right) \div (-1)$$

Solution :

$$(i) \left(\frac{-4}{9} \right) \div \left(\frac{-4}{9} \right)$$

$$= \left(\frac{-4}{9} \right) \times \left(\frac{9}{-4} \right) \quad [\text{Multiplicative inverse of divisor } \left(\frac{-4}{9} \right) = \left(\frac{9}{-4} \right)]$$

$$= \frac{(-4) \times 9}{9 \times (-4)} = \frac{-36}{-36} = 1$$

$$(ii) \left(\frac{1}{-6} \right) \div (-1)$$

$$= \left(\frac{1}{-6} \right) \div \left(\frac{-1}{1} \right)$$

$$= \left(\frac{1}{-6} \right) \times \left(\frac{1}{-1} \right) \quad [\text{multiplicative inverse of divisor } \left(\frac{-1}{1} \right) = \left(\frac{1}{-1} \right)]$$

$$= \frac{1 \times 1}{(-6) \times (-1)} = \frac{1}{6}$$

Try :

Simplify the following :

$$(i) \left(\frac{-7}{12} \right) \div \left(\frac{-2}{3} \right)$$

$$(ii) \left(\frac{-6}{15} \right) \div \frac{6}{15}$$

$$(iii) \left(\frac{8}{-35} \right) \div 1$$

Thus, we see that :

In dividing a rational number by another non-zero rational number, first rational number is multiplied by the multiplicative inverse of other rational number (division). That is

$$\text{If } \frac{p}{q} \text{ and } \frac{r}{s}, \text{ be two rational numbers, then } \frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} = \frac{ps}{qr}$$

Example : Product of two rational numbers is $\left(\frac{-8}{5}\right)$. If out of these one number is $\left(\frac{-6}{7}\right)$, then find the other number.

Solution : To obtain second number, we shall have to divide $\left(\frac{-8}{5}\right)$ by $\left(\frac{-6}{7}\right)$.

$$\begin{aligned}\text{Second number} &= \left(\frac{-8}{5}\right) \div \left(\frac{-6}{7}\right) \\ &= \left(\frac{-8}{5}\right) \times \left(\frac{7}{-6}\right) \quad [\text{multiplicative inverse of divisor } \left(\frac{-6}{7}\right) = \left(\frac{7}{-6}\right)] \\ &= \frac{-56}{-30} \\ &= \frac{28}{15}\end{aligned}$$

From above examples, following facts are to be noticed :

1. If x and y are two rational numbers and $y \neq 0$, then $x \div y$ is a rational number.
2. If x is any rational number, then $x \div 1 = x$, $x \div (-1) = -x$
3. For each non-zero rational number x , $x \div x = 1$, $x \div (-x) = -1$, $(-x) \div x = -1$

Exercise 1(g)

1. Divide first number by second number of the following pairs :

- (i) $\left(\frac{-8}{5}\right), \left(\frac{1}{-10}\right)$ (ii) $18, \left(\frac{-12}{7}\right)$ (iii) $(-8), \left(\frac{5}{-16}\right)$

(iv) $\left(\frac{65}{-14}\right), \left(\frac{-13}{7}\right)$

(v) $\frac{5}{12}, \left(\frac{-4}{9}\right)$

(iv) $\frac{3}{4}, (-9)$

2. By doing process of division tell whether the following statement are true or false :

(i) $\left(\frac{-1}{8}\right) \div \frac{3}{4}$ is a rational number. (ii) $\left(\frac{8}{-9}\right) \div \left(\frac{-4}{3}\right) = \left(\frac{-4}{3}\right) \div \left(\frac{8}{-9}\right)$

(iii) $\left(\frac{-15}{56}\right) \div 1 = 1 \div \left(\frac{-15}{56}\right)$ (iv) $\left(\frac{-9}{10}\right) \div \left(\frac{-9}{10}\right) = 1$

3. Simplify :

(i) $\frac{2}{3} \div \left(\frac{-4}{5}\right)$

(ii) $(-4) \div \left(\frac{-3}{5}\right)$

(iii) $\left(\frac{-6}{7}\right) \div (-15)$

(iv) $\left(\frac{-1}{8}\right) \div \frac{3}{4}$

(v) $\frac{15}{7} \div \left(\frac{-5}{7}\right)$

(vi) $\left(\frac{-7}{12}\right) \div \left(\frac{-2}{3}\right)$

4. The product of two rational numbers is $\left(\frac{-6}{7}\right)$. If out of these one number is $\left(\frac{-8}{5}\right)$, then find the second number.

5. The product of two rational number is $\left(\frac{-5}{6}\right)$. If out of these one number is $\left(\frac{-7}{12}\right)$, then find the second number.

6. By which number $\left(\frac{-4}{9}\right)$ be multiplied so that the product (-1) is obtained.

7. By which number $\frac{8}{9}$ be multiplied so that the product $\left(\frac{-16}{28}\right)$ is obtained.

Efficiency practice 1 (A)

1. Find the rational number by simplify the following :

(i) $\left(\frac{-2}{3}\right) + \left(-\frac{4}{5}\right) - \frac{6}{7}$

(ii) $\left[\left(\frac{2}{-3}\right) + \frac{1}{4}\right] \div 2$

(iii) $\left[\left(\frac{-3}{5}\right) \times \left(\frac{35}{-12}\right)\right] \div \frac{1}{14}$

(iv) $\left[\left(\frac{-5}{9}\right) - \left(\frac{1}{-3}\right)\right] \times \frac{5}{2}$

2. Multiply the sum of $\left(\frac{-12}{7}\right)$ and $\left(\frac{13}{5}\right)$ by their difference.

3. Divide the sum of $\left(\frac{1}{-2}\right)$ and $\left(\frac{-3}{7}\right)$ by their product.
4. Write true/false against following statements in your exercise-book:
 - (i) The multiplicative inverse of $\frac{1}{a}$ is a , if $a \neq 0$
 - (ii) Additive inverse of negative rational number cannot be positive rational number.
 - (iii) To divide 1 by zero is not possible.
 - (iv) Zero is not additive inverse of any rational number.

1.3. Absolute value of rational number

We know that absolute value of any integer is the numerical value of that integer. This integer is absolute from sign (+) or (-). As, Absolute value of $+5 = |+5| = 5$

$$\text{Absolute value of } -5 = |-5| = -(-5) = 5$$

$$\text{Absolute value of } 0 = |0| = 0$$

Thus if x is any integer, then absolute value of x is represented by $|x|$ and

$$|x| = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

Like absolute value of integers, the absolute value of rational number is also absolute from the sign (+) or (-).

$$\text{Absolute value of } \frac{5}{7} = \left|\frac{5}{7}\right| = \frac{5}{7}$$

$$\text{Absolute value of } -\frac{5}{7} = \left|-\frac{5}{7}\right| = \frac{5}{7}$$

From above example, we see that :

- (i) The absolute value of any rational number is never negative.
- (ii) The absolute value of each non-zero rational number is always positive.
- (iii) Absolute value of rational number 0 is 0.

Example 1. Find the values of the following :

$$(i) \left|\frac{3}{7}\right|$$

$$(ii) \left|\frac{-5}{12}\right|$$

$$(iii) \left| \frac{-12}{-7} \right|$$

$$(iv) |0|$$

Solution :

We can find the values of rational numbers by using the definition of absolute value.

$$(i) \left| \frac{3}{7} \right| = \frac{3}{7}$$

$$(ii) \left| \frac{-5}{12} \right| = \frac{5}{12}$$

$$(iii) \left| \frac{-12}{-7} \right| = \left| \frac{12}{7} \right| = \frac{12}{7}$$

$$(iv) |0| = 0$$

Try :

Find the values of the following :

$$(i) \left| \frac{13}{-8} \right|$$

$$(ii) \left| -\left(\frac{-7}{13} \right) \right|$$

Example 2. Savita verified the absolute values for the following relations taking $x = \frac{-4}{5}$ and $y = \frac{3}{7}$.

$$(i) |x + y| \leq |x| + |y|$$

$$(ii) |x \times y| = |x| \times |y|$$

$$(iii) |x \div y| = |x| \div |y|$$

Solution :

$$(i) x + y = \frac{-4}{5} + \frac{3}{7} = \frac{-28+15}{35} = \frac{-13}{35}$$

$$\therefore |x + y| = \left| \frac{-13}{35} \right| = \frac{13}{35}$$

$$\text{and } |x| = \left| \frac{-4}{5} \right| = \frac{4}{5}$$

$$|y| = \left| \frac{3}{7} \right| = \frac{3}{7}$$

$$|x| + |y| = \frac{4}{5} + \frac{3}{7} = \frac{28+15}{35} = \frac{43}{35}$$

$$\therefore \frac{13}{35} < \frac{43}{35}$$

$$\therefore |x + y| < |x| + |y|$$

$$(ii) \quad x \times y = \frac{-4}{5} \times \frac{3}{7} = \frac{-4 \times 3}{5 \times 7} = \frac{-12}{35} \dots\dots\dots$$

$$\therefore |x \times y| = \left| \frac{-12}{35} \right| = \frac{12}{35} \dots\dots\dots (i)$$

$$\text{and } |x| = \left| \frac{-4}{5} \right| = \frac{4}{5}, |y| = \left| \frac{3}{7} \right| = \frac{3}{7}$$

$$|x| \times |y| = \frac{4}{5} \times \frac{3}{7} = \frac{4 \times 3}{5 \times 7} = \frac{12}{35} \dots(ii)$$

From equations (i) and (ii)

$$|x \times y| = |x| \times |y|$$

$$(iii) \quad x = \frac{-4}{5}, y = \frac{3}{7}$$

$$|x| = \left| \frac{-4}{5} \right| = -\left(\frac{-4}{5} \right) = \frac{4}{5}$$

$$|y| = \left| \frac{3}{7} \right| = \frac{3}{7}$$

$$|x| \div |y| = \frac{4}{5} \div \frac{3}{7} = \frac{4}{5} \times \frac{7}{3} = \frac{28}{15} \dots\dots\dots (i)$$

$$|x \div y| = \left| \frac{4}{5} \div \frac{3}{7} \right|$$

$$= \left| \frac{-4}{5} \times \frac{7}{3} \right| = \left| \frac{-28}{15} \right| = \frac{28}{15} \dots\dots\dots (ii)$$

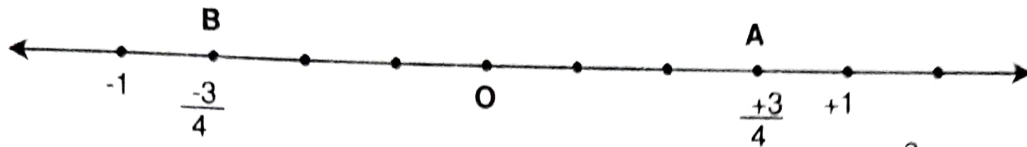
From equations (i) and (ii)

$$|x \div y| = |x| \div |y|$$

Example 3. Find the corresponding rational number having absolute value $\frac{3}{4}$.

Solution :

First Method : Amit draws a number line to find the rational number with absolute value $\frac{3}{4}$. Points A and B are marked on the number line such that they are situated at the distance of $\frac{3}{4}$ from point 0.



On number line, the point A is right side from 0 at a distance of $\frac{3}{4}$ and the point B is left side from 0 to distance of $\frac{3}{4}$, $\frac{3}{4}$ represents point A and $\frac{-3}{4}$ represents point B.

$$\text{Absolute value of } \frac{+3}{4} = \left| \frac{+3}{4} \right| = \frac{3}{4}$$

$$\text{Absolute value of } \frac{-3}{4} = \left| \frac{-3}{4} \right| = -\left(\frac{-3}{4} \right) = \frac{3}{4}$$

Therefore, absolute value of both rational numbers $\frac{+3}{4}$ and $\frac{-3}{4}$ is $\frac{3}{4}$.

Second Method :

Razia obtained the same answer by solving it from following method.

Solution : Let the number is x .

$$|x| = \frac{3}{4}$$

The value of x cannot be zero,

$$\text{If } x > 0, \text{ then } x = \frac{3}{4}$$

$$\text{If } x < 0, \text{ then } -x = \frac{3}{4}$$

$$\text{i.e. } x = -\frac{3}{4}$$

Thus, you have seen that $\frac{3}{4}$ and $-\frac{3}{4}$ are such two rational numbers whose absolute values are $\frac{3}{4}$.

1. For every positive rational number there are only two such rational numbers for which absolute value of each is r .
2. Absolute value of zero is zero.
3. Absolute value of each rational number is positive.

Exercise 1(h)

1. Find the absolute values of the following rational numbers:

(i) $\frac{15}{7}$ (ii) $\frac{-11}{8}$ (iii) $\frac{-19}{-23}$ (iv) $\frac{-8}{19}$

2. Simplify:

(i) $|\frac{3}{5} - \frac{2}{3}|$ (ii) $|\frac{-9}{7} + \frac{11}{3}|$ (iii) $|\frac{-3}{5} \times \frac{-7}{11}|$

Write the correct option of the answer in your exercise book:

3. If $x = -\frac{1}{2}$ and $y = \frac{3}{4}$, then the value of $|x| + |y|$ is :

(i) $\frac{7}{4}$ (ii) $-\frac{5}{7}$ (iii) $\frac{5}{4}$ (iv) $\frac{7}{8}$

4. The value of $|\frac{3}{-7}|$ is :

(i) $\frac{3}{7}$ (ii) $\frac{-3}{7}$ (iii) $\frac{3}{-7}$ (iv) $\frac{-3}{-7}$

5. If $x = \frac{-5}{7}$ and $y = \frac{3}{4}$, then the value of $|x| \times |y|$ is :

(i) $\frac{7}{28}$ (ii) $-\frac{15}{28}$ (iii) $\frac{15}{14}$ (iv) $\frac{15}{28}$

6. The absolute values of rational numbers $\frac{-5}{7}$ and $\frac{-5}{-7}$ are :

(i) $\frac{-5}{7}$ (ii) $\frac{5}{-7}$ (iii) $\frac{5}{7}$ (iv) $\frac{7}{5}$

7. Write appropriate sign $>$, $=$, $<$ in each square by writing in your exercise-book :

(i) $|- \frac{3}{4}|$ $|- \frac{3}{4} + \frac{5}{8}|$

(ii) $|- \frac{3}{8} + \frac{2}{3}|$ $|- \frac{3}{4} + \frac{22}{24}|$

(iii) $|-(-\frac{12}{25})|$ $|- \frac{2}{5}| \times |(-\frac{6}{5})|$

(iv) $|- \frac{3}{8} + \frac{1}{12}|$ $|- \frac{1}{2} + \frac{3}{4} + \frac{1}{8}|$

8. If $x = \frac{5}{9}$, $y = \frac{2}{3}$ then show that:

(i) $|x + y| = |x| + |y|$

(ii) $|x \times y| = |x| \times |y|$

9. If $x = \frac{15}{13}$, $y = \frac{-2}{11}$, then show that:

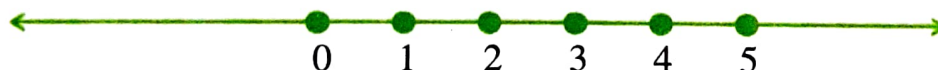
(i) $|x + y| < |x| + |y|$

(ii) $|x \times y| = |x| \times |y|$

10. Find two rational numbers whose absolute value is $\frac{1}{2}$.

1.4. Insertion of rational numbers between two different rational numbers :

It is not necessary that there exist any other integer between any two integers on the number line. See the following number line carefully:



There are only four integers 1, 2, 3 and 4 between 0 and 5 on the number line. But there is no any integer between 0 and 1.

Opposite of it there are many rational numbers between any two different rational numbers.

Let us see that how can we find many rational numbers between two rational numbers lie between 5 and 10. Similarly Rashmi wanted to find the number of integers between -3 and 3. She wrote integers -3, -2, -1, 0, 1, 2, 3 between -3 and 3.

Thus Rashmi obtained 5 integers between -3 and 3. You see that any integer does not lie between -3 and -2. There is no any integer between two consecutive integers.

It is clear that **number of integers between two integers are limited (finite)**

You can see that many rational numbers lie between two rational numbers.

Rashmi took two rational numbers $-\frac{2}{3}$ and $-\frac{1}{5}$ and changed these into numbers having same denominators.

$$\frac{-2}{3} = \frac{-10}{15} \text{ and } \frac{-1}{5} = \frac{-3}{15}$$

Rashmi had obtained numbers of following types between rational numbers $\frac{-10}{15}$ and $\frac{-3}{15}$.

$$\begin{array}{ccc} \frac{-10}{15} & \boxed{\frac{-9}{15} < \frac{-8}{15} < \frac{-7}{15} < \frac{-6}{15} < \frac{-5}{15} < \frac{-4}{15}} & \frac{-3}{15} \\ \text{or } \frac{-2}{3} & \boxed{\frac{-3}{5} < \frac{-8}{15} < \frac{-7}{15} < \frac{-2}{5} < \frac{-1}{3} < \frac{-4}{15}} & \frac{-1}{5} \end{array}$$

Therefore $\frac{-3}{5} < \frac{-8}{15}, \frac{-7}{15} < \frac{-2}{5} < \frac{-1}{3} < \frac{-4}{15}$ are rational numbers between $\frac{-2}{3}$ and $-\frac{1}{5}$.

Let us see that, there only above 6 rational numbers between $-\frac{2}{3}$ and $-\frac{1}{5}$?

Now find other rational numbers between $-\frac{2}{3}$ and $-\frac{3}{5}$.

Equivalent rational numbers of $-\frac{2}{3}$ and $-\frac{3}{5}$ are following:

$$\frac{-20}{30} \text{ and } \frac{-18}{30}$$

Therefore, $\frac{-20}{30} < \frac{-19}{30} < \frac{-18}{30}$

i.e. $\frac{-2}{3} < \frac{-19}{30} < \frac{-3}{5}$

Thus, we have obtained one more rational number between $-\frac{2}{3}$ and $-\frac{3}{5}$.

You can find as much rational numbers as you want by using this method.

See further $\frac{-2}{3}$ and $\frac{-3}{5}$

$$\frac{-3}{5} = \frac{-3 \times 30}{5 \times 30} = \frac{-90}{150}$$

and

$$\frac{-2}{3} = \frac{-2 \times 50}{3 \times 50} = \frac{-100}{150}$$

We find 9 rational numbers $\frac{-91}{150}, \frac{-92}{150}, \frac{-93}{150}, \dots, \frac{-99}{150}$ between equivalent numbers $\frac{-90}{150}$ and $\frac{-100}{150}$. (i.e. in between $\frac{-3}{5}$ and $\frac{-2}{5}$) Thus this list will never be exhausted.

Conclusion:

We can obtain infinitely many rational numbers between two rational numbers.

To find one rational number between two given different rational numbers :

Example 1 : Show that $\frac{1}{2} \times \left(-\frac{1}{3} + \frac{1}{2} \right)$ is situated between rational numbers $-\frac{1}{3}$ and $\frac{1}{2}$.

Solution :
$$\frac{1}{2} \left(-\frac{1}{3} + \frac{1}{2} \right) = \frac{1}{2} \left(\frac{-2+3}{6} \right) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

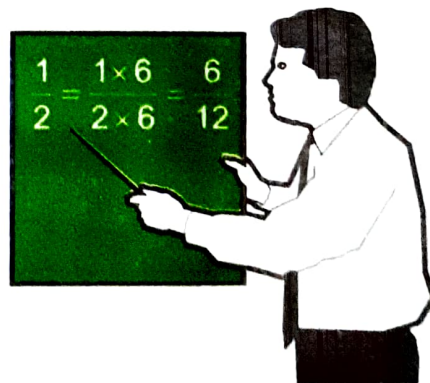
Now, we arrange $-\frac{1}{3}, \frac{1}{2}$ and $\frac{1}{12}$ in ascending order. L.C.M. of 3, 2 and 12 = 12.

$$-\frac{1}{3} = \frac{-1 \times 4}{3 \times 4} = \frac{-4}{12}$$

$$\frac{1}{2} = \frac{1 \times 6}{2 \times 6} = \frac{6}{12}$$

$$\frac{1}{12} = \frac{1 \times 1}{12 \times 1} = \frac{1}{12}$$

It is clear that $\frac{-4}{12} < \frac{1}{12} < \frac{6}{12}$



or
$$\frac{-1}{3} < \frac{1}{12} < \frac{1}{2}$$

From this, it is known that $\frac{1}{12}$ i.e. $\left\{ \frac{1}{2} \times \left(\frac{-1}{3} + \frac{1}{2} \right) \right\}$ lie in between rational numbers $\frac{-1}{3}$ and $\frac{1}{2}$.

Example 2. If $x \neq y$, then show that $\frac{x+y}{2}$ lie in the middle of x and y .

Solution : x and y are two different rational numbers. One of these will be necessarily greater than other.

Let $x > y$

On adding x to both sides

$$x + x > x + y$$

or, $2x > x + y$

or, $x > \frac{x+y}{2}$... (1)

Further, $x > y$

On adding y to both sides

$$x + y > y + y$$

or, $x + y > 2y$

or, $\frac{x+y}{2} > y$... (2)

On combining equations (1) and (2)

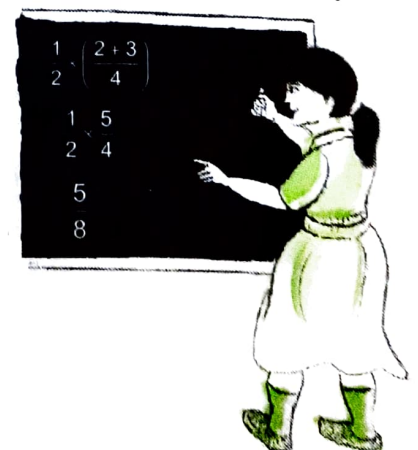
$$x > \frac{x+y}{2} > y$$

Therefore, $\frac{x+y}{2}$, is a rational number between rational numbers x and y .

Example 3. Find the rational numbers between $\frac{1}{2}$ and $\frac{3}{4}$.

Solution : Rational number between $\frac{1}{2}$ and $\frac{3}{4}$

$$= \frac{1}{2} \times \left(\frac{1}{2} + \frac{3}{4} \right)$$

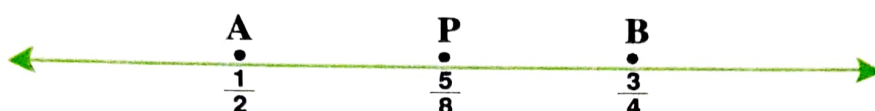


$$= \frac{1}{2} \times \left(\frac{2+3}{4} \right)$$

$$= \frac{1}{2} \times \frac{5}{4}$$

$$= \frac{5}{8}$$

Mark these rational numbers $\frac{1}{2}$ and $\frac{3}{4}$ on number line by point A and B



Now, rational number $\frac{5}{8}$ known between both is represented by point P.

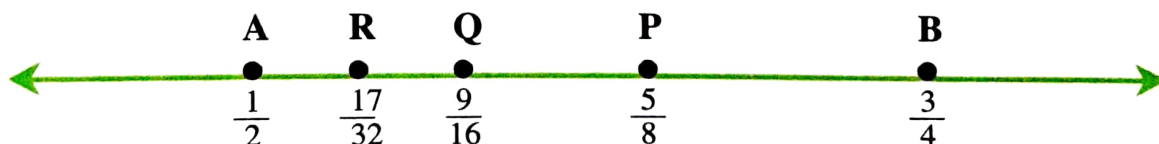
Using this process, other rational numbers can be obtained between $\frac{1}{2}$ and $\frac{5}{8}$.

As, rational number between $\frac{1}{2}$ and $\frac{5}{8} = \frac{1}{2} \times \left(\frac{1}{2} + \frac{5}{8} \right)$

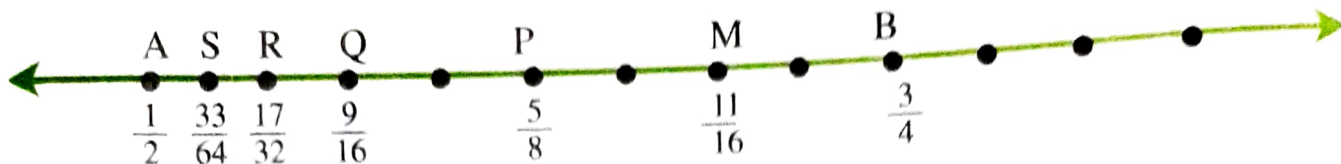
$$= \frac{1}{2} \times \left(\frac{4+5}{8} \right)$$

$$= \frac{1}{2} \times \frac{9}{8} = \frac{9}{16}$$

Rational number between $\frac{1}{2}$ and $\frac{5}{8}$ is $\frac{9}{16}$ which is represented by Q, further rational number between $\frac{1}{2}$ and $\frac{9}{16}$ is $\frac{17}{32}$ which is represented by point R.



Thus $\frac{17}{32}$ is between rational numbers $\frac{1}{2}$ and $\frac{9}{16}$ and $\frac{33}{64}$ is between rational numbers $\frac{1}{2}$ and $\frac{17}{32}$. Using this process many rational numbers can be obtained between $\frac{1}{2}$ and $\frac{3}{4}$.



Only five rational numbers are marked between rational numbers $\frac{1}{2}$ and $\frac{3}{4}$ on the above number line. In fact, there are infinite rational numbers between these.

As rational number $\frac{9}{16}$ is obtained between $\frac{1}{2}$ and $\frac{5}{8}$, similarly rational number between $\frac{5}{8}$ and $\frac{3}{4} = \frac{1}{2} \left(\frac{5}{8} + \frac{3}{4} \right) = \frac{11}{16}$ is also obtained. It is represented by point M on number line.

Thus we see that

If x and y are two different rational numbers, then

$$q_1 = \frac{1}{2}(x + y); q_2 = \frac{1}{2}(q_1 + y); q_3 = \frac{1}{2}(q_2 + y) \dots$$

etc. will be between x and y . Therefore, infinite rational numbers can be found between two different rational numbers.

Example 4. Find four rational numbers between $\frac{1}{6}$ and $\frac{1}{3}$.

Solution: Let q_1, q_2, q_3 and q_4 are four rational numbers between $\frac{1}{6}$ and $\frac{1}{3}$.

$$q_1 = \frac{1}{2} \left(\frac{1}{6} + \frac{1}{3} \right) = \frac{1}{2} \left(\frac{1+2}{6} \right) = \frac{1}{2} \times \frac{3}{6} = \frac{3}{12}$$

$$q_2 = \frac{1}{2} \left(q_1 + \frac{1}{3} \right) = \frac{1}{2} \left(\frac{3}{12} + \frac{1}{3} \right) = \frac{1}{2} \left(\frac{3+4}{12} \right) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$$

$$q_3 = \frac{1}{2} \left(q_2 + \frac{1}{3} \right) = \frac{1}{2} \left(\frac{7}{24} + \frac{1}{3} \right) = \frac{1}{2} \left(\frac{7+8}{24} \right) = \frac{1}{2} \times \frac{15}{24} = \frac{15}{48}$$

$$q_4 = \frac{1}{2} \left(q_3 + \frac{1}{3} \right) = \frac{1}{2} \left(\frac{15}{48} + \frac{1}{3} \right) = \frac{1}{2} \left(\frac{15+16}{48} \right) = \frac{1}{2} \times \frac{31}{48} = \frac{31}{96}$$

Therefore, four rational numbers between $\frac{1}{6}$ and $\frac{1}{3}$ are $\frac{3}{12}, \frac{7}{24}, \frac{15}{48}$ and $\frac{31}{96}$.

or $\frac{1}{4}, \frac{7}{24}, \frac{5}{16}$ and $\frac{31}{96}$

Try :

Find the rational number between $\frac{1}{5}$ and $\frac{7}{10}$ situated at one fourth distant from $\frac{1}{5}$.

Exercise 1 (i)

Choose out correct alternative from question 1 and 2 and write it in your exercise-book.

1. The rational number in the middle of -1 and $-\frac{1}{2}$ is:

(i) $-\frac{1}{2}$
(ii) $\frac{1}{2}$
(iii) $-\frac{3}{4}$
(iv) $\frac{3}{4}$
2. The rational number situated in the middle of -3 and 4 is:

(i) $\frac{1}{2}$
(ii) $\frac{-7}{2}$
(iii) $\frac{7}{2}$
(iv) $-\frac{1}{2}$
3. Find the rational number situated in the middle of -1 and 1 :
4. Find the rational number situated in the middle of $\frac{1}{3}$ and $\frac{1}{2}$.
5. Find the rational number situated in the middle of $\frac{-7}{8}$ and $\frac{7}{8}$.
6. Find the rational number situated in the middle of $\frac{-3}{5}$ and $\frac{8}{3}$.
7. Find the rational number situated in the middle of $\frac{-5}{4}$ and $\frac{-1}{6}$.
8. Find the rational number situated in the middle of $1\frac{3}{4}$ and $4\frac{3}{8}$.
9. Find the rational number situated in the middle of $-1\frac{2}{7}$ and $\frac{9}{2}$.
10. Express $\frac{1}{2}\left(\frac{-2}{3} + \frac{1}{4}\right)$ in the form of $\frac{p}{q}$ and show that this rational number is situated in the middle of $\frac{-2}{3}$ and $\frac{1}{4}$.

1.5. To express rational number in the form of decimal number :

We know to change fractions into decimal numbers. Like fractions, we can also change rational number into decimal number.

Example 1. Change the fractions $\frac{5}{8}$ and $\frac{7}{11}$ into decimals. (All fractions are rational numbers).

Solution:

$$\frac{5}{8} = 5 \div 8$$

$$\begin{array}{r} 0.625 \\ 8 \overline{) 5.0} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$$\therefore \frac{5}{8} = 0.625$$

$$\frac{7}{11} = 7 \div 11$$

$$\begin{array}{r} 0.6363 \\ 11 \overline{) 7.0} \\ \underline{66} \\ 40 \\ \underline{33} \\ 70 \\ \underline{66} \text{ so on} \\ 40 \\ \underline{33} \\ 7 \end{array}$$

$$\therefore \frac{7}{11} = 0.6363\ldots$$

We see that in division process of 5 by 8, the division-process ends after some finite steps while to divide 7 by 11, the division process does not end.

(A) Terminating (To come to an end) decimal number :

Example 2: Change rational numbers $\frac{3}{4}$, $\frac{7}{5}$ and $\frac{3}{20}$ in decimals.

Solution:

$$\frac{3}{4} = 3 \div 4$$

$$\begin{array}{r} 0.75 \\ 4 \overline{) 3.0} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

$$\frac{3}{4} = 0.75$$

$$\frac{7}{5} = 7 \div 5$$

$$\begin{array}{r} 1.4 \\ 5 \overline{) 7.0} \\ \underline{5} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

$$\frac{7}{5} = 1.4$$

$$\frac{3}{20} = 3 \div 20$$

$$\begin{array}{r} 0.15 \\ 20 \overline{) 3.0} \\ \underline{20} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

$$\frac{3}{20} = 0.15$$

Above rational numbers $\frac{3}{4}$, $\frac{7}{5}$ and $\frac{3}{2}$ are expressed in terminating decimals.

Try

Find the following rational numbers in decimals : $\frac{3}{8}$, $\frac{7}{20}$ and $\frac{11}{5}$

In all the above examples, obtained quotients are terminating decimals. See carefully the denominators of these:

We see that prime factors of their denominators are only 2 or 5 (or both).

Therefore, the prime factors of the denominators of numbers expressible in terminating decimals are only 2 or 5 (or both).

Try

Trace out the rational numbers expressible in terminating decimals from the following numbers :

$$\frac{1}{4}, \frac{13}{20}, \frac{5}{6}, \frac{5}{16}, \frac{2}{7}, \frac{13}{50}$$

(B) Infinite recurrent (non-terminating recurring) decimals numbers:

We see that in dividing 7 by 11, the group of digits 63 comes again and again and process of division goes on continuously. In this way the obtained quotient 0.6363... is non-terminating recurring.

Verify taking some other example like this:

Example 3. Change the rational numbers $\frac{1}{3}$ and $\frac{2}{11}$ in decimals.

Solution: $\frac{1}{3} = 1 \div 3$

$$\begin{array}{r} 0.333 \\ 3 \overline{) 1.0} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

so on

$$\therefore \frac{1}{3} = 0.333...$$

$$\frac{2}{11} = 2 \div 11$$

$$\begin{array}{r} 0.1818 \\ 11 \overline{) 2.0} \\ \underline{11} \\ 90 \\ \underline{88} \\ 20 \\ \underline{11} \\ 2 \end{array}$$

so on

$$\therefore \frac{2}{11} = 0.1818...$$

Try :

Change the following rational numbers in decimals :

$$\frac{4}{9}, \frac{1}{7}, \frac{5}{13}$$

In all above examples, one digit or group of digits comes repeatedly and process of division goes on continuously.

Consequently, such rational numbers whom on changing in decimals a groups of one or more than one digits comes repeatedly in decimal part and the process of division never finishes, are called non-terminating recurring decimal numbers.

In non-terminating recurring decimal numbers to express repeated continuously group of digits a bar (–) is marked above them or put a point on first and last digits.

As, $\frac{1}{3} = 0.333\ldots$ or $\frac{1}{3} = 0.\overline{3}$ or 0.3

$$\frac{2}{11} = 0.1818\ldots \text{ or } \frac{2}{11} = 0.\overline{18} \text{ or } 0.18$$

$$\frac{1}{7} = 0.142857\ldots \text{ or } \frac{1}{7} = 0.14\overline{2857} \text{ or } 0.142857$$

Example 4. Express $\frac{5}{21}$ in decimal.

Solution: $\frac{5}{21} = 5 \div 21$

$$\begin{array}{r} 0.238095 \\ 21 \overline{) 5.0} \\ \underline{42} \\ 80 \\ \underline{63} \\ 170 \\ \underline{168} \\ 200 \\ \underline{189} \\ 110 \\ \underline{105} \\ 5 \end{array}$$

so on

$$\frac{5}{21} = \overline{0.238095} \text{ or } 0.238095$$

Why did we draw a bar over the group of digits 238095 in the above decimal number?

At the time of starting the division process, we started it from digit 5 being divided. Now, as remainder is again obtained, we know that on dividing further, the repetition of the same group of digits will occur.

We draw a bar over 238095, or mark point on first and last digit to demonstrate the continuous repetition of group of digits

Non-terminating recurring decimal fractions are of two types.

1. Pure non-terminating recurring decimal number :

All examples given above are of pure non-terminating recurring decimal number.

2. Mixed non-terminating recurring decimal number :

See the following examples :

Exempl 5. Change rational number $\frac{1}{6}$ in decimal.

Solution:

$$\begin{array}{r} 0.166 \\ 6 \overline{) 1.0} \\ \underline{6} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

so on. Therefore, $\frac{1}{6} = 0.166\ldots$ or $0.1\overline{6}$

In this example, the digit just after decimal is 1, whose repetition is not occurring. But next digit 6 occurs again and again. It is called mixed non-terminating recurring decimal number.

To express negative rational numbers in the form of decimal numbers:

We learned to change positive rational numbers in the form of decimal numbers. Can we change the negative rational numbers in the form of decimal numbers?

Example 6. Express $-\frac{2}{5}$ in the form of decimal numbers.

Solution: Decimal representation of $\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4$

$\therefore$ Decimal representation of $-\frac{2}{5} = -0.4$

Example 7. Change $\frac{-24}{7}$ in decimal number.

Solution : Let change $\frac{-24}{7}$ in decimal number.

$$\begin{array}{r} 3.428571 \\ 7 \overline{) 24} \\ \underline{21} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 70 \\ \underline{49} \\ 10 \\ \underline{7} \\ 3 \end{array}$$

$$\begin{array}{r} 3.428571 \\ 7 \overline{) 24} \\ \underline{21} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \end{array}$$



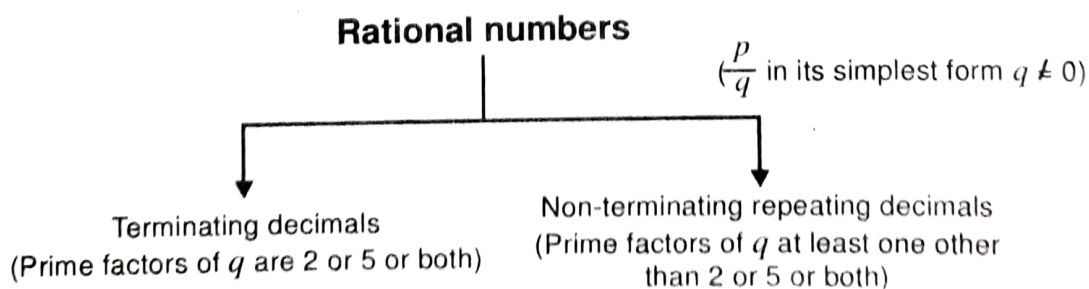
$$\therefore \frac{24}{7} = 3.\overline{428571}$$

so on.

$$-\frac{24}{7} = -3.\overline{428571}$$

We see that there is no terminating decimal in rational numbers, if in their simplest form $\frac{p}{q}$, besides 2 and 5 there are other prime factor also in the factors of q . Consequently these are non-terminating recurring decimals.

The classification of rational numbers is as following.



Exercise 1(j)

1. Express the following rational numbers in the form of decimal:

$$\frac{11}{3}, \frac{17}{5}, \frac{19}{6}, \frac{3}{8}$$

2. Change the following rational numbers in the form of decimal:

$$\frac{-5}{4}, \frac{-15}{2}, \frac{-16}{5}, \frac{-5}{9}$$

3. Which numbers of the following rational numbers can be expressed in the form of terminating decimals?

$$\frac{1}{4}, \frac{11}{8}, \frac{-27}{4}, \frac{-33}{10}, \frac{2}{7}, \frac{6}{15}$$

4. If the sum of $\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$ changes in decimal number, then will this be terminating or non-terminating?
5. Which of the following rational numbers cannot be represented in the form of terminating decimal numbers?

$$\frac{24}{7}; \frac{5}{6}; \frac{7}{15}; \frac{3}{5}; 5\frac{3}{4}$$

6. Mark (✓) for true and (✗) for false against following statements in your exercise-book:

- (a) Representation of $\frac{1}{25}$ can be done in the form of a terminating decimal number.
- (b) The representation of $\frac{1}{9}$ can be done in the form of a terminating decimal number.
- (c) There is no difference between 0.6 and 0.60000000.
- (d) $\frac{3}{7}$ is not non terminating recurring in the form of its decimal number.

1.6. To express terminating decimal number in the form of rational number:

See carefully the table of place value of decimal notation system and express the decimal numbers 0.15, 1.5, 0.625, 12.05 and 2.125 in the form of rational numbers.

Decimal number	Hundred	Tens	Unit	Decimal point	One Tenth	One Hundredth	One Thousandth
	100	10	1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
		Integer			Decimal or fractional part		
0.15			0	•	1	5	
1.5			1	•	5		
0.625			0	•	6	2	5
12.05		1	2	•	0	5	
2.125			2	•	1	2	5

Solution: (1) $0.15 = 1 \text{ tenth} + 5 \text{ hundredth}$

$$= \frac{1}{10} + \frac{5}{100}$$

$$= \frac{10}{100} + \frac{5}{100} = \frac{15}{100} = \frac{3}{20}$$

(2) $1.5 = 1 \text{ unit} + 5 \text{ tenth}$

$$= 1 + \frac{5}{10}$$

$$= 1\frac{5}{10} = 1\frac{1}{2} = \frac{3}{2}$$

(3) $0.625 = 6 \text{ tenth} + 2 \text{ hundredth} + 5 \text{ thousandth}$

$$= \frac{6}{10} + \frac{2}{100} + \frac{5}{1000}$$

$$= \frac{600}{1000} + \frac{20}{1000} + \frac{5}{1000}$$

$$= \frac{625}{1000} = \frac{5}{8}$$

(4) $12.05 = 1 \text{ tens} + 2 \text{ unit} + 0 \text{ tenth} + 5 \text{ hundredth}$

$$= 12 + \frac{0}{10} + \frac{5}{100}$$

$$= 12 + \frac{0+5}{10} = 12 + \frac{5}{100} = \frac{1205}{100} = \frac{241}{20}$$

(5) $2.125 = 2 \text{ unit} + 1 \text{ tenth} + 2 \text{ hundredth} + 5 \text{ thousandth}$

$$= 2 + \frac{1}{10} + \frac{2}{100} + \frac{5}{1000}$$

$$= 2 + \frac{100}{100} + \frac{20}{1000} + \frac{5}{1000}$$

$$= 2 + \frac{125}{1000} = \frac{2125}{1000} = \frac{17}{8}$$

Try :

Express the following decimal numbers in the form of rational numbers $\frac{p}{q}$.
 0.3, 0.016, 1.45

From above example, we find that:

If $0.r$ and $0.r.s$ are decimal numbers, where r and s are digits, then the values of these be respectively $\frac{r}{10}$ and $\frac{r.s}{100}$ on changing them in the form of rational numbers $\frac{p}{q}$.

Exercise 1(k)

1. Express the following decimal numbers in the form of rational numbers $\frac{p}{q}$:

0.35, 0.750, 2.15, 7.010, 10.10, 0.015, 1.05, 2.25.

2. Express the following decimal numbers in the form of rational number $\frac{p}{q}$:

2.25, 10.5, 8.624, 16.375

Efficiency practice 1 (B)

1. If $x = -\frac{3}{5}$, $y = -\frac{4}{7}$, then show that:

(i) $|x \times y| = |x| \times |y|$

(ii) $|x \div y| = |x| \div |y|$

2. Find all those rational numbers whose absolute value is $\frac{5}{6}$.
3. Find that rational number whose absolute value is zero.
4. Which of the following rational numbers can be expressed in the form of terminating decimal numbers?

$$\frac{11}{8}, 4\frac{3}{45}, \frac{-33}{80}, \frac{24}{7}, \frac{5}{6}$$

5. Which rational numbers in the following can be expressed in the form of non terminating recurring decimal numbers?

$$\frac{5}{7}, \frac{1}{9}, \frac{3}{5}, \frac{17}{30}, \frac{-5}{28}, \frac{-413}{605}$$

6. Expressed the following rational numbers in the form of terminating or non-terminating decimal numbers:

$$\frac{-9}{40}, \frac{-5}{9}, \frac{35}{16}, -2\frac{4}{11}, \frac{-88}{7}$$

7. Change the following rational numbers in fractions:

(i) 0.015

(ii) 0.84

(iii) 12.625

We have learnt

- For finding the sum of rational numbers having equal denominators, we find the sum of their numerator and the denominator remains same as : $\frac{p}{q} + \frac{r}{q} = \frac{p+r}{q}$
- For adding two rational numbers with unequal positive denominators, firstly find the L.C.M of both denominators and then change the both rational numbers into two equivalent rational numbers with denominators equal to the L.C.M. of the denominators of both rational numbers. Find the sum like serial number-1.
- Process of addition in rational numbers satisfies closure, commutative and associative properties.
- We add additive inverse of the rational number to be subtracted for the subtracting from a rational number to other rational number.

5. Process of subtraction in rational numbers satisfies the closure property but does not satisfy commutative and associative properties.
6. To multiply two rational numbers, we multiply the numerators and denominators of these numbers separately, as

$$\frac{p}{q} \times \frac{r}{s} = \frac{p \times r}{q \times s} = \frac{pr}{qs}$$

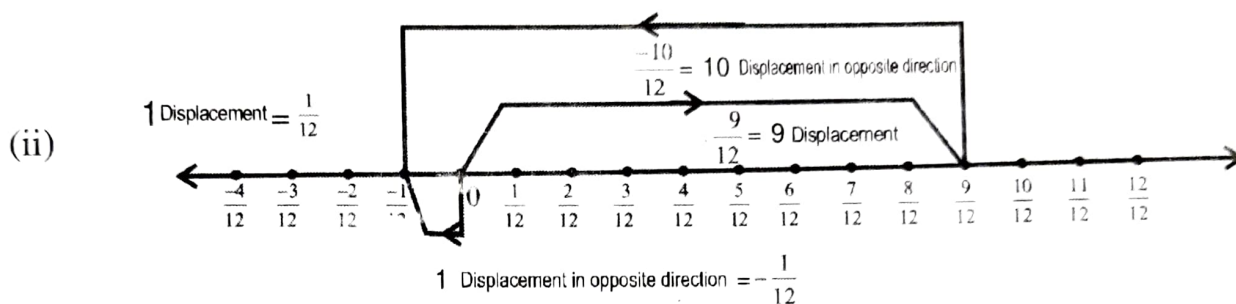
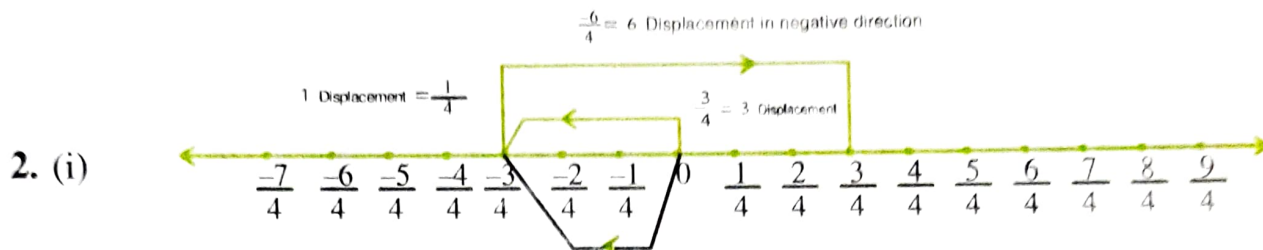
7. The process of multiplication in rational numbers satisfies closure, commutative and associative properties.
8. Distributive property of multiplication over addition applicable as in integers.
9. In rational numbers, 'zero' is the identity element of addition.
10. In rational numbers, '1' is the identity element of multiplication.
11. The additive inverse of rational number $\frac{p}{q}$ is $-\frac{p}{q}$.
12. The multiplicative inverse of rational number $\frac{p}{q}$ is $\frac{1}{\left(\frac{p}{q}\right)} = \left(\frac{p}{q}\right)^{-1} = \frac{q}{p}$. It is also called the reciprocal of the rational number.
13. To divide any rational number by another non-zero rational number, multiply first rational number to the reciprocal of the other (non-zero) rational number.
14. The absolute value of any rational number is never negative.
15. There are infinite rational numbers between two rational numbers
16. The prime factors of the denominators of which rational numbers are only 2 or 5 or both, they can always be changed in terminating decimal.
17. The prime factors of the denominators of rational numbers are also other prime factors besides 2 and 5, then they can be changed in non-terminating recurring decimal.



ANSWERS

Exercise 1 (a)

1. (i) $\frac{2}{5}$, (ii) $\frac{2}{5}$;



3. (i) $\left(\frac{-1}{48}\right)$, (ii) $\frac{5}{33}$, 4. (i) $\frac{7}{9}$, (ii) $\left(\frac{-7}{3}\right)$

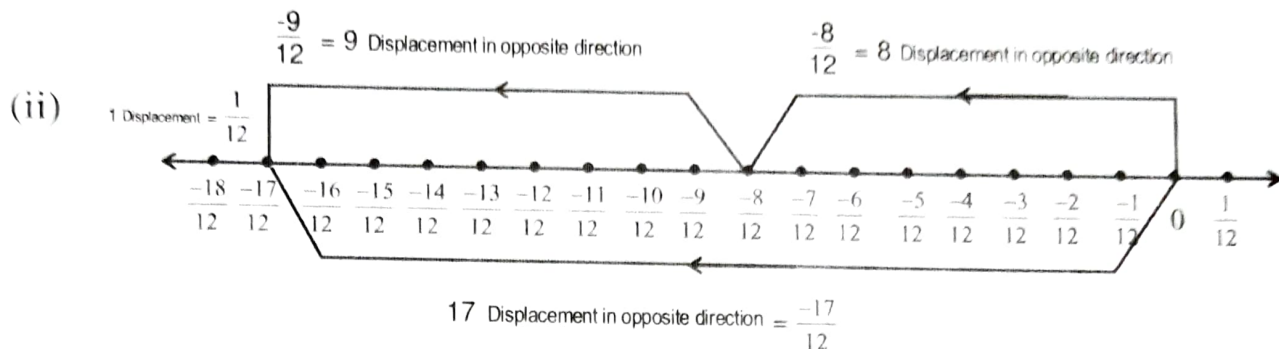
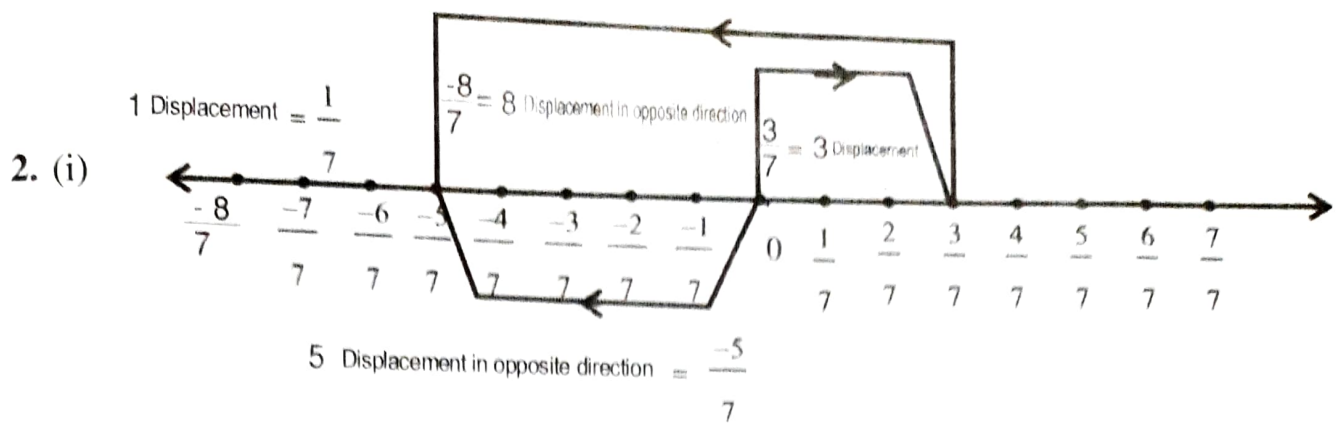
Exercise 1 (b)

1. (i) True, (ii) False, (iii) True, (iv) False; 4. $\left(\frac{-15}{11}\right)$; 5. (i) $\left(\frac{5}{-11}\right)$, (ii) $\frac{5}{6}$, 6. (i) $\frac{35}{72}$, (ii)

$\frac{-1}{12}$, 7. (i) $\left(\frac{-5}{7}\right)$, (ii) $\left(\frac{8}{5}\right)$

Exercise 1 (c)

1. (i) (-1) , (ii) $\frac{7}{3}$, (iii) 1 , (iv) $\left(\frac{-7}{3}\right)$



3. (i) -1 (ii) $-\frac{34}{9}$; (iii) $\frac{11}{4}$; (iv) $-\frac{3}{2}$; 4. (i) $\left(-\frac{68}{63}\right)$, (ii) $\frac{23}{18}$; 5. $\left(-\frac{19}{7}\right)$, 6. (i) $\left(-\frac{1}{10}\right)$, (ii) $\frac{65}{72}$; 7. (i) False, (ii) False, (iii) True, (iv) True.

Exercise 1 (d)

1. (i) (-12) , (ii) $\frac{56}{9}$, (iii) 48 , (iv) $\left(-\frac{5}{6}\right)$; 2. (i) (-28) , (ii) (-6) , (iii) $\frac{32}{15}$, (iv) $\frac{11}{15}$;

Exercise 1 (e)

1. (i) True, (ii) True, (iii) False, (iv) True; 3. (i) $\frac{2}{11}$, Commutative properties of multiplication, (ii) $\left(-\frac{11}{3}\right), \frac{2}{3}$, Associative properties of multiplication (iii) $\frac{2}{5}, \left(-\frac{5}{7}\right)$ Distributive properties of multiplication 4. (i) (-2) , (ii) $\frac{2}{25}$

Exercise 1 (f)

1. (i) False, (ii) True, (iii) False, (iv) True, (v) False, (vi) True, (vii) True, (viii) True; 2. (i) 0 , (ii) $\left(-\frac{4}{5}\right)$, (iii) (-2) , (iv) -2 ; 3. (i) $\frac{3}{2}$, (ii) $\frac{3}{4}$, (iii) $\left(-\frac{4}{5}\right)$, (iv) $\left(\frac{7}{6}\right)$ or $\left(-\frac{7}{6}\right)$, (v) 1 , (vi) 0 ; 4. (i) $\left(-\frac{1}{2}\right)$, (ii) $\frac{3}{8}$, (iii) 0 , (iv) $\left(-\frac{4}{7}\right)$, 5. (i) $\frac{13}{8}$, (ii) $\left(-\frac{19}{6}\right)$, (iii)

$$\left(\frac{-16}{17}\right), \text{ (iv) } \left(\frac{-1}{9}\right), \text{ (v) } \frac{1}{17}, \text{ (vi) } \frac{10}{9}, 6.0$$

Exercise 1 (g)

1. (i) 16, (ii) $\left(\frac{-21}{2}\right)$, (iii) $\frac{128}{5}$, (iv) $\frac{5}{2}$, (v) $\frac{-15}{16}$, (vi) $\left(\frac{-1}{12}\right)$; 2. (i) True, (ii) False, (iii) False, (iv) True, 3. (i) $\left(\frac{-5}{6}\right)$, (ii) $\frac{20}{3}$, (iii) $\frac{2}{35}$, (iv) $\left(\frac{-1}{6}\right)$, (v) (-3) , (vi) $\frac{7}{8}$; 4. $\frac{15}{28}$; 5. $\frac{10}{7}$; 6. $\frac{9}{4}$; 7. $\left(\frac{-9}{14}\right)$;

Efficiency practice 1 (A)

1. (i) $\left(\frac{-244}{105}\right)$ (ii) $\left(\frac{-5}{24}\right)$ (iii) $\frac{49}{2}$ (iv) $-\frac{5}{9}$ 2. $\left(\frac{-4681}{1225}\right)$ 3. $\left(\frac{-13}{3}\right)$ 4. (i) True, (ii) False, (iii) True, (iv) True,

Exercise 1 (h)

1. (i) $\frac{15}{7}$, (ii) $\frac{11}{8}$, (iii) $\frac{19}{23}$, (iv) $\frac{8}{19}$; 2. (i) $\frac{1}{15}$, (ii) $\frac{50}{21}$, (iii) $\frac{21}{55}$, 3. (iii) $\frac{5}{4}$; 4. (i) $\frac{3}{7}$; 5. (iv) $\frac{15}{28}$; 6. (iii) $\frac{5}{7}$; 7. (i) $>$; (ii) $>$, (iii) $=$, (iv) $<$; 10. $\frac{1}{2}, \frac{-1}{2}$

Exercise 1 (i)

1. (iii) $-\frac{3}{4}$; 2. (iv) $\frac{1}{2}$; 3. 0; 4. $\frac{5}{12}$; 5. 0; 6. $\frac{31}{30}$; 7. $\frac{-17}{24}$; 8. $\frac{49}{16}$; 9. $\frac{45}{28}$; 10. $\frac{-5}{24}$;

Exercise 1 (j)

1. $3.\bar{6}$, 3.4, $3.1\bar{6}$, 0.375; 2. -1.25, -7.5, -3.2, $-0.\bar{5}$; 3. $\frac{1}{4}, \frac{11}{8}, \frac{-27}{4}, \frac{-33}{10}, \frac{6}{15}$; 4. non-terminating decimal; 5. $\frac{24}{7}, \frac{5}{6}, \frac{7}{15}, \frac{3}{7}$; 6. (a) $\sqrt{\quad}$ (b) $\times$ (c) $\sqrt{\quad}$ (d) $\times$

Exercise 1 (k)

1. $\frac{7}{20}, \frac{3}{4}, \frac{43}{20}, \frac{701}{100}, \frac{101}{10}, \frac{3}{200}, \frac{21}{20}, \frac{9}{4}$; 2. $\frac{9}{4}, \frac{21}{2}, \frac{69}{8}, \frac{131}{8}$

Efficiency practice 1 (B)

2. $\frac{-5}{6}$ and $\frac{5}{6}$; 3. 0, 4, $\frac{11}{8}, \frac{-33}{80}$ 5. $\frac{5}{7}, \frac{1}{9}, \frac{17}{30}, \frac{-5}{28}, -\frac{413}{605}$; 6. -0.225, $-0.\bar{5}$, 2.1875, $-2.\bar{36}$, $-12.\overline{571428}$; 7. (i) $\frac{3}{200}$, (ii) $\frac{21}{25}$; (iii) $\frac{101}{8}$;

